Investment Performance Evaluation

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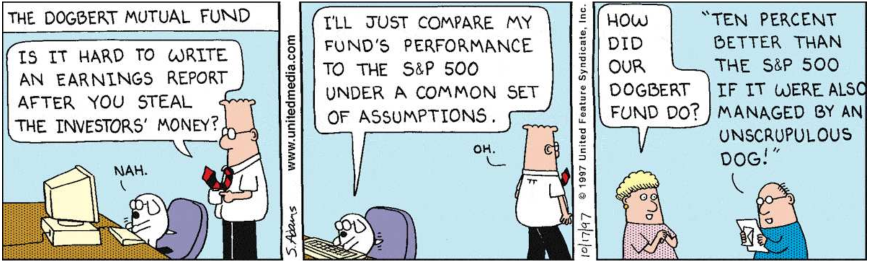


Intro

"Performance question": How well did a given equity portfolio perform?

We observe the actual portfolio return.

How "good" was this return?

Need: A theoretical framework.

Mean Variance framework: The classical measures: Sharpe, Treynor and Jensen.

Intro ctd

Many alternatives.

- Modifications of the classical measures.
  Example: Jensen alpha - based on the CAPM.
  Alternative: Alpha measure using alternative models for required returns.
- Alternatives bringing more information into the evaluation of the portfolio.
  What if not only using returns?
  Another piece of information one can potentially bring into the analysis is the actual portfolio decisions, when stocks are bought and sold.

Illustration using CAPM

illustrate – picking overvalued stocks with reference to the CAPM. According to the CAPM, *all* securities should plot on the security market line (SML).

$$E[\tilde{r}_j] = r_f + (E[\tilde{r}_m] - r_f)\beta_j$$

Use the SML to identify "*mispriced*" securities.

Exercise

Suppose your investment company estimates the beta for Westinghouse to be 1.20 and the dividend growth rate to be 10%. The current yield on a one-year T-bill is 8.0% and the market risk premium is estimated to be 7.0%. Westinghouse is expected to pay a dividend of $3.50 next year.

1. If these estimates are correct, what should be the market price of Westinghouse's stock?
2. If you observe a price of Westinghouse of 52, what is your recommendation?

Exercise Solution

1. To find the current market price, Use CAPM to find the discount rate for Westinghouse:

$$r = r_f + (E[\tilde{r}_m] - r_f)\beta = 0.08 + 0.07 \cdot 1.20 = 16.4\%$$

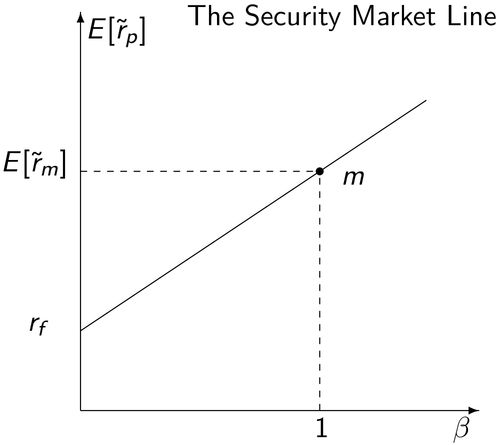
   The estimated market price

$$P_0 = \frac{E[D_1]}{r - g} = \frac{\$3.50}{0.164 - 0.10} = \$54.69$$

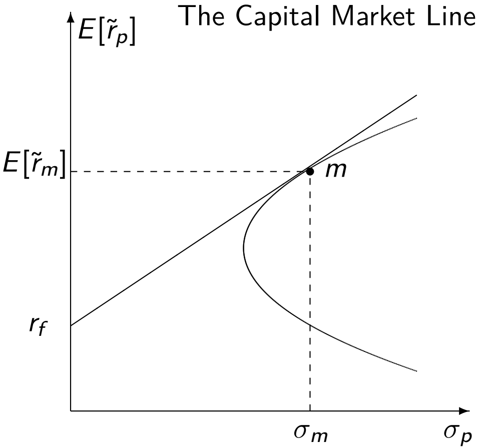
   If the actual market price is less than \$54.69, the stock is undervalued, and if greater than \$54.69, the stock is overvalued.

2. An observed price of 52, the stock is undervalued, buy.

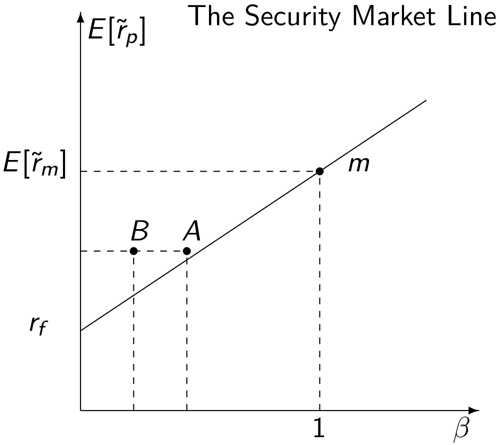
Benchmark: SML



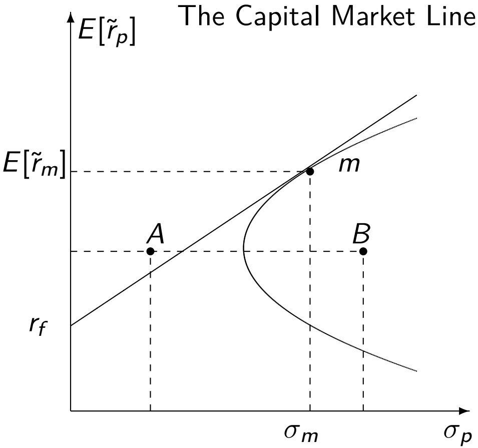
Benchmark: CML



Mispricing in SML



Mispricing in CML



Conceptually - what is performance valuation?

