Do households understand the benefits of diversification when adding stocks to their portfolios?

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Overview

Diversification

My ventures are not in one bottom trusted,
Nor to one place, nor is my whole estate
Upon the fortune of this present year.
Therefore my merchandise makes me not sad.

Shakespeare, Merchant of Venice, Act 1.

Object of study: *Diversification* in household portfolios.

Specifically: What *equities* do households have in their portfolios?

Question to be investigated: Do households choices reveal an understanding of the benefits of diversification?

Intro

Starting point: Individual investor

Standard investment advice – Spread your portfolio.

- Across assets
  - Stocks
  - Bonds
  - etc

Within your stock portfolio

- Spread your investment across sectors.

Motivation - Current evidence

- Asset pricing theory:
  Idiosyncratic risk diversified away, not priced in equilibrium
  $\rightarrow$ theory rely on diversification type arguments
- Household finance.
  Current evidence showing that households do not diversify
  - Few households hold equity (non-participation)
  - Actual household portfolios contains very few stocks.
  - Experimental evidence: Lack of understanding of diversification.

  $\rightarrow$ empirical evidence not consistent with an understanding of diversification

Motivation - Current evidence

Isseues with current empirical studies.

Both

- Limited participation
- Too few securities

sensitive to argument based on:

- Wealth constraint
- Transaction cost

Question:

Is there an alternative way of empirically ask whether households understand diversification?

That is less prone to the wealth/costs counterargument?

Intro

My suggested alternative empirical investigation:
Have data on household portfolio *changes*.
Look at marginal decision:

- A stock is added to a portfolio.

Ask:

- Is the chosen stock "good" in diversification terms?

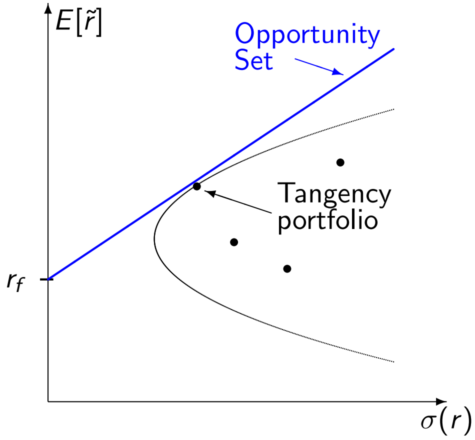
Setting

Households decision problem

A household currently has a portfolio p.
Thinks about adding one more stock to the portfolio.
How to think about the optimal choice?
Concider a mean-variance setting.

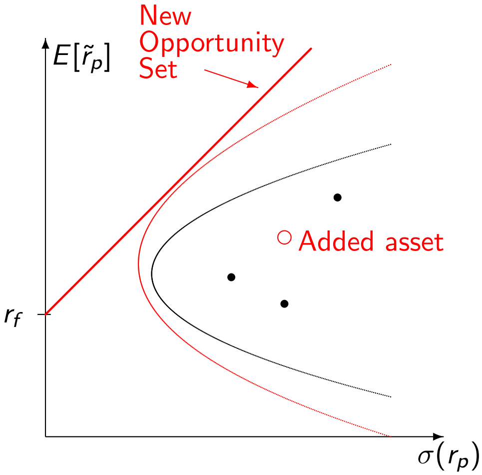
Setting

Current situation



Setting

After one added stock



Setting

Suggest possible empirical strategy

Search across possible stocks to add to the portfolio.
Choose the one with the best improvement to the mean-variance tradeoff.
(Highest Sharpe Ratio)
Is this the one households choose?

Why is solving for the optimal stock to add not a good idea?

The inputs (expected returns, variances) needs to be estimated.
Optimal weights highly sensitive to estimates (Best and Grauer, 1991)
Estimates of expected returns in particular noisy (Merton, 1980).

Setting

Simpler question to ask – Correlation

Stocks that expand the mean-variance tradeoff will have low correlation with the current portfolio.
$\rightarrow$ test whether added stocks have low correlation.

