Interest Rates

Introduction

This lecture: *interest rates*.

Finance: interest rates appear in many settings.

$\rightarrow$ view of *complexity*

But: there is a simple, underlying structure.

Key points:

- Economic interpretation of interest rates – it is the *price* for moving consumption between periods.
- Key *usage* of interest rates in corporate finance: Discounting future cash flows.
  - Need to know the rules for discounting.

Introduction ctd

Key points:

- Key *problem* with interest rates: There are many of them
  Interest rates vary with
  - *horizon* of cash flows
  - *risk* of cash flows
  - and change over time.
- Key *skill* to be acquired: Use data from financial markets to find the *relevant* interest rate.

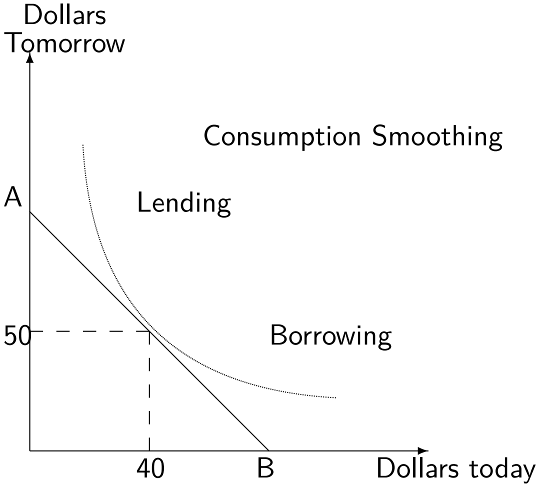
Exercise

Suppose a consumer only lives for two periods, with projected income today of $40, and $50 next period.

1. What financial service can help this consumer moving consumption between periods?
2. What is this consumers maximum consumption today?
3. What is this consumers maximum consumption next period?

Exercise solution

Financial service: The ability to borrow and lend at an interest rate r.



Note in this picture that the slope of the "budget constraint" equals $-(1 + r)$.

Exercise solution

Point A represents the maximum amount of *future* consumption.

$$A = 50 + 40(1 + r)$$

If for example the interest rate is 25%,

$$A = 50 + 40(1 + r) = 50 + 40 \cdot 1.25 = 100$$

Point B represent the maximum amount of *current* consumption.

$$B = 40 + \frac{50}{1 + r}$$

If for example the interest rate is 25%,

$$B = 40 + \frac{50}{1 + r} = 40 + \frac{50}{1.25} = 80$$

With well-functioning, competitive capital markets any possible consumption pattern along the line AB is possible.

Financial fixed income markets

The financial markets offer a bewildering array of opportinities to transfer money between periods.

From the point of view of consumers:

- Borrowing
  - From a bank - “fixed” interest rate depending on probability of repayment
  - Consumer debt
  - Credit card debt
  - ...
- Lending / Investing
  - Bank placements
  - Fixed income assets
  - ...

Financial fixed income markets

From the point of view of corporations:

- Borrowing
  - Bond issuance
  - Bank borrowing
  - ...
- Lending / Investing
  - Bank placements
  - Buy Fixed income assets
  - ...

Summary of discounting

1. Method for evaluating current value of stream of future cashflows
   Present Value
2. Subtracting present cost
   Net Present Value
   or: Value added

Summary of discounting

Future Value

$$FV_t = PV(1+r)^t$$

Present Value

$$PV = \frac{FV_t}{(1+r)^t}$$

Discount factor

$$d_t = 1/(1+r)^t$$

Summary of discounting

Multiple cash flows – present value additive

$$PV = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t}$$

$$NPV = PV - C_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t} - C_0$$

The internal rate of return: the discount rate r that makes the $NPV = 0$.

Summary of discounting - special cases

A *perpetuity*: fixed income for each year for the indefinite future.

$$PV = \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^t} \; = \; \frac{C}{r}$$

A *growing perpetuity*: a fixed income for each year that grows at a constant rate g.

$$PV = \sum_{t=1}^{\infty} \frac{C_1(1+g)^{t-1}}{(1+r)^t} \; = \; \frac{C_1}{r-g}$$

An *annuity*: pays a fixed (constant) amount each year for a specified finite number of years (T).

$$\begin{aligned} PV &= \sum_{t=1}^{T} \frac{C}{(1-r)^t} \\ &= C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right] \end{aligned}$$

An addition wrinkle: Multiple payment dates during the year

Interest rates are sometimes paid more than once a year.

Exercise

Corporations A and B have both issued bonds with face values of 100 and interest rates of 10%. Both bonds mature in a year. Bond A pays coupon once a year, bond B pays coupon twice a year.

- Which of these bonds have a higher price?
- What would the rate on bond B have to be to make a buyer indifferent between bond A and bond B?

Exercise Solution

- Which is more valuable?
  Consider the cash flows from these two bonds:
  Bond A:
  Coupon 10 after a year
  Bond B:
  Coupon 5 after a half year.
  Coupon 5 after a year.
  One always prefer to get cash as soon as possible.
  With bond B you get more money earlier, it is more valuable.

Exercise Solution

- You want to find what coupon rate would make you indifferent.
  Let r_2 be the (biannual) interest rate in question. It need to satisfy (why)?

  $$(1 + r_2)(1 + r_2) = (1 + r_1)$$

  $$r_2 = \sqrt{1 + 0.1} - 1 = 0.489 = 4.89\%$$

  The stated interest rate on the bond paying interest twice a year would have to be

  $$2 * r_2 = 9.78\%$$

Translating interest rates

A key difference: The *period* the interest rate is valid for.
Compounding : frequency with which interest is paid.

$$FV_t = PV \left(1 + \frac{r}{m}\right)^{mt}$$

where

- $r =$ Annual interest rate.
- $m =$ number of compounding intervals per year.