Idea: Given an investment, how does it compare to a *possible alternative* investment strategies?
To implement this idea, boils down to finding a *benchmark* for the comparison.

Benchmark

A benchmark is a measuring tape, a portfolio that is an alternative investment opportunity.
Good benchmarks should be

- Unambiguous
- Tradeable
- Measurable
- Appropriate
- Reflective of current investment opinions
- Specified in advance.

Desirable properties of performance measures

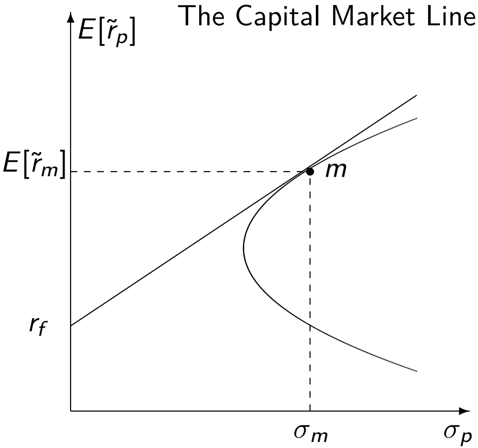
Chen and Knez (1996): Desirable properties of performance measures.

- Fit. Capture strategies relevant for uninformed investors. Have zero performance for simple strategies feasible for such investors.
- Be Scalable. Linear combinations of manager measures should equal the measure for the linear combination of manager portfolios
- Be continuous. Close skills/strategies should have close performance measures.
- Exhibit monotonicity. Assign higher measures for more skilled managers.

An added desirable property is manipulation-proofness. See Goetzmann, Ingersoll, Spiegel, and Welch (2007)

The Sharpe Ratio

How far is an asset p from the Capital Market Line?



The Sharpe Ratio ctd

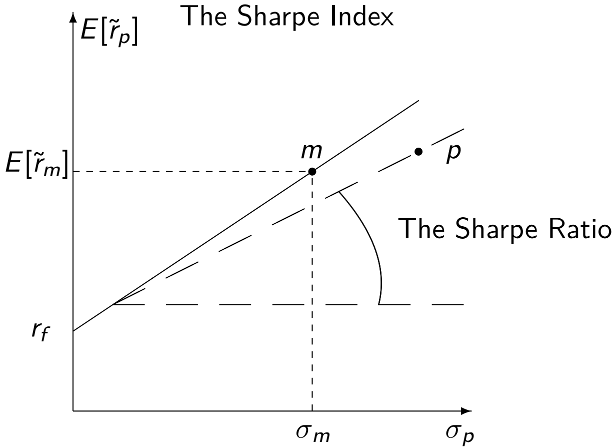
$$r_p - r_f = S\sigma_p$$

Here S is the slope of the line from the risk free rate through p. From the equation for this line solve for S:

$$S = \frac{r_p - r_f}{\sigma_p}$$

The Sharpe Ratio ctd

Sharpe index used for comparisons.
For example to the market



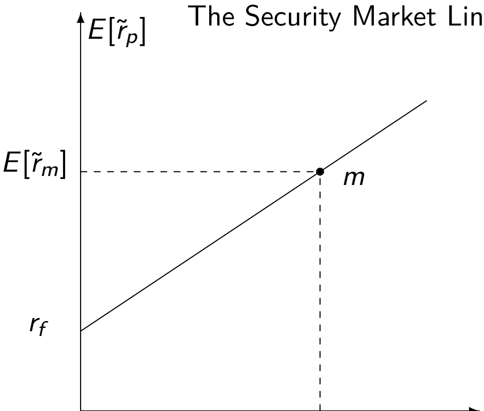
Sharpe is primarily used for undiversified portfolios.

The Treynor Measure

The Treynor measure T_p of a portfolio p is defined as

$$T_p = \frac{r_p - r_f}{\beta_p}$$

The Treynor measure is the slope of the line from r_f in mean-beta-space. To see that, consider a mapping in $E[r] - \beta$ space



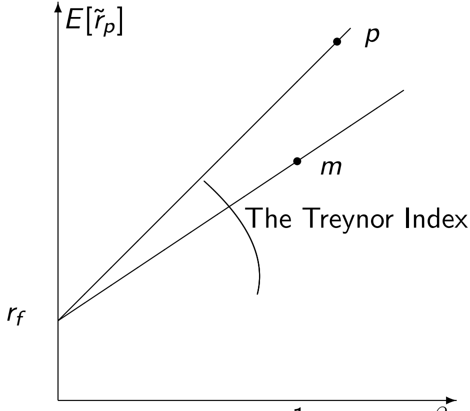
The Treynor Measure

The equation for the line starting at r_f

$$r_p - r_f = T\beta,$$

$$T = \frac{r_p - r_f}{\beta_p}$$

The Treynor Index



The Jensen alpha

Does the return on a portfolio/asset exceed its *required* return?
Jensens alpha is the difference between actual performance and required return