Even simpler question to ask – Industry sector

Low correlation stocks likely to be in different industry sectors than current portfolio.
Test whether added stock likely to be in new industry sector.
Matches advice given by investment advisors.

Alternative hypothesis (base case)

Stocks are chosen radomnly among available stocks at the exchange.

Methods – Sectors

Stocks at the OSE grouped into industry sectors.
For each portfolio:

1. What is the probability that a random stock will be in a "new" sector?
2. What is the empirical proportion of portfolios (i.e. empirical probability) in which the chosen stock is in a "new" sector?

If households follows advice that they should prefer "new" sectors, the second probability should be higher.

Methods – Correlation

Alternatively (Closer to Mean-Variance intuition)
test whether a chosen stock has lower correlation with the current portfolio than a random choice.
Construct test statistic D.

- $\mathcal{M}$ – Set of all available stocks (the crossection).
- r_i the return of stock i
- $|\mathcal{M}| = m$

Household portfolio

- Household portfolio p is a set of stocks $\mathcal{P} \subset \mathcal{M}$.
- The household portfolio is characterized by the weights ω_i of the assets in the portfolio. $r_p = \sum_{i \in \mathcal{P}} \omega_i r_i$
- $|\mathcal{P}| = n$.

Stocks not in housholds portfolio

- $\mathcal{R}$ – the set of stocks *not* in the households portfolio, i.e. $\mathcal{R} = \mathcal{M} \setminus \mathcal{P}$.

Methods – Correlation

Correlation of "new" stock $j \notin \mathcal{P}$ with household portfolio p

$$\text{corr}(r_j, r_p) = \text{corr}\left(r_j, \sum_{i \in \mathcal{P}} \omega_i r_i\right) = \sum_{i \in \mathcal{P}} \omega_i \text{corr}\left(r_j, r_i\right)$$

New stock j chosen randomly among all all possible stocks $j \in \mathcal{R}$:

$$\begin{aligned} E\left[\text{corr}(r_j, r_p)\right] &= \frac{1}{m-n} \sum_{j \in \mathcal{R}} \text{corr}\left(r_j, r_p\right) \\ &= \frac{1}{m-n} \sum_{j \in \mathcal{R}} \sum_{i \in \mathcal{P}} \omega_i \text{corr}\left(r_j, r_i\right) \end{aligned}$$

Does a chosen stock j have lower correlation than random stock?

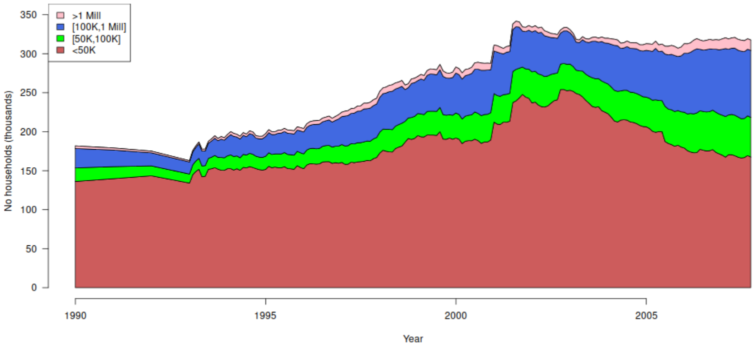
$$D = \text{corr}(r_j, r_p) - E[\text{corr}(r_j, r_p)]$$

Under a null that the household does not consider diversification properties $E[D] = 0$.
If households pick stocks with lower correlation $E[D] < 0$.

Data

Firms listed on the Oslo Stock Exchange (OSE) 1992–2007.
All listed Norwegian companies tracked their equity owners through a electronic centralized registry. (VPS)
Data in paper: Monthly snapshots of these ownership data.
Unique ids, allowing construction of portfolio for all owners, follow evolution over time.
Owners anonymized, no information beyond type of owner

