Lecture Notes – Investment Performance Evaluation

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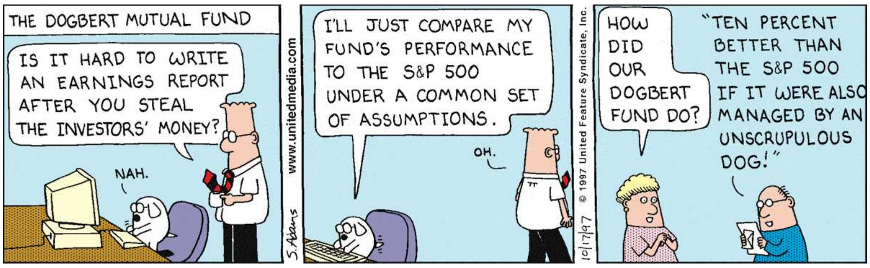

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1 Introduction

A common quesion in finance is the "performance question":

How well did a given investment/portfolio/mutual fund "perform"?

The empirical challenge: When we observe an actual portfolio return, we need a way to ask how good this return was.

This is a nontrivial exercise, one needs to establish a theoretical framework that allows one to say whether the performance is good or not.

The classical measures are developed relative to the CAPM, in particular the three measures named after their developers: Sharpe, Treynor and Jensen.

There are however many alternatives to these.

Some of the alternatives are modifications of the classical measures. For example, the classical Jensen alpha is calculated based on the CAPM. One can however calculate a alpha measure using alternative models for expected returns.

Other alternatives bring more information into the evaluation of the portfolio. The classical measures are based on only observing returns. Another piece of information one can potentially bring into the analysis is the actual portfolio decisions, when stocks are bought and sold.

1.1 Performance evaluation of managed portfolios

The application one typically use this for is looking at *portfolio managers*.

The business of evaluating the performance of a portfolio manager has developed a rich set of methodologies for testing whether a manager is skilled or not.

The goal is to identify whether the manager has a skill that goes beyond simple, well known strategies that can easily be implemented by unskilled investors. For example, portfolio tilts towards small stocks should not necessarily be viewed as skill.

The methods can be grouped into two major approaches

1. Returns-based performance evaluation
2. Portfolio holdings-based performance evaluation

Pros and cons.

Returns-based:

1. Rely on less information
2. Returns are often available at higher frequencies than other information

Portfolio holdings-based

1. Will more clearly identify skill
2. Require more information than returns-based measures.

2 Illustration using CAPM

Let us illustrate by discussing the classical issue of picking overvalued stocks with reference to the CAPM.

According to the CAPM, *all* securities should plot on the security market line (SML). If so expected return for every security should satisfy:

$$E[\tilde{r}_j] = r_f + (E[\tilde{r}_m] - r_f)\beta_j$$

Many security analysts use the SML to identify 'mis-priced' securities. Obviously, these analyst believe that markets are inefficient, but also think that betas provide a good measure of the risk of an individual security. The following example illustrates the procedure used by security analysts to identify 'mis-priced' securities.

Exercise 1.

Suppose your investment company estimates the beta for Westinghouse to be 1.20 and the dividend growth rate to be 10%. The current yield on a one-year T-bill is 8.0% and the market risk premium is estimated to be 7.0%. Westinghouse is expected to pay a dividend of $3.50 next year.

1. If these estimates are correct, what should be the market price of Westinghouse's stock?
2. If you observe a price of Westinghouse of 52, what is your recommendation?

Solution to Exercise 1.

1. To find the current market price, Use CAPM to find the discount rate for Westinghouse:

$$r = r_f + (E[\tilde{r}_m] - r_f)\beta = 0.08 + 0.07 \cdot 1.20 = 16.4\%$$

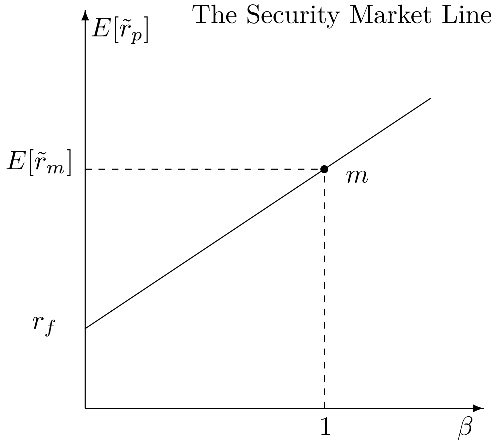
   The estimated market price

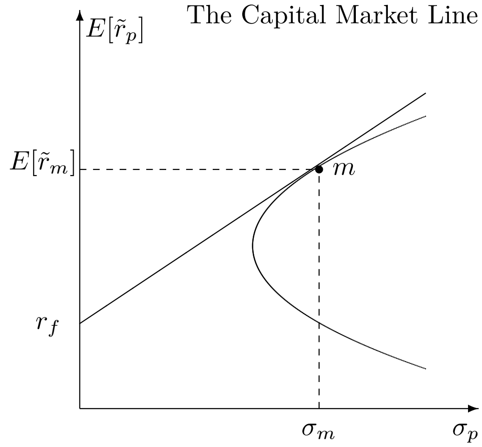
$$P_0 = \frac{E[D_1]}{r-g} = \frac{\$3.50}{0.164 - 0.10} = \$54.69$$

   If the actual market price is less than \$54.69, the stock is undervalued, and if greater than \$54.69, the stock is overvalued.

2. An observed price of 52, the stock is undervalued, buy.

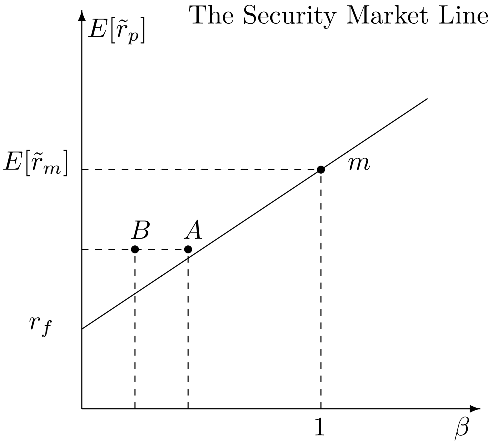
The CAPM offers two benchmarks against which the performance of an investment manager can be judged, the CML and the SML.