$$\alpha_p = r_p - \text{reqiured return} = r_p - \hat{r}_p$$

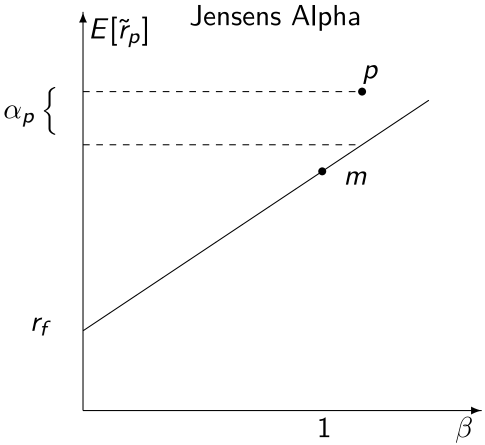
To estimate requred return: need an asset pricing model
classical asset pricing model: CAPM

$$\hat{r}_p = (r_f + \beta_p(r_m - r_f))$$

Calculate Alpha:

$$\alpha_p = r_p - (r_f + \beta_p(r_m - r_f))$$

The Jensen alpha



Exercise

Given the following information about the return on a stock ABC, the S&P market index and risk free returns.

	Rate of Return		
Year	ABC	S&P 500	T-bills
1	14%	12%	7
2	10	7	7.5
3	19	20	7.7
4	−8	−2	7.5
5	23	12	8.5
6	28	23	8
7	20	17	7.3
8	14	20	7
9	−9	−5	7.5
10	19	16	8
Average	13%	12%	7.6%
Standard Deviation	12.4%	9.4%	0.5%
Geometric Mean	12.3%	11.6%	7.6%
$\text{cov}(r_{ABC}, r_m)$	0.0107		

Exercise

1. Calculate the beta of ABC stock.
2. Calculate the Sharpe measure for ABC stock. Compare it to the market and draw a diagram illustrating its placing in mean – standard deviation space.
3. Calculate the Treynor measure for ABC stock. Compare it to the marke and draw a diagram illustrating its placing in mean – beta space.
4. Calculate Jensen's Alpha for ABC stock and draw a diagram illustrating its placing in mean – beta space.

Solution

Beta

$$\beta_{ABC,m} = \frac{\text{cov}(r_{ABC}, r_m)}{\text{var}(r_m)} = 1.20375$$

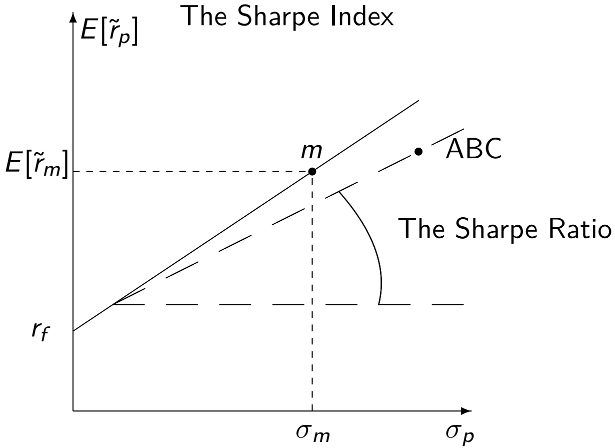
Sharpe index:

$$S_{ABC} = \frac{r_{ABC} - r_f}{\sigma_{ABC}} = \frac{0.13 - 0.076}{0.124} = 0.43$$

Sharpe Ratio for the market portfolio m:

$$S_m = \frac{r_m - r_f}{\sigma_m} = \frac{0.12 - 0.076}{0.094} = 0.468$$

Solution


The Sharpe Index
$E[\tilde{r}_p]$
$E[\tilde{r}_m]$
m
ABC
The Sharpe Ratio
r_f
σ_m
σ_p

Solution

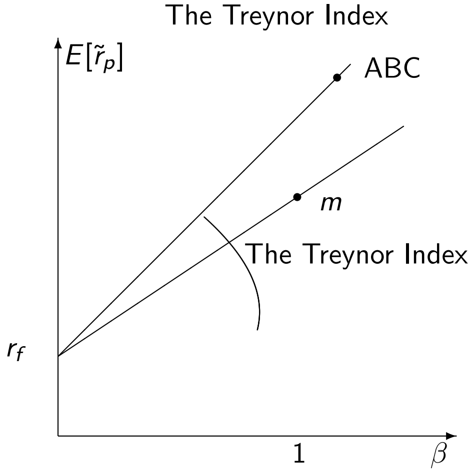
Treynor Index:

$$T_{ABC} = \frac{r_{ABC} - r_f}{\beta_{ABC}} = \frac{0.13 - 0.076}{1.20375} = 0.04485$$

Treynor Index of the market:

$$T_m = \frac{r_m - r_f}{\beta_m} = \frac{0.12 - 0.076}{1} = 0.044$$

Solution

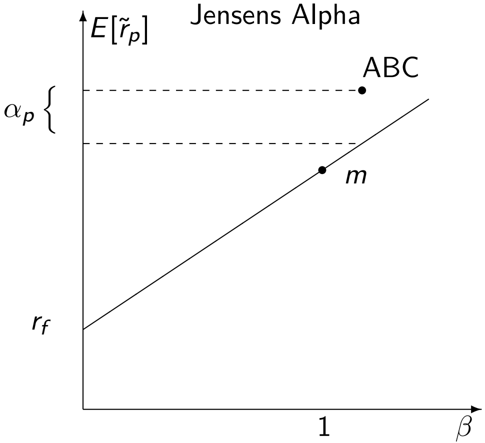


Solution

Jensens alpha:

$$\alpha_{ABC} = \overline{r_{ABC}} - E[r_{ABC}] = \bar{r}_{ABC} - (r_f + \beta_{A,m}(r_m - r_f))$$

$$= 0.13 - (0.076 + 1.20375(0.12 - 0.076)) = 0.00103 = 0.103\%$$



Some relations beween performance measures

Useful intuition –linking measures.