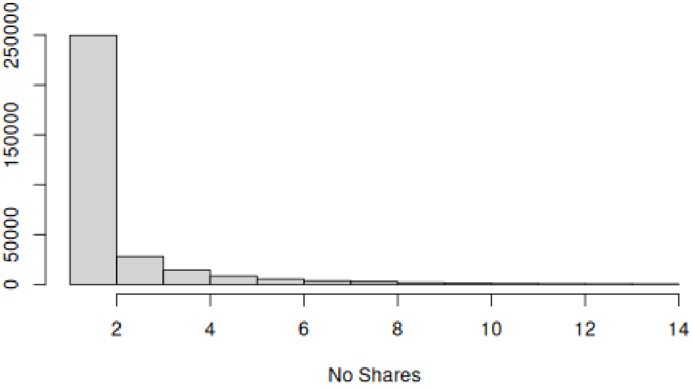
Number of households



The figure illustrates number of households in the sample. On each date we group households by their equity wealth, where we group the equity portfolio value into the bins: $< 50K$, $(50K, 100K]$, $(100K, 1\text{Mill}]$, $> 1\text{Mill}$ NOK.

Data

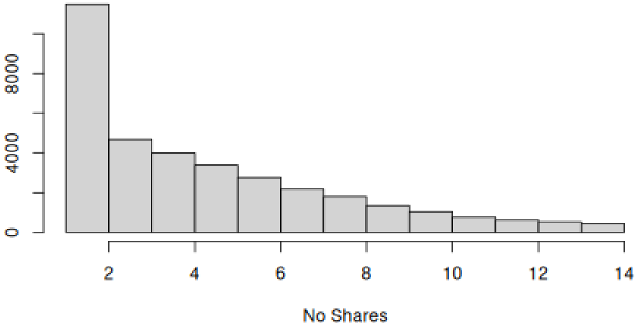
Number of shares in household portfolios



December 1998. Households with portfolios < 15 shares.

Data

Number of shares in household portfolios
(Higher valued portfolios)



December 1998. Households w portfolio value $>$ 100 thousand NOK (portolios $<$ 15 shares)

Data

Equity market data

- Monthly returns
  $\rightarrow$ estimate correlations using most recent five years of past data
- GICS codes – 10 different industries.

Results

Estimation
All cases where households add one stock to an existing portfolio.
Sample: ≈ 1.1 million observations.

Results – sectors

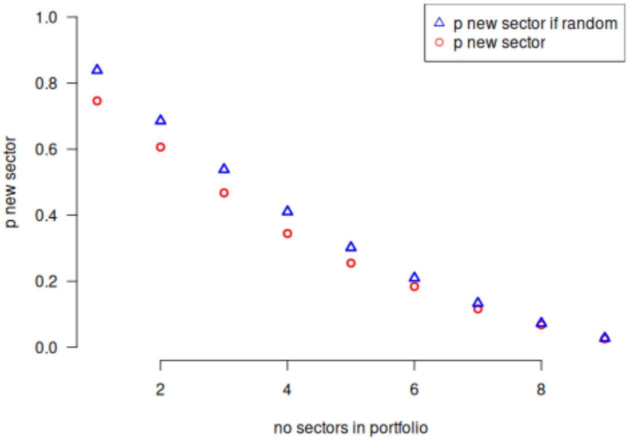
Likelihood of choosing a new sector

	mean
p(New)	55.8%
p(New) if Random	62.7%

First line: Likelihood of the new stock being in a different GICS sector when added by a household. Second line: Corresponding Likelihood if stock is added randomly.

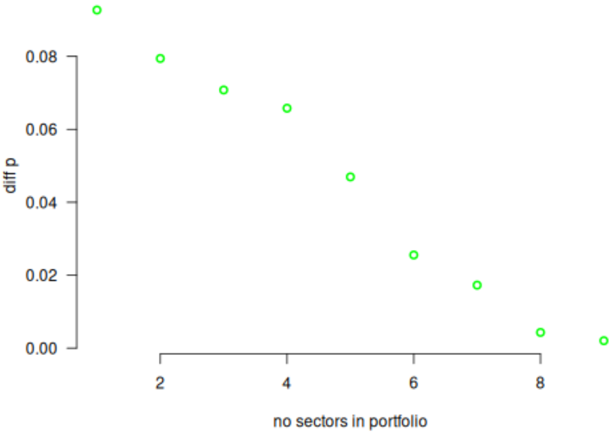
Results – Sectors

Probability of adding news sector depending on number of sectors in current portfolio