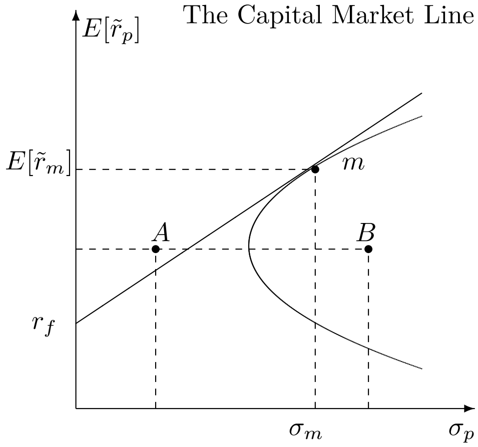




If an investment manager consistently outperforms the SML, this would be evidence that the manager has the ability to pick undervalued securities. Clearly, you would want such an investment manager on your team.

But outperforming the SML may *not* be sufficient to qualify as a 'good' investment manager. Part of the reason investors hold mutual funds is for their diversification services. If a large fraction of your wealth is tied up in a single mutual fund, you want them to be well-diversified. Thus, for investment managers who are supplying diversification benefits to investors, the SML is an incomplete benchmark for performance. They should provide high rewards for risk as measured by the variance. In other words, the investment manager should be expected to compare favourably to the CML, which gives the tradeoff between risk and return for well-diversified portfolios.





Which of the two mutual funds would you rather invest in, A or B? Why?

3 Conceptually - what is performance valuation?

Idea: Given an investment, how does it compare to a *possible alternative* investment strategies?

To implement this idea, boils down to finding a *benchmark* for the comparison.

3.1 Benchmark

A benchmark is a measuring tape, a portfolio that is an alternative investment opportunity.

Good benchmarks should be

- Unambiguous
- Tradeable
- Measurable
- Appropriate
- Reflective of current investment opinions
- Specified in advance.

3.2 Desirable properties of performance measures

Chen and Knez (1996): Desirable properties of performance measures.

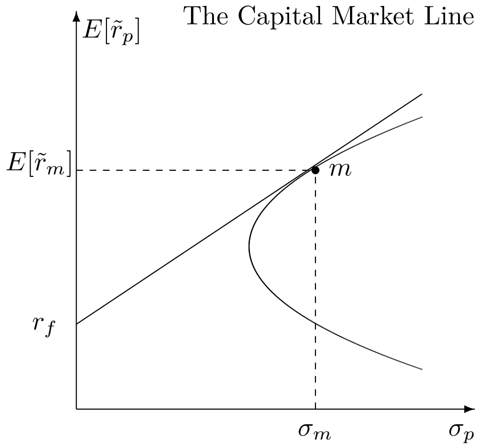
- Fit. Capture strategies relevant for uninformed investors. Have zero performance for simple strategies feasible for such investors.
- Be Scalable. Linear combinations of manager measures should equal the measure for the linear combination of manager portfolios
- Be continuous. Close skills/strategies should have close performance measures.
- Exhibit monotonicity. Assign higher measures for more skilled managers.

An added desirable property is manipulation-proofness. See Goetzmann, Ingersoll, Spiegel, and Welch (2007)

3.3 The Sharpe Ratio

The Sharpe ratio is an answer to the question: How far is an asset p from the Capital Market Line?

Recall the capital market line as the mapping of the opportunity set in mean-standard-deviation space.



In such a figure, on can calculate the line from r_f through any portfolio p as:

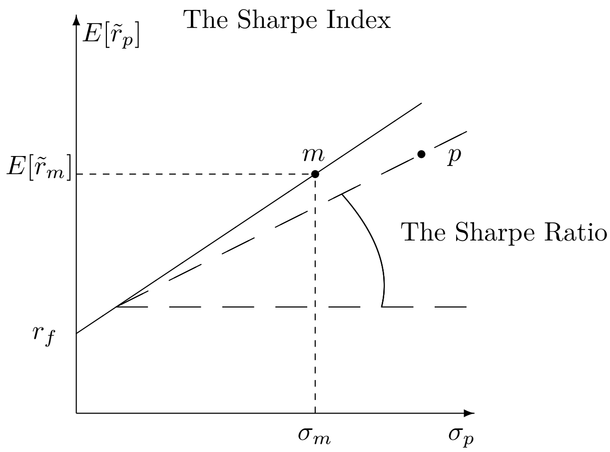
$$r_p - r_f = S\sigma_p$$

Here S is the slope of the line from the risk free rate through p.

From the equation for this line solve for S:

$$S = \frac{r_p - r_f}{\sigma_p}$$

This is the Sharpe inde. The Sharpe Index is not informative by itself, only when compared to something, such as the market index:



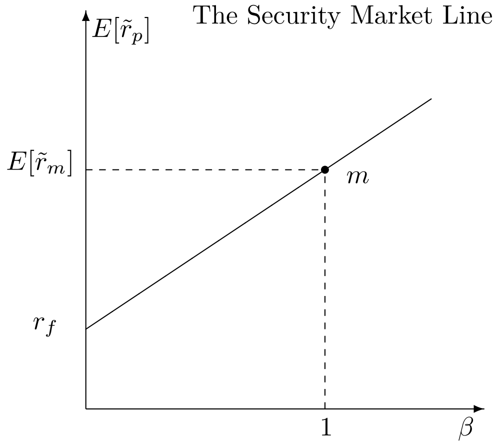
Sharpe is primarily used for undiversified portfolios.