Treynor measure and Jensen's alpha

for positive beta assets, positive alpha assets will have a Treynor index above the Treynor index of the market.

Jensen's alpha and the Sharpe ratio.

Positive alpha assets will have a Sharpe ratio above that of the market as long as the correlation between p and m is not too low.

Exercise

The Treynor index of an asset p is $T = \frac{E[r_p]-r_f}{\beta_p}$.
Jensen's alpha α_p for the same asset is

$$\alpha_p = E[r_p] - (r_f + \beta_p(E[r_m] - r_f)) \, .$$

Show that

$$T_p = \frac{\alpha_p}{\beta_p} + T_m$$

where T_m is the Treynor measure of the market.

Solution

$$
\begin{aligned}
T_p &= \frac{r_p - r_f}{\beta_p} \\
&= \frac{\alpha_p - \alpha_p + r_p - r_f}{\beta_p} \\
&= \frac{\alpha_p - (r_p - (r_f + \beta_p(r_m - r_f))) + r_p - r_f}{\beta_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\beta_p(r_m - r_f)}{\beta_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{r_m - r_f}{1} \\
T_p &= \frac{\alpha_p}{\beta_p} + T_m
\end{aligned}
$$

Observe the implication: for positive beta assets, positive alpha assets will have a Treynor index above the Treynor index of the market.

Exercise

The Sharpe index of an asset p is $S_p = \frac{E[r_p] - r_f}{\sigma_p}$. The Jensen alpha α_p of the same asset is

$$\alpha_p = E[r_p] - (r_f + \beta_p(E[r_m] - r_f)) \,.$$

Show that

$$S_p = \frac{\alpha_p}{\beta_p} + \rho(r_p, r_m) S_m$$

where $\rho(r_p, r_m)$ is the correlation between asset p and the market m.

Solution

$$
\begin{aligned}
S_p &= \frac{r_p - r_f}{\sigma_p} \\
&= \frac{\alpha_p - \alpha_p + r_p - r_f}{\sigma_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\beta_p(r_m - r_f)}{\sigma_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\frac{\mathrm{cov}(r_p, r_m)}{\mathrm{var}(r_m)}(r_m - r_f)}{\sigma_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\frac{\mathrm{cov}(r_p, r_m)}{\sigma(r_m)\sigma(r_p)}(r_m - r_f)}{\sigma_m} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\rho(r_p, r_m)(r_m - r_f)}{\sigma_m} \\
&= \frac{\alpha_p}{\beta_p} + \rho(r_p, r_m) S_m
\end{aligned}
$$

Solution

$$S_p = \frac{\alpha_p}{\beta_p} + \rho(r_p, r_m) S_m$$

Positive alpha assets will have a Sharpe ratio above that of the market as long as the correlation between p and m is not too low.

In practice, we need to use a computer tool to do performance calculations. Obviously much of this can be done in Excel and similar spreadsheets. But spreadsheets is not the best tool to do this kind of analysis. We will instead look at two alternatives.

- `Matlab` and similar matrix tools. This is the best tool for doing and learning the calculations. The bad part about Matlab is that it is hard to get the *data* lined up and into the matrix handler. In current practice we are therefore seeing a move away from Matlab, replacing it with:
- `R`, which is a tool for statistical analysis. This tool is much easier to get data into, and it can do most of the tasks you use Matlab for. It is therefore taking over for Matlab in many "quant shops"
- `Julia`, a more efficient matlab.

Exercise

You are given historical returns of two different equities, r_A and r_B, as well as the market return r_m, and the risk free rate r_f.

```
  rA,      rB,      rm,     rf
 0.10,    0.05,    0.01,   0.01
 0.20,    0.03,   -0.05,   0.01
-0.10,   -0.01,   -0.05,   0.01
 0.13,    0.03,    0.10,   0.01
 0.24,    0.04,    0.14,   0.0140
-0.08,   -0.05,   -0.02,   0.02
-0.15,   -0.02,       0,   0.02
 0.15,    0.12,    0.10,   0.01
 0.45,    0.15,    0.05,   0.01
-0.10,   -0.10,    0.04,   0.02
 0.01,    0.01,    0.03,   0.01
-0.05,   -0.01,    0.01,   0.01
 0.20,    0.11,    0.05,   0.02
-0.05,    0.12,    0.05,   0.01
```

Exercise

Use matlab/octave to calculate

- Sharpe measures
- Treynor measures
- Jensen alphas (relative to the CAPM)

Exercise Solution

```
> rets = dlmread("../data/example.txt",",",1,0);
rets =
   0.10000   0.05000   0.01000   0.01000
   0.20000   0.03000  -0.05000   0.01000
  -0.10000  -0.01000  -0.05000   0.01000
   0.13000   0.03000   0.10000   0.01000
   0.24000   0.04000   0.14000   0.01400
  -0.08000  -0.05000  -0.02000   0.02000
  -0.15000  -0.02000   0.00000   0.02000
   0.15000   0.12000   0.10000   0.01000
   0.45000   0.15000   0.05000   0.01000
  -0.10000  -0.10000   0.04000   0.02000
   0.01000   0.01000   0.03000   0.01000
  -0.05000  -0.01000   0.01000   0.01000
   0.20000   0.11000   0.05000   0.02000
  -0.05000   0.12000   0.05000   0.01000
```