Results – Sectors

Probability of adding news sector depending on number of sectors in current portfolio
Differences in probability



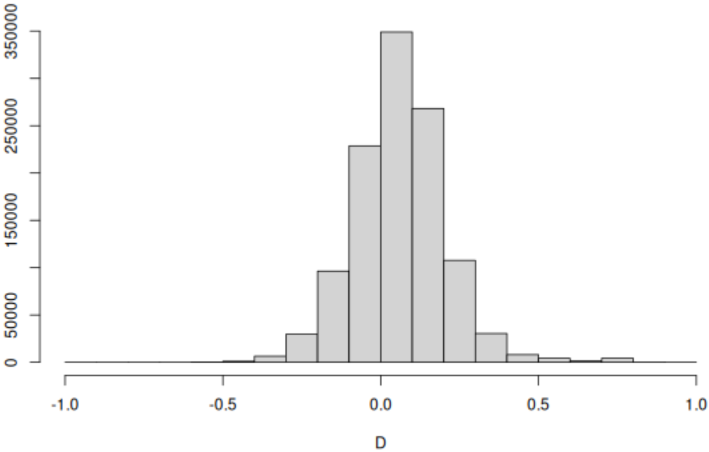
Results – correlation

Characterize test statistic D:

	D
Q1	-0.03
Median	0.06
Mean	0.06
Q3	0.15
n	1136313

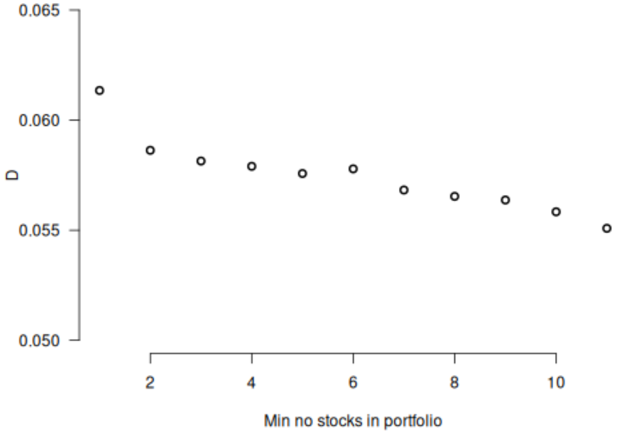
$\rightarrow \overline{D} > 0$.

Distribution of test statistic D



Results – Correlation

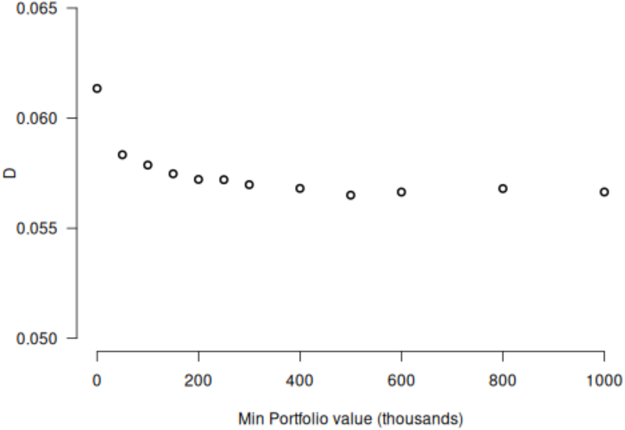
Conditioning on properties of current portfolio



Varying minimum number of stocks in current portfolio

Results – Correlation

Conditioning on properties of current portfolio



Varying minium value of current portfolio.

Conclusion

So, households do not seem to account for diversification in their choices .
(At least relative to the proposed heuristics)

App: Stock Market Participation

Empirically, Investment in the stock market has much higher expected return than risk free investments (equity premium).

$\rightarrow$ All households should have an investment in the stock market.

However, only a small fraction of households do have equity investments.