3.4 The Treynor Measure

The Treynor measure T_p of a portfolio p is defined as

$$T_p = \frac{r_p - r_f}{\beta_p}$$

The Treynor measure is the slope of the line from r_f in mean-beta-space. To see that, consider a mapping in $E[r] - \beta$ space



The equation for the line starting at r_f is

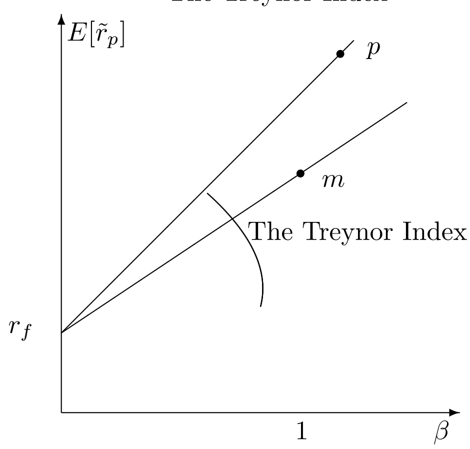
$$r_p - r_f = T\beta,$$

where T is the slope of this line.

Solving for T we find

$$T = \frac{r_p - r_f}{\beta_p}$$

A Treynor index is not meaningful by itself, only when compared to an alternative (benchmark) investment, such as the market portfolio.



3.5 The Jensen alpha

Alpha is an attempt to answer the question: Does the return on a portfolio/asset exceed its *required* return?

Jensens alpha is the difference between actual performance and required return

$$\alpha_p = r_p - \text{reqiured return} = r_p - \hat{r}_p$$

To find an estimate of requred return an asset pricing model is required.

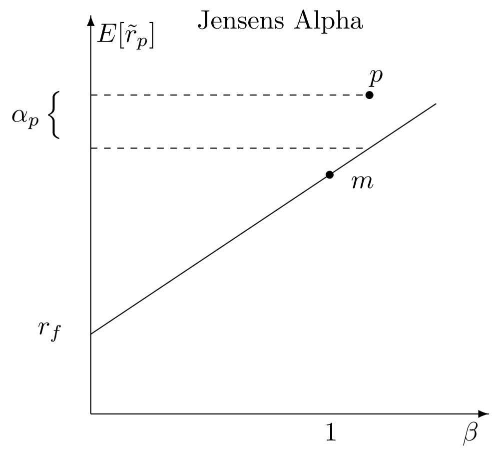
The classical such asset pricing model is the CAPM, which is what Jensen used

$$\hat{r}_p = (r_f + \beta_p(r_m - r_f))$$

Alpha is then

$$\alpha_p = r_p - (r_f + \beta_p(r_m - r_f))$$

Graphically:



3.6 Example calculation

Exercise 2.

Given the following information about the return on a stock ABC, the S&P market index and risk free returns.

	Rate of Return		
Year	ABC	S&P 500	T-bills
1	14%	12%	7
2	10	7	7.5
3	19	20	7.7
4	-8	-2	7.5
5	23	12	8.5
6	28	23	8
7	20	17	7.3
8	14	20	7
9	-9	-5	7.5
10	19	16	8
Average	13%	12%	7.6%
Standard Deviation	12.4%	9.4%	0.5%
Geometric Mean	12.3%	11.6%	7.6%
cov(r_{ABC}, r_m)	0.0107		

1. Calculate the beta of ABC stock.
2. Calculate the Sharpe measure for ABC stock. Compare it to the market and draw a diagram illustrating its placing in mean – standard deviation space.
3. Calculate the Treynor measure for ABC stock. Compare it to the marke and draw a diagram illustrating its placing in mean – beta space.
4. Calculate Jensen's Alpha for ABC stock and draw a diagram illustrating its placing in mean – beta space.

Solution to Exercise 2.

1. Beta

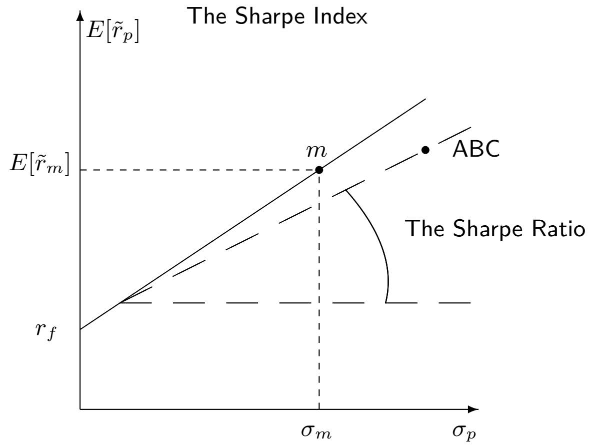
$$\beta_{ABC,m} = \frac{\text{cov}(r_{ABC}, r_m)}{\text{var}(r_m)} = 1.20375$$

2. Sharpe index:

$$S_{ABC} = \frac{r_{ABC} - r_f}{\sigma_{ABC}} = \frac{0.13 - 0.076}{0.124} = 0.43$$

This can be compared to the Sharpe Ratio for the market portfolio m:

$$S_m = \frac{r_m - r_f}{\sigma_m} = \frac{0.12 - 0.076}{0.094} = 0.468$$

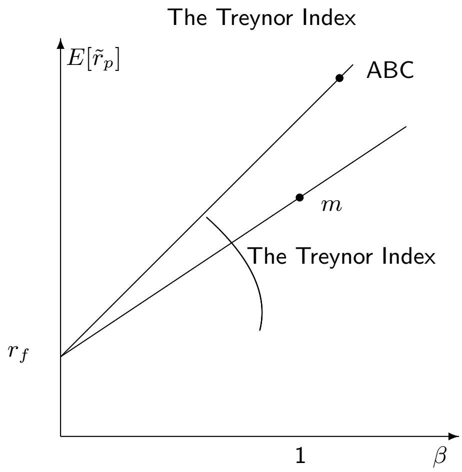


3. Treynor Index:

$$T_{ABC} = \frac{r_{ABC} - r_f}{\beta_{ABC}} = \frac{0.13 - 0.076}{1.20375} = 0.04485$$