Exercise Solution

```
> rA = rets(:,1);
> rB = rets(:,2);
> rm = rets(:,3);
> rf = rets(:,4);
```

Calculating Sharpe Measures

```
> sA = mean(rA-rf)/std(rA)
sA =  0.32043
> sB = mean(rB-rf)/std(rB)
sB =  0.28515
> sm = mean(rm-rf)/std(rm)
sm =  0.35502
```

Exercise Solution

Treynor measure, first need to estimate beta

```
> betaA = cov(rA,rm)/var(rm)
betaA =  1.3418
> betaB = cov(rB,rm)/var(rm)
betaB =  0.52031
> betam = 1
betam =  1
```

Then can calculate

```
> tA = mean(rA-rf)/betaA
tA =  0.040778
> tB = mean(rB-rf)/betaB
tB =  0.039262
> tm = mean(rm-rf)/betam
tm =  0.019714
```

Exercise Solution

Alpha measure

```
> alphaA = mean(rA - (rf + betaA*(rm-rf)))
alphaA =  0.028262
> alphaB = mean(rB - (rf + betaB*(rm-rf)))
alphaB =  0.010171
```

Let us do the same example using R as the tool

Exercise

You are given historical returns of two different equities, r_A and r_B, as well as the market return r_m, and the risk free rate r_f.

```
   rA,      rB,      rm,     rf
 0.10,    0.05,    0.01,   0.01
 0.20,    0.03,   -0.05,   0.01
-0.10,   -0.01,   -0.05,   0.01
 0.13,    0.03,    0.10,   0.01
 0.24,    0.04,    0.14,   0.0140
-0.08,   -0.05,   -0.02,   0.02
-0.15,   -0.02,       0,   0.02
 0.15,    0.12,    0.10,   0.01
 0.45,    0.15,    0.05,   0.01
-0.10,   -0.10,    0.04,   0.02
 0.01,    0.01,    0.03,   0.01
-0.05,   -0.01,    0.01,   0.01
 0.20,    0.11,    0.05,   0.02
-0.05,    0.12,    0.05,   0.01
```

Use R to calculate

- Sharpe measures
- Treynor measures
- Jensen alphas (relative to the CAPM)

Solution

```
> data <- read.table("../data/example.txt",
                     header=TRUE,sep=",")
> head(data)
     rA    rB    rm    rf
1  0.10  0.05  0.01 0.010
2  0.20  0.03 -0.05 0.010
3 -0.10 -0.01 -0.05 0.010
...

> rA <- data$rA
> rB <- data$rB
> rm <- data$rm
> rf <- data$rf
```

Sharpe Ratio

```
> sA <- mean(rA-rf)/sd(rA-rf)
> print(sA)
[1] 0.3175401
> sB <- mean(rB-rf)/sd(rB-rf)
> print(sB)
[1] 0.2767519
> sm <- mean(rm-rf)/sd(rm-rf)
> print(sm)
[1] 0.352184
```

Beta

```
> betaA <- cov(rA,rm)/var(rm)
> print(betaA)
[1] 1.341768
> betaB <- cov(rB,rm)/var(rm)
> print(betaB)
[1] 0.5203136
> betam <- 1
```

Treynor

```
> tA <- mean(rA-rf)/betaA
> print(tA)
[1] 0.04077777
> tB <- mean(rB-rf)/betaB
> print(tB)
[1] 0.03926204
> tm <- mean(rm-rf)/1
> print(tm)
[1] 0.01971429
```

Alpha

```
> alphaA <- mean(rA - (rf + betaA*(rm-rf)))
> print(alphaA)
[1] 0.0282623
> alphaB <- mean(rB - (rf + betaB*(rm-rf)))
> print(alphaB)
[1] 0.01017096
```

Summarizing the calculated numbers

	r_A	r_B	r_m
Sharpe	0.318	0.277	0.352
β	1.342	0.520	
Treynor	0.041	0.039	0.020
α	0.028	0.010	

The appraisal ratio

The appraisal ratio is calculated by dividing Jensens alpha by the variance of the unsystematic risk of the portfolio

$$\text{Appraisal Ratio} = \frac{\alpha_p}{\sigma(e_P)}$$

e_P can be calculated as the residual of the regression

$$e_{pt} = r_{pt} - (\alpha_i + \beta_i r_{mt})$$

The M^2 measure

Measure introduced by Franco Modigliani
focus: total variability.
A managed portfolio p is mixed with a position in the risk free asset
$\rightarrow$ "adjusted" portfolio have the same volatility as the market.

Example

Managed portfolio p with total variability $1.5 \times \sigma_m$. The "adjusted" portfolio p^*

- investing a weighte w in p
- weight $(1 - w)$ in the risk free asset

such that the portfolio has the same standard deviation as the market:

$$w\sigma_p + (1 - w)\sigma(r_f) = w\sigma_p + (1 - w) \cdot 0 = w\sigma_P = \sigma_m$$

or

$$w = \frac{\sigma_m}{\sigma_p} = \frac{\sigma_m}{1.5\sigma_m} = \frac{1}{1.5} = 0.67$$

By investing two thirds in p and one third in the risk free asset, achieve the same volatility as the market. Since P^* and m have the same volatility, see how well P is performing by comparing the returns.

$$M^2 = r_{P^*} - r_m$$

Exercise

Given the following data:

	P	m
Average return	35%	28%
Beta	1.2	1
Standard Deviation	42%	30%
Nonsystematic risk ($\sigma(e)$)	18%	0

The T-bill rate during the period was 6%.