$\rightarrow$ *Stock Market Participation Puzzle*.

App: Stock Market Participation

The canonical portfolio problem Suppose an investor has wealth W_0 that can be invested in two assets

- A risk free asset (bank account) with return R_f.
- A risky asset (equity) with (random) return $\tilde{R}$.

The choice variable of the investor is the fraction ω to be invested in the risky asset.
The decision problem is to maximize the expected utility of end-of-period wealth

$$\tilde{W} = W_0 \left((1 - \omega) R_f + \omega \tilde{R} \right)$$

Optimization problem

$$\max_{\omega} E \left[u \left(\tilde{W} \right) \right] = \max_{\omega} E \left[u \left((1 - \omega) R_f + \omega \tilde{R} \right) \right]$$

App: Stock Market Participation

Theorem

Suppose the expected return $E[\tilde{R}]$ of the risky asset is greater than the risk free rate R_f. Then it is never an optimal strategy to put all your wealth in the risk free asset.

Proof.

The first-order condition for optimization:

$$\frac{\partial}{\partial\omega} E\left[u\left(W_0\left((1-\omega)R_f + \omega\tilde{R}\right)\right)\right] = E\left[u'(\tilde{W})\left(\tilde{R} - R_f\right)\right] = 0$$

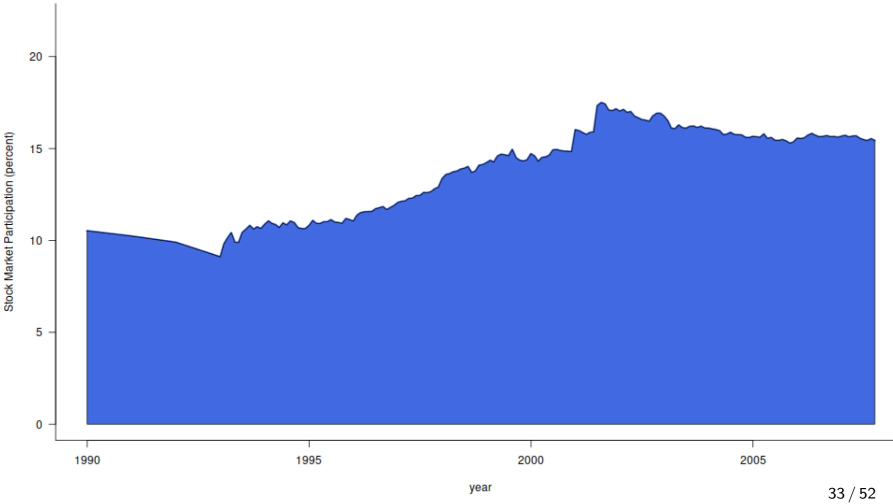
If $\omega = 0$ this simplifies to (since $W_0 R_f$ is nonrandom)

$$E\left[u'\left(W_0 R_f\right)\left(\tilde{R} - R_f\right)\right] = u'\left(W_0 R_f\right) E\left[\left(\tilde{R} - R_f\right)\right] = 0$$

This can never be satisfied since $E[\tilde{R} - R_f] > 0$ (by assumption) □

App: Stock Market Participation

Stock market participation of Norwegian Households



App: Stock Market Participation

What can explain the lack of participation?

- Transaction costs of entering the stock market.
- The stock market is not the only risky asset available to a household.
  Prime example: Owning your house. Background risk.
- Cash-in-advance type of constraints.
- Lack of understanding
  - Do not understand that expected returns are higher for equities?
  - Lack of financial sophistication.

App: Mean Variance Analysis

Go through the development of the mean-variance approach to portfolio choice.

