

PROBLEM SET: Interest

**Exercise 1.**

[1]

The effective annual rate (EAR) for a loan with a stated APR of 8% compounded monthly is closest to:

- A) 7.72%.
- B) 8.00%.
- C) 8.30%.
- D) 8.66%.

**Exercise 2.**

Your bank is paying 9% nominal annual interest, with monthly compounding. If you deposit kr 2,500 now, how much will it be worth in one year?

- 1. kr 2,335
- 2. kr 2,735
- 3. kr 3,885
- 4. kr 6,050
- 5. I choose not to answer.

**Exercise 3.**

*EZ and Sleasy* [2]

EZ Credit is offering personal loans at 15% compounded quarterly. Sleasy Savings is offering a rate of 14.5% compounded monthly.

- 1. For a potential borrower, which opportunity is more attractive?

**Exercise 4.**

*Borrowing* [2]

You must pay a creditor \$6,000 one year from now, \$5,000 two years from now, \$4,000 three years from now, \$2,000 four years from now, and a final \$1,000 five years from now. You would like to restructure the loan into five equal annual payments due at the end of each year. The agreed interest rate is 6% (compounded annually).

- 1. What is the annual payment when the loan is restructured as five equal annual payments?

**Exercise 5.**

*Mr Miser* [1]

Mr Miser, who is 35 years old, has just inherited €5,000 and decides to use the windfall towards his retirement. He places the money in a bank which promises a return of 10% per year until his planned retirement at age 65.

- 1. If his funds earn 10% interest (compounded annually), how much will he have at retirement?

**Exercise 6.**

*Bahamas* [2]

Suppose you are saving for a trip to the Bahamas in two years and will need \$2000 at that time. The rate at which you can invest is 10%.

1. How much will you need to invest today to have enough money to make your trip two years from now?

**Exercise 7.**

*Lottery* [4]

You have just won the state lottery and are given the choice of receiving \$1 million immediately or \$100,000 per year for the next 20 years. The first of these payments is to be paid immediately, with subsequent payments made at one year intervals. The opportunity cost of capital is  $r = 8\%$ .

1. Which payment alternative should you choose?

**Exercise 8.**

Your bank account pays interest with an EAR of 5%. What is the APR quote for this account based on semiannual compounding? What is the APR with monthly compounding?

**Exercise 9.**

*Acme* [4]

You are head of cash management for Acme Industries. The current 3 month commercial paper rate is 6.5% per year, compounding quarterly. The current 6 month rate is 7.1% per year compounding quarterly. (Note - Commercial Paper is very short term zero coupon debt issued by large corporations. It is very low risk). Acme expects to receive a 10 million dollar payment in 6 months from a large customer. Unfortunately, it needs the money in three months to proceed on its plans to modernize a plant. Your corporate economics department is predicting short term interest rates will rise over the next two months.

1. Describe a series of transactions Acme could make today to guarantee an interest rate at which it could borrow against the \$10 million over the 3 month period prior to receipt of the funds.
2. What is the guaranteed rate of interest? (Show your calculations) Compound quarterly.

**Exercise 10.**

*Term structure* [2]

The term structure is upward sloping.

1. Is the yield on a ten year coupon bond higher than the ten year zero rate.

**Exercise 11.**

[2]

The six month zero rate is 8% with semi-annual compounding. The price of a one-year bond that provides a coupon of 6% per annum semi-annually is 97.

1. What is the one year zero rate? (Use discrete, annual compounding)

## Solutions

PROBLEM SET: Interest

### Solution to Exercise 1.

[1]

Answer: C

Explanation:  $EAR = (1 + APR/k)^k - 1 = (1 + .08/12)^{12} - 1 = .083$  or 8.3%

### Solution to Exercise 2.

2,735

(b) is correct

### Solution to Exercise 3.

*EZ and Sleasy* [2]

1. Let us calculate the effective interest rate for the two borrowing opportunities: EZ Credit:

$$r = \left(1 + \frac{0.15}{4}\right)^4 - 1 = 15.86\%$$

Sleasy Savings

$$r = \left(1 + \frac{0.145}{12}\right)^{12} - 1 = 15.50\%$$

Take Sleasy, it has a lower annualized interest rate.