1. Calculate the M^2 measure for the portfolio P.

Exercise solution

The M^2 measure.

What weight to get a portfolio of P and risk free asset with the same standard deviation? $w\sigma_P = \sigma_m$, giving $w = \sigma_m/\sigma_p = 0.3/0.42 = 0.714$. with this weight, calculate return

$$r_{P*} = wr_P + (1-w)r_f = 0.7140.35 + (1-0.714)0.06 = 0.267$$

Comparing this to the market return gives the M^2 measure

$$M^2 = r_{P*} - r_m = 0.267 - 0.28 = -0.013 = -1.3\%$$

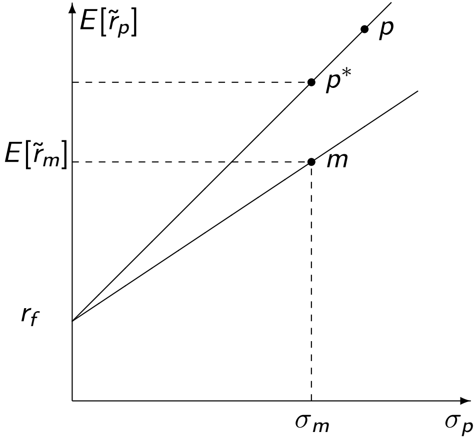
Exercise

Demonstrate the following relationship between M^2 and the Sharpe measure S_p for a portfolio p:

$$M^2 = (S_p - S_m)\sigma_m$$

Solution

$$M^2 = R_{P^*} - R_M = S_p\sigma_m - S_m\sigma_m = (S_p - S_m)\sigma_m$$



Find the portfolio p^* with the same risk as the market
make the comparison for that portfolio.

Market timing

The classical measures are measures of asset selection: Does the picked asset(s) show superior performance?
implemented using historical averages.
Implicit assumption: Portfolio Risk is no not changing
For some funds this assumption not fulfilled.
The typical example: **market timers**.

Market timing

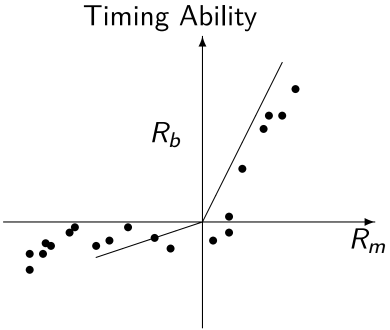
Timing:
Periodically shifting between broad asset classes, such as

- Stocks
- Bonds
- Cash

Based on predictions of which asset class will perform best next period
For example, suppose the fund only invests in two assets: bonds and stocks.

Market timing

If one is able to predict periods when stocks were doing better, and be in stocks:



- R_b – return on bonds
- R_m – return on stocks

Market timing

How can one measure timing?
Suggested regressions:
Treynor-Mazry:

$$r_p - r_f = a + b(r_m - r_f) + c(r_m - r_f)^2 + \varepsilon_p$$

Henriksson-Merton

$$r_p - r_f = a + b(r_m - r_f) + c(r_m - r_f)1_{\{r_m > r_f\}} + \varepsilon_p$$

A positive estimate of c in these regression are indications of timing abilities.

Exercise

Given: historical percentage excess returns (returns in excess of the risk free rate) for 2 portfolios, P, Q and a benchmark M.

time	$r_P - r_f$	$r_Q - r_f$	$r_M - r_f$	
1	3.58	2.81	2.2	0
2	-4.91	-1.15	-8.41	
3	6.51	2.53	3.27	
4	11.13	37.09	14.41	
5	8.78	12.88	7.71	
6	9.38	39.08	14.36	
7	-3.66	-8.84	-6.15	
8	5.56	0.83	2.74	
9	-7.72	0.85	-15.27	
10	7.76	12.09	6.49	
11	-4.01	-5.68	-3.13	
12	0.78	-1.77	1.41	

1. Determine whether there is evidence of timing ability for the two portfolios by calculating the Theynor-Mazy and Henriksson-Merton measures.

Exercise Solution

Need to do some regressions here, I will be using Matlab to go through this.
Read in the data

```
> erP = [ 3.58 -4.91 6.51 11.13 8.78 9.38 -3.66 5.56 -7.72
> erQ = [ 2.81 -1.15 2.53 37.09 12.88 39.08 -8.84 0.83 0.85
> erM = [ 2.2 -8.41 3.27 14.41 7.71 14.36 -6.15 2.74 -15.27
```

Will first estimate the Treynor-Marzuy regression

$$r_P - r_f = \alpha_P + b_P(r_m - r_f) + c_P(r_m - r_f)^2 + e_P$$

Regressing the portfolio P on a constant, the excess market return and the excess market return squared:

```
> X=[ones(12,1) erM' (erM.*erM)']
>  bP = inv(X'*X)*X'*erP'
bP =
   1.7778189
   0.6982808
  -0.0020865
```

gives the following estimates:

$$\alpha_P = 1.77,\ b_P = 0.698 \text{ and } c_P = -0.002$$

Since the c_P is negative, and almost equal to zero, there is little evidence of timing ability for portfolio P.