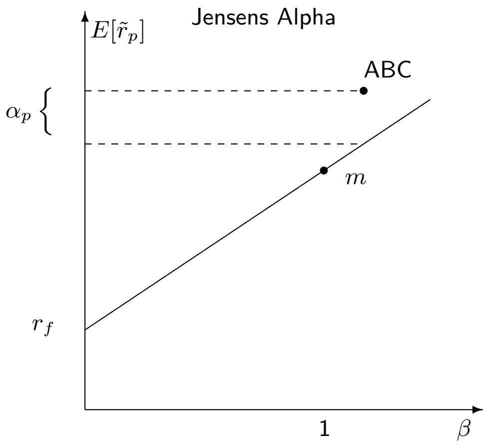
This is to be compared to the Treynor Index of the market:

$$T_m = \frac{r_m - r_f}{\beta_m} = \frac{0.12 - 0.076}{1} = 0.044$$



4. Jensens alpha:

$$\alpha_{ABC} = \overline{r_{ABC}} - E[r_{ABC}] = \overline{r}_{ABC} - (r_f + \beta_{A,m}(r_m - r_f))$$
$$= 0.13 - (0.076 + 1.20375(0.12 - 0.076)) = 0.00103 = 0.103\%$$



3.7 Some relations beween performance measures

There is some useful intuition to be had in terms of comparisons using several of these measures.

Let us first link the Treynor measure and Jensen's alpha, and show that for positive beta assets, positive alpha assets will have a Treynor index above the Treynor index of the market.

Exercise 3.

The Treynor index of an asset p is $T = \frac{E[r_p]-r_f}{\beta_p}$.
Jensen's alpha α_p for the same asset is

$$\alpha_p = E[r_p] - (r_f + \beta_p(E[r_m] - r_f)).$$

Show that

$$T_p = \frac{\alpha_p}{\beta_p} + T_m$$

where T_m is the Treynor measure of the market.

Solution to Exercise 3.

$$\begin{aligned}
T_p &= \frac{r_p - r_f}{\beta_p} \\
&= \frac{\alpha_p - \alpha_p + r_p - r_f}{\beta_p} \\
&= \frac{\alpha_p - (r_p - (r_f + \beta_p(r_m - r_f))) + r_p - r_f}{\beta_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\beta_p(r_m - r_f)}{\beta_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{r_m - r_f}{1} \\
T_p &= \frac{\alpha_p}{\beta_p} + T_m
\end{aligned}$$

Observe the implication: for positive beta assets, positive alpha assets will have a Treynor index above the Treynor index of the market.

Let us also look at the relationship between Jensen's alpha and the Sharpe ratio. Positive alpha assets will have a Sharpe ratio above that of the market as long as the correlation between p and m is not too low.

Exercise 4.

The Sharpe index of an asset p is $S_p = \frac{E[r_p]-r_f}{\sigma_p}$. The Jensen alpha α_p of the same asset is

$$\alpha_p = E[r_p] - (r_f + \beta_p(E[r_m] - r_f)).$$

Show that

$$S_p = \frac{\alpha_p}{\beta_p} + \rho(r_p, r_m)S_m$$

where $\rho(r_p, r_m)$ is the correlation between asset p and the market m.

Solution to Exercise 4.

$$
\begin{aligned}
S_p &= \frac{r_p - r_f}{\sigma_p} \\
&= \frac{\alpha_p - \alpha_p + r_p - r_f}{\sigma_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\beta_p(r_m - r_f)}{\sigma_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\frac{\text{cov}(r_p, r_m)}{\text{var}(r_m)}(r_m - r_f)}{\sigma_p} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\frac{\text{cov}(r_p, r_m)}{\sigma(r_m)\sigma(r_p)}(r_m - r_f)}{\sigma_m} \\
&= \frac{\alpha_p}{\beta_p} + \frac{\rho(r_p, r_m)(r_m - r_f)}{\sigma_m} \\
&= \frac{\alpha_p}{\beta_p} + \rho(r_p, r_m) S_m
\end{aligned}
$$

Positive alpha assets will have a Sharpe ratio above that of the market as long as the correlation between p and m is not too low.

4 Computer tools for doing calculations

In practice, we need to use a computer tool to do performance calculations. Obviously much of this can be done in Excel and similar spreadsheets. But spreadsheets is not the best tool to do this kind of analysis. We will instead look at two alternatives.

- Matlab and similar matrix tools. This is the best tool for doing and learning the calculations. The bad part about Matlab is that it is hard to get the *data* lined up and into the matrix handler. In current practice we are therefore seeing a move away from Matlab, replacing it with:
- R, which is a tool for statistical analysis. This tool is much easier to get data into, and it can do most of the tasks you use Matlab for. It is therefore taking over for Matlab in many “quant shops”

Let us first look at an example of using Matlab to do a performance analysis.

Exercise 5.

You are given historical returns of two different equities, r_A and r_B, as well as the market return r_m, and the risk free rate r_f.

```
   rA,      rB,      rm,     rf
 0.10,    0.05,    0.01,   0.01
 0.20,    0.03,   -0.05,   0.01
-0.10,   -0.01,   -0.05,   0.01
 0.13,    0.03,    0.10,   0.01
 0.24,    0.04,    0.14,   0.0140
-0.08,   -0.05,   -0.02,   0.02
-0.15,   -0.02,       0,   0.02
 0.15,    0.12,    0.10,   0.01
 0.45,    0.15,    0.05,   0.01
-0.10,   -0.10,    0.04,   0.02
 0.01,    0.01,    0.03,   0.01
-0.05,   -0.01,    0.01,   0.01
 0.20,    0.11,    0.05,   0.02
-0.05,    0.12,    0.05,   0.01
```

Use matlab/octave to calculate

- Sharpe measures

- Treynor measures
- Jensen alphas (relative to the CAPM)