The Canonical Portfolio Problem Revisited Instead of a single risky asset, suppose we have n risky assets, where risky asset i has return $\tilde{R}_i$. Let ω_j be the fraction of wealth invested in the risky asset j. The end of period wealth can be calculated as

$$\tilde{W} = W_0 \left(R_f \left(1 - \sum_j \omega_j \right) + \sum_j \omega_j \tilde{R}_j \right)$$

App: Mean Variance Analysis

Framework for developing "best" portfolio Split into two decisions:

1. Chose the "best" combination of risky assets.
   $\rightarrow$ Portfolio p of risky assets, with returns $R_p = \sum_i \omega_i \tilde{R}_i$.
2. Then combine this risky portfolio with risk free borrowing and lending. Let ω be the fraction of your initial wealth invested in the risky portfolio
   End of period wealth:

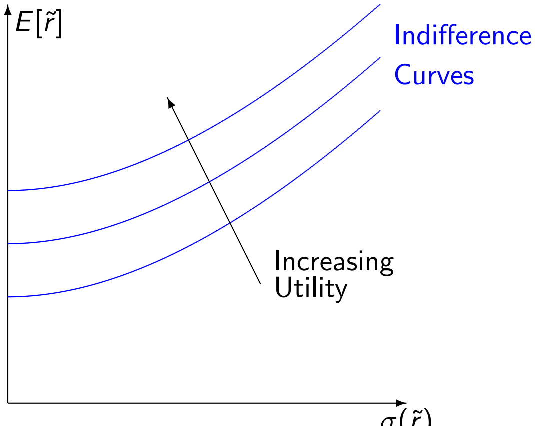
$$\tilde{W} = W_0 \left(R_f \left(1 - \omega\right) + \omega \tilde{R}_p \right)$$

App: Mean Variance Analysis

Need preference assumptions

Investors care only about expected returns and variances of the portfolio p:

$$u(\cdot) = u\left(E[r_p], \sigma^2(r_p)\right)$$



App: Mean Variance Analysis

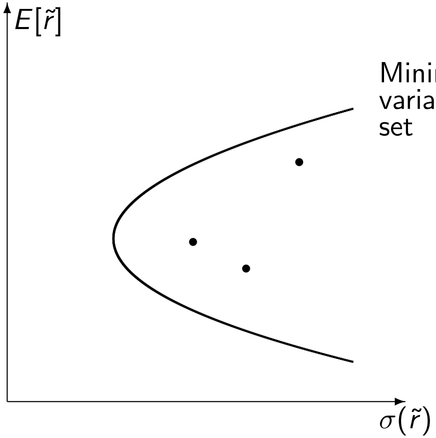
What are the possibilities?

$$E[r_p] = \sum_i \omega_i E[R_i]$$

$$\begin{aligned}
\sigma^2(r_p) &= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \text{cov}\left(\tilde{R}_i, \tilde{R}_j\right) \\
&= \sum_i \sum_j \omega_i \omega_j \text{corr}\left(\tilde{R}_i, \tilde{R}_j\right) \sigma\left(\tilde{R}_i\right) \sigma\left(\tilde{R}_j\right)
\end{aligned}$$

Minimum Variance Set

Varying weights allow construction of set of best possible weight, that minimize variance for a given expected returns.

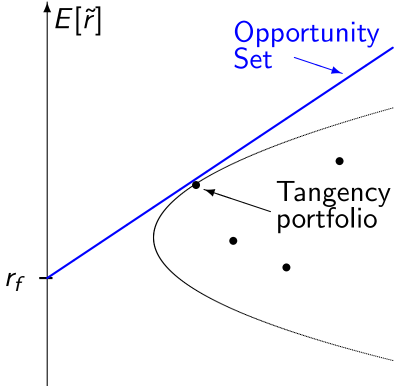


$\rightarrow$ Set of “best possible” risky assets.