Doing the same procedure for portfolio Q:

```
bQ = inv(X'*X)*X'*erQ'
bQ =
  -2.30096
   1.29892
   0.10408
```

We find some more evidence of timing here

$$\alpha_Q = -2.30, \; b_Q = 1.29 \text{ and } c_Q = 0.104$$

Next want to use the Henrikson-Merton methods, instead of the quadratic term on excess market return, use a dummy for whether the exess market return is positive.

$$r_P - r_f = \alpha_P + b_P(r_m - r_f) + c_P 1_{r_m - r_f > 0} + e_P$$

```
> X=[ones(12,1), erM', erM'.*(erM>0)']
X =
    1.00000    2.20000    2.20000
    1.00000   -8.41000    0.00000
    1.00000    3.27000    3.27000
    1.00000   14.41000   14.41000
    1.00000    7.71000    7.71000
    1.00000   14.36000   14.36000
    1.00000   -6.15000    0.00000
    1.00000    2.74000    2.74000
    1.00000  -15.27000    0.00000
    1.00000    6.49000    6.49000
    1.00000   -3.13000    0.00000
    1.00000    1.41000    1.41000
```

Doing the regression for P and Q:

```
> bP=inv(X'*X)*X'*erP'
bP =
   1.783669
   0.719809
  -0.044759
> bQ=inv(X'*X)*X'*erQ'
bQ =
  -7.41661
  -0.49959
   3.60384
```

We find no evidence in favour of timing for P ($c_P = -0.044$), but we do for Q ($c_Q = 3.603$).

Alternative alpha measures - the link to asset pricing

The alpha measure is the difference between the actual performance of a portfolio p and required return of an "otherwise equivalent" portfolio p^*.

$$\alpha_p = r_p - \text{reqiured return} = r_p - \hat{r}_{p^*}$$

In this calculation there are several choices involved.

- Finding an "otherwise equivalent" portfolio. This is typically called the "benchmark" portfolio. A usual requirement in practice is that benchmarks should be an investable trading strategy.
- Finding the *required return* on this portfolio. This involves choosing an asset pricing model.

Jensen's original alpha is calculated using the market portfolio as the benchmark portfolio, and the CAPM as an asset pricing model. However, any other asset pricing model can be used instead of the CAPM.

A common asset pricing model is the the Fama-French 3 factor model. Fama and French (1992, 1993).

$$E[r_{pt}] = r_{f,t} + (E[r_{m,t}] - r_{f,t})\beta_i + b_i^{hml} HML_t + b_i^{smb} SMB_t$$

where R_{pt} is the month-t return on a the managed portfolio (net return minus T-bill return); RMRF_t is the month-t excess return on a value-weighted aggregate market proxy portfolio; and SMB_t , HML_t and UMD_t are month-t return on value-weighted zero-investment factor-mimicking portfolios for size, book-to-market (BTM) equity, and one-year momentum in stock returns, respectively.
Using this instead of the CAPM, would calculate the alpha for a portfolio p as:

$$\alpha_{p,t} = r_{p,t} - \Big(r_{f,t} + \beta_i \left(r_{m,t} - r_{f,t} \right) + b_i^{hml} HML_t + b_i^{smb} SMB_t \Big)$$

One reason for the popularity of this model as a benchmark is the provision by Ken French of these factors on his homepage. These factors applies to the cross-section of US stock returns.
For other market places similar pricing factors applies, factors that captures predictable variation in asset returns.

Exercise

On the course homepage you will find returns for *Folketrygdfondet*, a Pension Fund controlled by the Ministry of Finance, primarily investing in the Norwegian equity markets. Consider data for 1998 to 2014, and use the norwegian equity part of the portfolio. With this data, do a performance analysis using one factor and three factor models

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + \varepsilon_t$$

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + b_s SMB_t + b_h HML_t + \varepsilon_t$$

Consider both an equally weighted and a value weighted market index.

Exercise Solution

You read in the data and align it.
Show reading the FTF data:

```
library(zoo)
datadir <- "/home/bernt/data/2015/folketrygdfondet/"
filename <- paste(datadir,"folketrygdfondet_1998_2014.csv",
data <- read.zoo(filename,format="%m/%d/%Y",skip=1,header=T
rets <- as.numeric(coredata(data$SPN))
SpnRets <- zoo(rets/100.0,order.by=as.yearmon(index(data))
head(SpnRets)
```

Exercise Solution

The resulting time series are summarized as

Statistic	N	Mean	St. Dev.	Min	Max
eRp	195	0.005	0.063	−0.245	0.141
eRmew	195	0.010	0.051	−0.188	0.119
eRmvw	195	0.014	0.061	−0.221	0.162
SMB	195	0.006	0.042	−0.171	0.133
HML	195	−0.001	0.046	−0.166	0.093

Exercise Solution

Doing the regressions. One factor model

```
eRp <- SpnRets - Rf
data <- merge(eRp,eRmew,eRmvw,all=FALSE)
eRp <- data$eRp
eRmEW <- data$eRmew
eRmVW <- data$eRmvw

regrEW <- lm(eRp ~ eRmEW)
regrVW <- lm(eRp ~ eRmVW)
```

Exercise Solution

Doing the regressions, Three factor model

```
data <- merge(eRp,eRmew,eRmvw,SMB,HML,all=FALSE)
eRp <- data$eRp
eRmEW <- data$eRmew
eRmVW <- data$eRmvw
SMB <- data$SMB
HML <- data$HML

regrEW3 <- lm(eRp ~ eRmEW+SMB+HML)
regrVW3 <- lm(eRp ~ eRmVW+SMB+HML)
```