Solution to Exercise 5.

```
rets = dlmread("../data/example.txt",",",1,0);
rets
rA = rets(:,1);
rB = rets(:,2);
rm = rets(:,3);
rf = rets(:,4);
sA = mean(rA-rf)/std(rA)
sB = mean(rB-rf)/std(rB)
sm = mean(rm-rf)/std(rm)
betaA = cov(rA,rm)/var(rm)
betaB = cov(rB,rm)/var(rm)
betam = 1
tA = mean(rA-rf)/betaA
tB = mean(rB-rf)/betaB
tm = mean(rm-rf)/betam
alphaA = mean(rA - (rf + betaA*(rm-rf)))
alphaB = mean(rB - (rf + betaB*(rm-rf)))
```

results in the following output

```
rets =
   0.10000   0.05000   0.01000   0.01000
   0.20000   0.03000  -0.05000   0.01000
  -0.10000  -0.01000  -0.05000   0.01000
   0.13000   0.03000   0.10000   0.01000
   0.24000   0.04000   0.14000   0.01400
  -0.08000  -0.05000  -0.02000   0.02000
  -0.15000  -0.02000   0.00000   0.02000
   0.15000   0.12000   0.10000   0.01000
   0.45000   0.15000   0.05000   0.01000
  -0.10000  -0.10000   0.04000   0.02000
   0.01000   0.01000   0.03000   0.01000
  -0.05000  -0.01000   0.01000   0.01000
   0.20000   0.11000   0.05000   0.02000
  -0.05000   0.12000   0.05000   0.01000

sA =  0.32043
sB =  0.28515
sm =  0.35502
betaA =  1.3418
betaB =  0.52031
betam =  1
tA =  0.040778
tB =  0.039262
tm =  0.019714
alphaA =  0.028262
alphaB =  0.010171
```

Let us do the same example using R as the tool

Exercise 6.

You are given historical returns of two different equities, r_A and r_B, as well as the market return r_m, and the risk free rate r_f.

```
   rA,     rB,     rm,    rf
 0.10,   0.05,   0.01,  0.01
```

```
 0.20,    0.03,   -0.05,   0.01
-0.10,   -0.01,   -0.05,   0.01
 0.13,    0.03,    0.10,   0.01
 0.24,    0.04,    0.14,   0.0140
-0.08,   -0.05,   -0.02,   0.02
-0.15,   -0.02,       0,   0.02
 0.15,    0.12,    0.10,   0.01
 0.45,    0.15,    0.05,   0.01
-0.10,   -0.10,    0.04,   0.02
 0.01,    0.01,    0.03,   0.01
-0.05,   -0.01,    0.01,   0.01
 0.20,    0.11,    0.05,   0.02
-0.05,    0.12,    0.05,   0.01
```

Use R to calculate

- Sharpe measures
- Treynor measures
- Jensen alphas (relative to the CAPM)

Solution to Exercise 6.

```
> ## example performance measure calculation
> data <- read.table("example.txt", header=TRUE,sep=",")
> head(data)
     rA    rB    rm    rf
1  0.10  0.05  0.01 0.010
2  0.20  0.03 -0.05 0.010
3 -0.10 -0.01 -0.05 0.010
4  0.13  0.03  0.10 0.010
5  0.24  0.04  0.14 0.014
6 -0.08 -0.05 -0.02 0.020
> rA <- data$rA
> rB <- data$rB
> rm <- data$rm
> rf <- data$rf
>                                          # Sharpe Ratio
> sA <- mean(rA-rf)/sd(rA-rf)
> sB <- mean(rB-rf)/sd(rB-rf)
> sm <- mean(rm-rf)/sd(rm-rf)
> print(c(sA,sB,sm))
[1] 0.3175401 0.2767519 0.3521840
>
>                                          #Beta
> betaA <- cov(rA,rm)/var(rm)
> betaB <- cov(rB,rm)/var(rm)
> betam <- 1
> print(c(betaA,betaB,betam))
[1] 1.3417676 0.5203136 1.0000000
>                                          #Treynor
> tA <- mean(rA-rf)/betaA
> tB <- mean(rB-rf)/betaB
> tm <- mean(rm-rf)/1
> print(c(tA,tB,tm))
[1] 0.04077777 0.03926204 0.01971429
>                                          #Alpha
> alphaA <- mean(rA - (rf + betaA*(rm-rf)))
> alphaB <- mean(rB - (rf + betaB*(rm-rf)))
> print(c(alphaA,alphaB))
[1] 0.02826230 0.01017096
```

Summarizing the calculated numbers

	r_A	r_B	r_m
Sharpe	0.318	0.277	0.352
β	1.342	0.520	
Treynor	0.041	0.039	0.020
α	0.028	0.010	

This example is also illustrated `Julia` with code next to these lecture notes.

4.1 The appraisal ratio

The appraisal ratio is calculated by dividing Jensens alpha by the variance of the unsystematic risk of the portfolio

$$\frac{\alpha_p}{\sigma(e_P)}$$

e_P can be calculated as the residual of the regression

$$e_{pt} = r_{pt} - (\alpha_i + \beta_i r_{mt})$$

5 The M^2 measure

Let us now look at a measure introduced by Franco Modigliani, M^2.

It focus is on total variability. A managed portfolio p is mixed with a position in the risk free asset to make the "adjusted" portfolio have the same volatility as the market.

Suppose the managed portfolio p has a total variability equal to $1.5 \times \sigma_m$. The "adjusted" portfolio p^* is found by investing a weighte w in p and a weight $(1-w)$ in the risk free asset, such that the portfolio has the same standard deviation as the market:

$$w\sigma_p + (1-w)\sigma(r_f) = w\sigma_p + (1-w)\cdot 0 = w\sigma_P = \sigma_m$$

or

$$w = \frac{\sigma_m}{\sigma_p} = \frac{\sigma_m}{1.5\sigma_m} = \frac{1}{1.5} = 0.67$$

By investing two thirds in p and one third in the risk free asset, achieve the same volatility as the market. Since P^* and m have the same volatility, see how well P is performing by comparing the returns.

$$M^2 = r_{P^*} - r_m$$

Exercise 7.