### Solution to Exercise 4.

*Borrowing* [2]

First calculate the present value

$t$	$C_t$
0	0
1	6000
2	5000
3	4000
4	2000
5	1000

$$PV = \frac{6000}{(1+0.06)^1} + \frac{5000}{(1+0.06)^2} + \frac{4000}{(1+0.06)^3} + \frac{2000}{(1+0.06)^4} + \frac{1000}{(1+0.06)^5} = 15800.3$$

Then spread it out over five years using the five year annuity factor.

Annuity factor( $r=0.06$   $n=5$ ) = 4.21236

$$C = \frac{15800}{4.212} = 3751$$

### Solution to Exercise 5.

*Mr Miser* [1]

$$5000(1.1)^{30} = 87247$$

### Solution to Exercise 6.

*Bahamas* [2]

1. Present value

$$\begin{aligned}FV_2 &= \$2000 \\r &= 10\% \\PV &= \frac{\$2000}{1.10^2} = 1653\end{aligned}$$

### Solution to Exercise 7.

Lottery [4]

$$1. \quad \begin{array}{cccccccc}t = & 0 & 1 & 2 & \dots & 18 & 19 \\C_t = & 100 & 100 & 100 & \dots & 100 & 100\end{array}$$

$$\begin{aligned}PV &= 100,000 + \sum_{t=1}^{19} \frac{100,000}{(1.08)^t} \\&= 100,000 \\&\quad + 100,000 \left[ \frac{1}{0.08} - \frac{1}{0.08 \cdot 1.08^{19}} \right] \\&= 100,000 + 100,000 \cdot 9.6036 \\&= 1,060,360\end{aligned}$$

Take the alternative that pays you \$100,000 in 20 equal annual installments.

### Solution to Exercise 8.

Using the formula for converting from an EAR to an APR quote:

$$\left(1 + \frac{APR}{k}\right)^k = 1.05$$

Solving for the APR:

$$APR = (1.05^{\frac{1}{k}} - 1)k$$

With annual payments  $k = 1$ , so  $APR = 5\%$

With semiannual payments  $k = 2$ , so  $APR = 4.939\%$

With monthly payments  $k = 12$ , so  $APR = 4.889\%$

### Solution to Exercise 9.

Acme [4]

1. Acme can today

- Borrow 10 mill for 6 months, interest 7.1%.
- Invest 10 mill for 3 months, interest 6.5%.

This sequence of transactions fixes the interest for the remaining 3 months.

2. Find the implied interest rate from:

$$\left(1 + \frac{0.071}{4}\right)^2 = \left(1 + \frac{0.065}{4}\right) \left(1 + \frac{r}{4}\right)$$

solve for  $r$ :

$$\left(1 + \frac{r}{4}\right) = \frac{\left(1 + \frac{0.071}{4}\right)^2}{\left(1 + \frac{0.065}{4}\right)}$$

giving

$$r = 7.7\%$$

### Solution to Exercise 10.

*Term structure* [2]

1. No, the ten year zero rate is higher.

### Solution to Exercise 11.

[2]

Semi-annual compounding means interests are paid twice a year, so the annual coupon of 6% is paid in twice-annual installments of  $6\%/2 = 3\%$ . Similarly, the 8% annual rate translates to 4% for the half-year.

1. Finding the one year zero rate:

$$P = 97 = \frac{3}{1.04} + \frac{103}{1 + r(0, 1)}$$

$$97 - \frac{3}{1.04} = \frac{103}{1 + r(0, 1)}$$

$$r(0, 1) = 9.44\%$$