PROBLEM SET: Mean Variance (revision)

Exercise 1.

Choosing asset [5]

Consider the following probability distributions for the returns of two assets 1 and 2.

r_1	Prob(r_1)	r_2	Prob(r_2)
10	0.1	22	0.2
25	0.4	23	0.5
30	0.3	24	0.2
32	0.2	50	0.1

1. Suppose you could only invest in either 1 or 2. Suppose also that you choose on the basis of mean-variance efficiency. Which asset would you pick?

2. Suppose now that you choose on the basis of expected utility maximization, and $U(w) = \ln(w)$. Which asset will you now pick?

Exercise 2.

Portfolio risk [1]

Assume that every asset has the same expected return. Furthermore, all assets have the same covariance with each other. As the number of assets in the portfolio grows, which becomes more important: Variance or covariance? Why?

Exercise 3.

Norwegian Stocks [6]

You want to combine the three stocks Norsk Hydro, Orkla and Statoil. Using historical monthly return data from 2001 to 2008, you estimate the historcal monthly return as

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0.0093
0.0083
0.0110
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and the covariance matrix as

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0.0086 0.0048 0.0052
0.0048 0.0074 0.0029
0.0052 0.0029 0.0057
```

Suppose the historical average returns can be used to estimate future expected returns.

1. For the following portfolios, calculate the portfolio expected return and standard deviation:

w_{Hydro}	0.5	0	1/3
w_{Orkla}	0.5	0.5	1/3
$w_{Statoil}$	0	0.5	1/3

2. Calculate the correlations between the three assets.

Exercise 4.

Suppose you hold a portfolio of two stocks. The relevant information on these two stocks is given below.

Stock	Weight	Variance	Expected return
1	0.6	0.04	0.12
2	0.4	0.09	0.20

1. Compute the expected return and variance of your portfolio assuming ρ_{12} is 0, -1 and 1.
2. Sketch how the portfolio variance and expectation would vary for these three cases.
3. Find the set of portfolio weights that minimizes the portfolio variance