Exercise Solution

The results are summarized as

	Model 1	Model 2	Model 3	Model 4
(Intercept)	-0.005^{*}	-0.008^{***}	-0.001	-0.007^{***}
	(0.002)	(0.001)	(0.002)	(0.001)
eRmEW	1.076^{***}		0.981^{***}	
	(0.041)		(0.030)	
eRmVW		0.988^{***}		0.959^{***}
		(0.019)		(0.022)
SMB			-0.534^{***}	-0.092^{**}
			(0.036)	(0.031)
HML			0.001	0.018
			(0.032)	(0.025)
Adj. R^2	0.776	0.936	0.896	0.938
Num. obs.	195	195	195	195

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

The four-factor model

The standard benchmark for academics adds one more factor, *UMD*, creating a four-factor model of Carhart (1997).

$$R^e_{p,t} = \alpha + \beta\left(r_{m,t} - r_{f,t}\right) + b^{SMB}\mathrm{SMB}_t + b^{HML}\mathrm{HML}_t + b^{UMD}\mathrm{UMD}_t + \varepsilon$$

The factor *UMD* is related to momentum, it is using a prior-performance sort using returns over the last year.

Relative performance evaluation

The above are the classical ways of measuring performance. An alternative way of looking at performance which has become popular among institutional investors is *relative performance*, where one looks at whether a fund differs from a precipecified index.
The difference between the fund and the benchmark is called *tracking error*.
One may also want to look at the volatility of the tracking error.

Relative performance evaluation –The Information Ratio

Let r_b be the return on some benchmark, and r_p. The information ratio is the tracking error divided by the standard deviation of the tracking error.

$$IR = \frac{r_p - r_b}{\sigma(r_p - r_b)}$$

Exercise

Two measures used for portfolio performance evaluation is the Information Ratio, defined as

$$IR_p = \frac{r_p - r_b}{\sigma(r_p - r_b)}$$

and the Sharpe Ratio

$$S_p = \frac{r_p - r_f}{\sigma_p}$$

where p sigifies the portfolio of interest, and b is a benchmark portfolio.

1. In an article William Sharpe claims that the Sharpe Ratio can be viewed as a special case of the Information Ratio. How can this be justified?

Exercise Solution

1. If the benchmark portfilio b is the risk free asset

$$IR_p = \frac{r_p - r_b}{\sigma(r_p - r_b)} = \frac{r_p - r_f}{\sigma(r_p - r_f)} = \frac{r_p - r_f}{\sigma(r_p)} = S_p$$

Holdings-based analysis

Do not just consider the portfolio returns, we use the complete records of the asset composition of the portfolios.
What can this achieve?

- It may alleviate the sensitivity of returns bases measures to choice of benchmark (the Roll critique).
- This approach may deal with nontrivial shifts in style allocations.
- One can look at performance before trading costs (which are incorporated in returns).
- One can decompose the sources of value added by a manager.
- Holdings-based analysis leads to more precise identification of manager ability, as observing performance on a security-by security basis increases the number of observations of ability.

Holdings-based analysis

holdings-bases measures $\rightarrow$ the covariance between lagged weights and current returns.

$$PHM_t = \text{cov}(w_{t-1}, R_t)$$

Intuition:
A skilled manager will have portfolio weights that move in the same direction as future returns.
Grinblatt and Titman (1993):

$$GT_t = \sum_j (w_{j,t-1} - w_{j,t-2}) R_{j,t}$$

Averaged across time

Stochastic Discount Factors and weight measures

Generate intuition
General relationship

$$E_t[m_{t+1}R_{t+1}|Z_t] = 1$$

where R_{+1} is the vector of primitive asset returns, m is the stochastic discount factor, and Z_t is conditioning information. For a given portfolio p, Alpha is calculated as

$$\alpha_p = E_t[m_{t+1}R_{p,t+1}|Z_t] - 1$$

Stochastic Discount Factors and weight measures

asset manager: chooses a set of weights w_t
weights – function of the asset manager's information set Ω_t

$$w_t = w_t(\Omega_t)$$

The next period portfolio return $R_{p,t+1}$ is then

$$R_{p,t+1} = w_t(\Omega_t) R_{t+1}$$

Plugging this into the alpha calculation

$$\alpha_p = E_t[m_{t+1} w_t(\Omega_t) R_{t+1} | Z_t] - 1$$

from the definition of covariance

$$\text{cov}(m_{t+1}R_{t+1}, w_t(\Omega_t))$$

$$= E[m_{t+1}R_{t+1}w_t(\Omega_t)] - E[m_{t+1}R_{t+1}]E[w_t(\Omega_t)]$$

From the fundamental pricing relation

$$E[m_{t+1}R_{t+1}] = 1$$

the second term in the covariance is equal to 1
and we can express alpha as

$$\alpha_p = \text{cov}(m_{t+1}R_{t+1}, w_t(\Omega_t)|Z_t)$$

Interpretation: alpha — the covariance between the weights with the risk-adjusted returns

Summarizing – performance evaluation

Performance evaluation: Does the return on an investment justify its risk?

Classical Performance evaluation.

- Sharpe Ratio

$$S_p = \frac{r_p - r_f}{\sigma_p}$$

- Treynor Ratio

$$T_p = \frac{r_p - r_f}{\beta_p}$$

- Jensen's alpha

$$\alpha_p = r_p - (r_f + \beta_p(r_m - r_f))$$

- Appraisal Ratio

$$AR_p = \frac{\alpha_p}{\sigma(e_p)}$$

Summarizing – performance evaluation

Additional empirical measurement

- Alpha with alternative performance measures.
- Market Timing
- Modigliani's M^2 measure
- Relative performance evaluation – information ratio
- Covariance (holdings based) measures.

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