Given the following data:

	P	m
Average return	35%	28%
Beta	1.2	1
Standard Deviation	42%	30%
Nonsystematic risk ($\sigma(e)$)	18%	0

The T-bill rate during the period was 6%.

1. Calculate the M^2 measure for the portfolio P.

Solution to Exercise 7.

1. The M^2 measure.

   What weight to get a portfolio of P and risk free asset with the same standard deviation? $w\sigma_P = \sigma_m$, giving $w = \sigma_m/\sigma_p = 0.3/0.42 = 0.714$. with this weight, calculate return

   $$r_{P^*} = wr_P + (1-w)r_f = 0.7140.35 + (1-0.714)0.06 = 0.267$$

   Comparing this to the market return gives the M^2 measure

   $$M^2 = r_{P^*} - r_m = 0.267 - 0.28 = -0.013 = -1.3\%$$

Exercise 8.

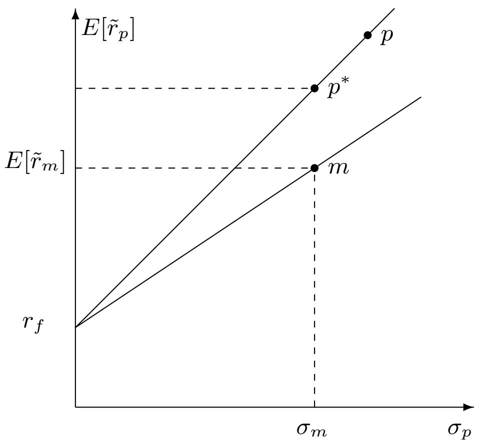
Demonstrate the following relationship between M^2 and the Sharpe measure S_p for a portfolio p:

$$M^2 = (S_p - S_m)\sigma_m$$

Solution to Exercise 8.

$$M^2 = R_{P*} - R_M = S_p\sigma_m - S_m\sigma_m = (S_p - S_m)\sigma_m$$

This is easiest seen in a diagram.



when you want to evaluate p, you first find the portfolio p^* with the same risk as the market, and make the comparison for that portfolio.

6 Market timing

The classical measures are measures of asset selection: Does the picked asset(s) show superior performance? They are implemented using historical averages to estimate. Implicit assumption when doing so: Risk of portfolio does not change during the estimation period.

For some funds this assumption not fulfilled.

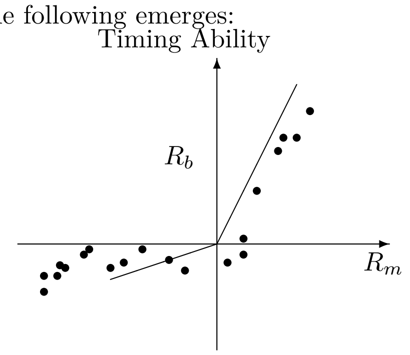
The typical example: market timers. Typical timing: Periodically shifting between broad asset classes, such as

- Stocks
- Bonds
- Cash

Based on estimates of which asset class will perform best next period

For example, suppose the fund only invests in two assets: bonds and stocks.

If one is able to predict periods when stocks were doing better, and be in stocks then, a picture like the following emerges:



where R_b is the return on bonds, and R_m is the return on stocks.

How can one measure this?

Suggested regressions: Treynor-Mazry:

$$r_p - r_f = a + b(r_m - r_f) + c(r_m - r_f)^2 + \varepsilon_p$$

Henriksson-Merton

$$r_p - r_f = a + b(r_m - r_f) + c(r_m - r_f)1_{\{r_m > r_f\}} + \varepsilon_p$$

A positive estimate of c in these regression are indications of timing abilities.

Exercise 9.

You are given the historical percentage excess returns (returns in excess of the risk free rate) for 2 portfolios, P, Q and a benchmark M.

time	$r_P - r_f$	$r_Q - r_f$	$r_M - r_f$	
1	3.58	2.81	2.2	0
2	-4.91	-1.15	-8.41	
3	6.51	2.53	3.27	
4	11.13	37.09	14.41	
5	8.78	12.88	7.71	
6	9.38	39.08	14.36	
7	-3.66	-8.84	-6.15	
8	5.56	0.83	2.74	
9	-7.72	0.85	-15.27	
10	7.76	12.09	6.49	
11	-4.01	-5.68	-3.13	
12	0.78	-1.77	1.41	

1. Determine whether there is evidence of timing ability for the two portfolios by calculating the Theynor-Mazy and Henriksson-Merton measures.

Solution to Exercise 9.

1. We need to do some regressions here, I will be using Matlab to go through this.

   Read in the data

```
> erP = [ 3.58 -4.91 6.51 11.13 8.78 9.38 -3.66 5.56 -7.72 7.76 -4.01 0.78];
> erQ = [ 2.81 -1.15 2.53 37.09 12.88 39.08 -8.84 0.83 0.85 12.09 -5.68 -1.77 ];
> erM = [ 2.2 -8.41 3.27 14.41 7.71 14.36 -6.15 2.74 -15.27 6.49 -3.13 1.41];
```

   Will first estimate the Treynor-Marzuy regression

$$r_P - r_f = \alpha_P + b_P(r_m - r_f) + c_P(r_m - r_f)^2 + e_P$$

   Regressing the portfolio P on a constant, the excess market return and the excess market return squared:

```
> X=[ones(12,1) erM' (erM.*erM)']
>  bP = inv(X'*X)*X'*erP'
bP =
   1.7778189
   0.6982808
  -0.0020865
```

   gives the following estimates:

$$\alpha_P = 1.77,\ b_P = 0.698 \text{ and } c_P = -0.002$$

   Since the c_P is negative, and almost equal to zero, there is little evidence of timing ability for portfolio P.