Add risk free asset to risky portfolio p

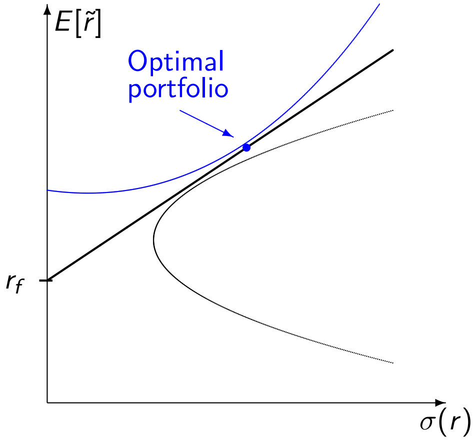
$$E\left[\tilde{R}\right] = R_f\left(1-\omega\right) + \omega E\left[\tilde{R}_p\right]$$

$$\sigma(\tilde{R}) = \omega\sigma\left(\tilde{R}\right)$$



App: Mean Variance Analysis

Find the optimal portfolio



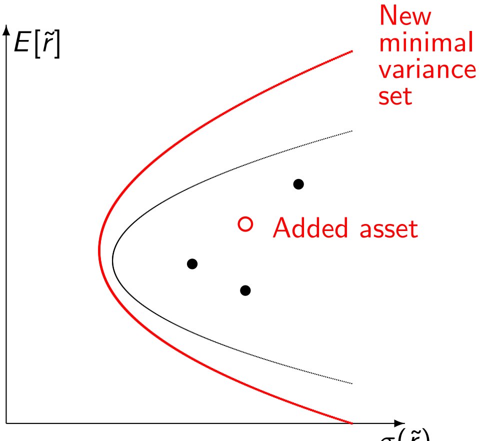
App: Mean Variance Analysis

Now for the analysis of this paper

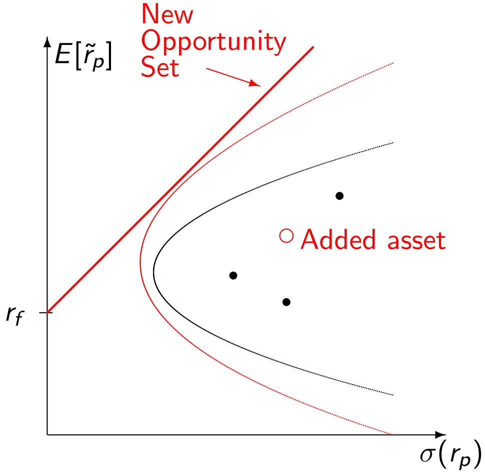
Previous picture: Optimal solution given a set of risky assets in portfolio p.

What if you add the possibility of investing in one additional asset?

What happens to the minimum variance set of risky assets?



Improvement in Decision Relevant MV optimal set



App: Mean Variance Analysis

Decision problem facing investor
You can add one stock (only) to your portfolio.
How would you choose the stock.
From the picture: The stock that pushes the resulting mean-variance optimal portfolio the most to the Northwest in the picture.

App - Simulation evidence - Norway

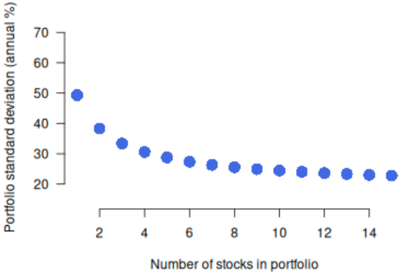
Experiment: Use historical returns for OSE for period we investigate.
Construct simulated portfolios.
Ask: If we simulate the heuristics:

- Pick stocks with low correlation
- Pick stocks in “new” industry sectors

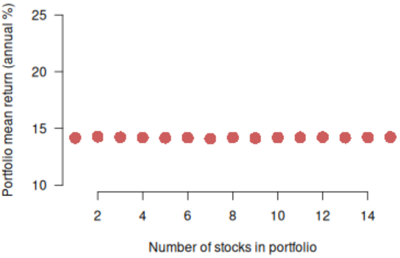
What effect (if any) does it have on simulated portfolios