   Doing the same procedure for portfolio Q:

```
bQ = inv(X'*X)*X'*erQ'
bQ =
  -2.30096
   1.29892
   0.10408
```

We find some more evidence of timing here

$$\alpha_Q = -2.30,\ b_Q = 1.29 \text{ and } c_Q = 0.104$$

Next want to use the Henrikson-Merton methods, instead of the quadratic term on excess market return, use a dummy for whether the exess market return is positive.

$$r_P - r_f = \alpha_P + b_P(r_m - r_f) + c_P 1_{r_m - r_f > 0} + e_P$$

```
> X=[ones(12,1), erM', erM'.*(erM>0)']
X =
    1.00000    2.20000    2.20000
    1.00000   -8.41000    0.00000
    1.00000    3.27000    3.27000
    1.00000   14.41000   14.41000
    1.00000    7.71000    7.71000
    1.00000   14.36000   14.36000
    1.00000   -6.15000    0.00000
    1.00000    2.74000    2.74000
    1.00000  -15.27000    0.00000
    1.00000    6.49000    6.49000
    1.00000   -3.13000    0.00000
    1.00000    1.41000    1.41000
```

Doing the regression for P and Q:

```
> bP=inv(X'*X)*X'*erP'
bP =
   1.783669
   0.719809
  -0.044759
> bQ=inv(X'*X)*X'*erQ'
bQ =
  -7.41661
  -0.49959
   3.60384
```

We find no evidence in favour of timing for P ($c_P = -0.044$), but we do for Q ($c_Q = 3.603$).

7 Alternative alpha measures - the link to asset pricing

The alpha measure is the difference between the actual performance of a portfolio p and required return of an "otherwise equivalent" portfolio p^*.

$$\alpha_p = r_p - \text{required return} = r_p - \hat{r}_{p^*}$$

In this calculation there are several choices involved.

- Finding an "otherwise equivalent" portfolio. This is typically called the "benchmark" portfolio. A usual requirement in practice is that benchmarks should be an investable trading strategy.
- Finding the *required return* on this portfolio. This involves choosing an asset pricing model.

Jensen's original alpha is calculated using the market portfolio as the benchmark portfolio, and the CAPM as an asset pricing model. However, any other asset pricing model can be used instead of the CAPM.

7.1 The Fama French model

A common asset pricing model is the the Fama-French 3 factor model. Fama and French (1992, 1993).

$$E[r_{pt}] = r_{f,t} + (E[r_{m,t}] - r_{f,t})\beta_i + b_i^{hml} HML_t + b_i^{smb} SMB_t$$

where R_{pt} is the month-t return on a the managed portfolio (net return minus T-bill return); RMRF_t is the month-t excess return on a value-weighted aggregate market proxy portfolio; and SMB_t , HML_t and UMD_t are month-t return on value-weighted zero-investment factor-mimicking portfolios for size, book-to-market (BTM) equity, and one-year momentum in stock returns, respectively.

Using this instead of the CAPM, would calculate the alpha for a portfolio p as:

$$\alpha_{p,t} = r_{p,t} - \left(r_{f,t} + \beta_i \left(r_{m,t} - r_{f,t}\right) + b_i^{hml} HML_t + b_i^{smb} SMB_t\right)$$

One reason for the popularity of this model as a benchmark is the provision by Ken French of these factors on his homepage. These factors applies to the cross-section of US stock returns.

For other market places similar pricing factors applies, factors that captures predictable variation in asset returns.

Exercise 10.

On the course homepage you will find returns for *Folketrygdfondet*, a Pension Fund controlled by the Ministry of Finance, primarily investing in the Norwegian equity markets. The file "folketrygdfondet_1998_2014.csv" contains data for 1998 to 2014. In this file, the first data column (labeled SPN), contains data for the norwegian equity part of the portfolio. With this data, do a performance analysis using one factor and three factor models

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + \varepsilon_t$$

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + b_s SMB_t + b_h HML_t + \varepsilon_t$$

Consider both an equally weighted and a value weighted market index.

Solution to Exercise 10.

You read in the data and align it.

Show reading the FTF data:

```
library(zoo)
datadir <- "/home/bernt/data/2015/folketrygdfondet/"
filename <- paste(datadir,"folketrygdfondet_1998_2014.csv",sep="")
data <- read.zoo(filename,format="%m/%d/%Y",skip=1,header=TRUE,sep=",")
rets <- as.numeric(coredata(data$SPN))
SpnRets <- zoo(rets/100.0,order.by=as.yearmon(index(data)))
head(SpnRets)
```

The resulting time series are summarized as

Statistic	N	Mean	St. Dev.	Min	Max
eRp	195	0.005	0.063	−0.245	0.141
eRmew	195	0.010	0.051	−0.188	0.119
eRmvw	195	0.014	0.061	−0.221	0.162
SMB	195	0.006	0.042	−0.171	0.133
HML	195	−0.001	0.046	−0.166	0.093

Doing the regressions. One factor model

```
eRp <- SpnRets - Rf
data <- merge(eRp,eRmew,eRmvw,all=FALSE)
eRp <- data$eRp
eRmEW <- data$eRmew
eRmVW <- data$eRmvw

regrEW <- lm(eRp ~ eRmEW)
regrVW <- lm(eRp ~ eRmVW)
```

Doing the regressions, Three factor model

```
data <- merge(eRp,eRmew,eRmvw,SMB,HML,all=FALSE)
eRp <- data$eRp
eRmEW <- data$eRmew
eRmVW <- data$eRmvw
SMB <- data$SMB
```

```
HML <- data$HML

regrEW3 <- lm(eRp ~ eRmEW+SMB+HML)
regrVW3 <- lm(eRp ~ eRmVW+SMB+HML)
```

The results are summarized as

	Model 1	Model 2	Model 3	Model 4
(Intercept)	-0.005^{*}	-0.008^{***}	-0.001	-0.007^{***}
	(0.002)	(0.001)	(0.002)	(0.001)
eRmEW	1.076^{***}		0.981^{***}	
	(0.041)		(0.030)	
eRmVW		0.988^{***}		0.959^{***}
		(0.019)		(0.022)
SMB			-0.534^{***}	-0.092^{**}
			(0.036)	(0.031)
HML			0.001	0.018
			(0.032)	(0.025)
Adj. R^2	0.776	0.936	0.896	0.938
Num. obs.	195	195	195	195

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

7.2 The four-factor model

The standard benchmark for academics adds one more factor, UMD, creating a four-factor model of Carhart (1997).

$$R^e_{p,t} = \alpha + \beta\left(r_{m,t} - r_{f,t}\right) + b^{SMB}\text{SMB}_t + b^{HML}\text{HML}_t + b^{UMD}\text{UMD}_t + \varepsilon_{pt}$$

The factor UMD is related to momentum, it is using a prior-performance sort using returns over the last year.