Base case: Choose randomly n stocks in portfolio

Standard deviations

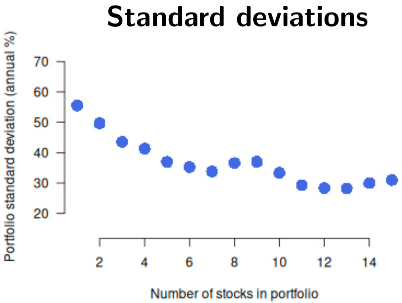


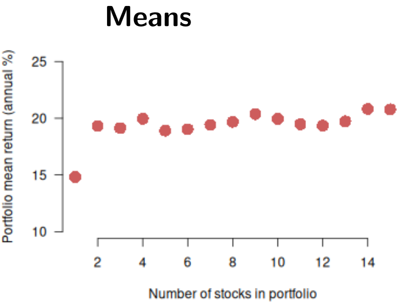
Means



App - Simulation evidence - Norway

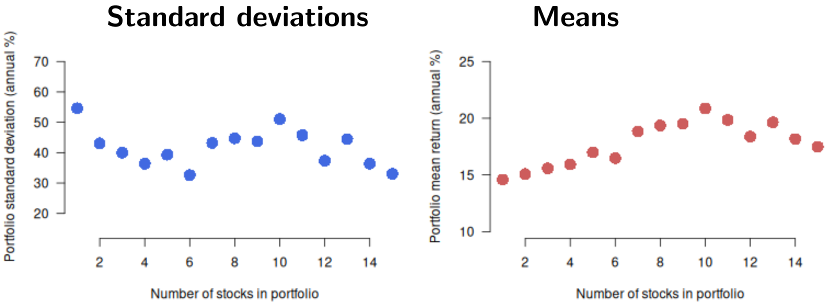
Simulation strategy: At each decision point, pick stock with lowest correlation with existing portfolio





App - Simulation evidence - Norway

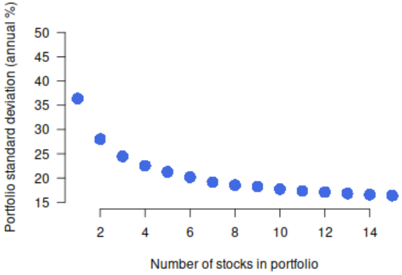
Simulation strategy: At each decision point, pick among stock in "new" industries relative to existing portfolio



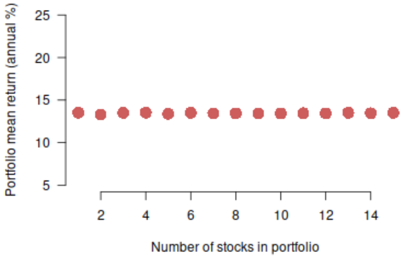
App: Simulations using US data

Base Case

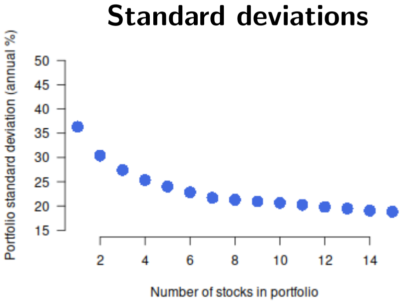
Standard deviations

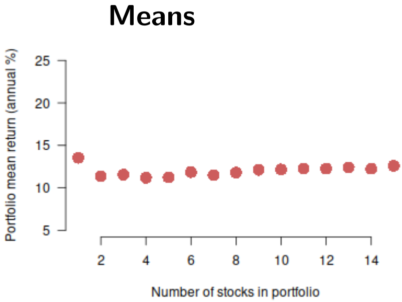


Means



Searching for low correlation when adding stocks

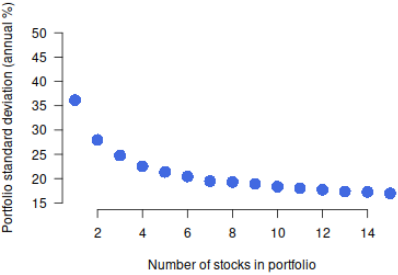




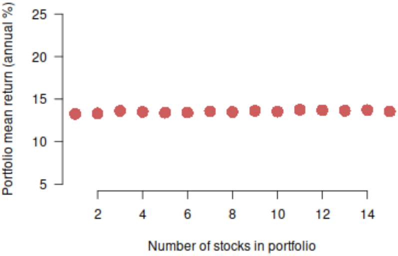
App: Simulations using US data

Adding “new” sectors to current portfolio

Standard deviations



Means



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