8 Relative performance evaluation

The above are the classical ways of measuring performance. An alternative way of looking at performance which has become popular among institutional investors is *relative performance*,

where one looks at whether a fund differs from a precipecified index.

The difference between the fund and the benchmark is called *tracking error*.

One may also want to look at the volatility of the tracking error.

8.1 The Information Ratio

Let r_b be the return on some benchmark, and r_p. The information ratio is the tracking error divided by the standard deviation of the tracking error.

$$IR = \frac{r_p - r_b}{\sigma(r_p - r_b)}$$

Exercise 11.

Two measures used for portfolio performance evaluation is the Information Ratio, defined as

$$IR_p = \frac{r_p - r_b}{\sigma(r_p - r_b)}$$

and the Sharpe Ratio

$$S_p = \frac{r_p - r_f}{\sigma_p}$$

where p sigifies the portfolio of interest, and b is a benchmark portfolio.

1. In an article William Sharpe claims that the Sharpe Ratio can be viewed as a special case of the Information Ratio. How can this be justified?

Solution to Exercise 11.

1. If the benchmark portfilio b is the risk free asset

$$IR_p = \frac{r_p - r_b}{\sigma(r_p - r_b)} = \frac{r_p - r_f}{\sigma(r_p - r_f)} = \frac{r_p - r_f}{\sigma(r_p)} = S_p$$

9 Holdings-based analysis

With this type of analysis we do not just consider the portfolio returns, we use the complete records of the asset composition of the portfolios.

What can this achieve?

- It may alleviate the sensitivity of returns bases measures to choice of benchmark (the Roll critique).
- This approach may deal with nontrivial shifts in style allocations.
- One can look at performance before trading costs (which are incorporated in returns).
- One can decompose the sources of value added by a manager.
- Holdings-based analysis leads to more precise identification of manager ability, as observing performance on a security-by security basis increases the number of observations of ability.

Generally, holdings-bases measures looks at the covariance between lagged weights and current returns.

$$PHM_t = \text{cov}(w_{t-1}, R_t)$$

The intuition is simple: A skilled manager will have portfolio weights that move in the same direction as future returns.

To implement this, consider the method proposed by Grinblatt and Titman (1993), which calculate the monthly performance measure

$$GT_t = \sum_j (w_{j,t-1} - w_{j,t-2})R_{j,t}$$

This is averaged across time to find a single measure for the analysis period.

$$GT = \sum_{t=3}^{T} G_t$$

9.0.1 Stochastic Discount Factors and weight measures

We can use the stochastic discount factor approach to get some intuition about what is going on when we do this. Start with the general relationship

$$E_t[m_{t+1}R_{t+1}|Z_t] = 1$$

where R_{+1} is the vector of primitive asset returns, m is the stochastic discount factor, and Z_t is conditioning information.

For a given portfolio p, Alpha is calculated as

$$\alpha_p = E_t[m_{t+1}R_{p,t+1}|Z_t] - 1$$

Now, an asset manager chooses a set of weights w_t using information available at time t. Think of the weights as a function of the asset manager's information set Ω_t

$$w_t = w_t(\Omega_t)$$

The next period portfolio return $R_{p,t+1}$ is then

$$R_{p,t+1} = w_t(\Omega_t)R_{t+1}$$

Plugging this into the alpha calculation

$$\alpha_p = E_t[m_{t+1} w_t(\Omega_t) R_{t+1} | Z_t] - 1$$

Now, use the definition of covariance

$$\text{cov}(m_{t+1} R_{t+1}, w_t(\Omega_t)) = E[m_{t+1} R_{t+1} w_t(\Omega_t)] - E[m_{t+1} R_{t+1}] E[w_t(\Omega_t)]$$

From the fundamental pricing relation

$$E[m_{t+1} R_{t+1}] = 1$$

which means that the second term in the covariance is equal to 1 (the sum of weights is equal to one), and we can express alpha as

$$\alpha_p = \text{cov}(m_{t+1} R_{t+1}, w_t(\Omega_t) | Z_t)$$

The interpretation is the the alpha is the covariance between the weights with the risk-adjusted returns of the assets, conditional on the asset managers information, summed across assets.

With this formulation one easily concludes that if the asset manager only uses public information, Ω_t is a subset of Z_t, and the alpha should be zero.

10 Summarizing – performance evaluation

Performance evaluation: Does the return on an investment justify its risk?

Classical Performance evaluation.

- Sharpe Ratio

$$S_p = \frac{r_p - r_f}{\sigma_p}$$

- Treynor Ratio

$$T_p = \frac{r_p - r_f}{\beta_p}$$

- Jensen's alpha

$$\alpha_p = r_p - (r_f + \beta_p (r_m - r_f))$$

- Appraisal Ratio

$$AR_p = \frac{\alpha_p}{\sigma(e_p)}$$

Additional empirical measurement

- Alpha with alternative performance measures.
- Market Timing
- Modigliani's M^2 measure
- Relative performance evaluation – information ratio
- Covariance (holdings based) measures.

11 Readings

Textbook treatments: Investment textbooks like Bodie, Kane, and Marcus (2021)

Recent academic summaries: Aragon and Ferson (2006), Ferson (2010) and Wermers (2011)

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