Where do spot rates/discount factors come from?

Example
Treasury securities, for example those issued by the Norwegian government.
Current bond prices (as of January 20, 2021) for the outstanding debt of the Norwegian state.

Where do spot rates/discount factors come from?

Norwegian treasury bond prices, January 20, 2021

EURONEXT Live markets

Navn, ISIN, ticker, indeks

Søk på nettstedet

NB

NAVN	UTSTEDER	MARKED	SLUTTDATO	KUPONGRENTE	SISTE	%	DATO/TID
NST474	NO0010572878	XOSL	2021-05-25	3.75%	% 101,731	-0,22%	30 Nov 2020
NST475	NO0010646813	XOSL	2023-05-24	2.0%	% 104,249	-0,04%	02 Dec 2020
NST476	NO0010705536	XOSL	2024-03-14	3.0%	% 108,76	0,03%	20 Nov 2020
NST477	NO0010732555	XOSL	2025-03-13	1.75%	% 105,37	-0,11%	04 Dec 2020
NST478	NO0010757925	XOSL	2026-02-19	1.5%	% 104,715	-0,41%	02 Dec 2020
NST479	NO0010786288	XOSL	2027-02-17	1.75%	% 106,665	0,07%	03 Dec 2020
NST480	NO0010821598	XOSL	2028-04-26	2.0%	% 109,285	-1,15%	30 Nov 2020
NST481	NO0010844079	XOSL	2029-09-06	1.75%	% 108,185	-0,04%	30 Nov 2020
NST482	NO0010875230	XOSL	2030-08-19	1.375%	% 105,025	0,25%	25 Nov 2020

Where do spot rates/discount factors come from?

To find implied *interest rates*:
What interest rate would deliver the quoted *bond price*?

$$B_0 = \sum_{t=1}^{T} \frac{E[C_t]}{(1+r_t)^t} + \frac{E[F_T]}{(1+r_T)^T}$$

Alternatively:
What *discount factors* d_t, would deliver the quoted bond prices

$$B_0 = \sum_{t=1}^{T} d_t E[C_t] + d_T E[F_T]$$

or

$$B_0 = \sum_{t=1}^{T} E[C_t] \left(\frac{1}{(1+r_t)^t} \right) + E[F_T] \left(\frac{1}{(1+r_T)^T} \right)$$

Translating:

$$d_t = \left(\frac{1}{(1+r_t)^t} \right)$$

Simpler example follows:

Exercise - estimating interest rates

A two-year Treasury bond with a face value of 1000 and an annual coupon payment of 8% sells for 982.50. A one-year T bill, with a face value of 100, and no coupons, sells for 90. Compounding is discrete, annual.

Given these market prices,

1. Find the implied one and two year interest rates.

Exercise Solution - estimating interest rates

To find the interest rates, first find the prices d_1 and d_2 of one dollar received respectively one and two years from now.
These two discount factors will produce the current prices, and hence it satisfies the following set of equtions.
Discount factors (prices):

$$\left[\begin{array}{l} 982.50 = d_1 80 + d_2 1080 \\ 90 = d_1 100 \end{array}\right]$$

Exercise Solution - estimating interest rates

Solving these equations we find prices d_1 and d_2

$$d_1 = \frac{90}{100} = 0.90$$

$$982.50 = 0.90 \times 80 + d_2 1080$$

$$d_2 = \frac{982.50 - 0.90 \times 80}{1080}$$

$$d_2 = 0.843$$

Summarizing

$$\left[\begin{array}{l} d_1 = 0.9 \\ d_2 = 0.84 \end{array} \right]$$

Exercise Solution - estimating interest rates

Then, translate from discount factors to interest rates:

$$d_1 = \frac{1}{1 + r_1}$$

$$r_1 = \frac{1}{d_1} - 1 = \frac{1}{0.9} - 1 = 0.11111 \approx 11\%$$

$$r_2 = \frac{1}{\sqrt{0.84}} - 1 = 0.09108945118 \approx 9\%$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \approx \begin{bmatrix} 11\% \\ 9\% \end{bmatrix}$$

Exercise Solution - estimating interest rates

For the technically interested (engineering students), these calculations is most compactly done in a matlab-like environment:

```
>> B=[982.50 90]
B =
   982.500    90.000
>> C=[80 1080;100 0]
C =
     80   1080
    100      0
>> d=inv(C)*B'
d =
   0.90000
   0.84306
>> r1=1/d(1)-1
r1 =  0.11111
>> r2=1/sqrt(d(2))-1
r2 =  0.089110
```

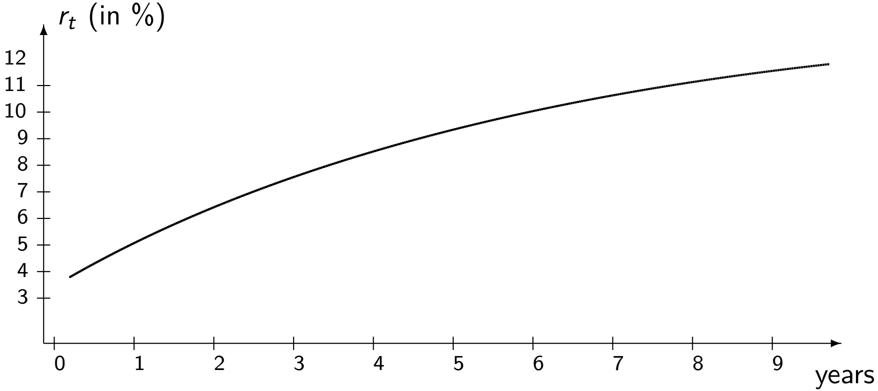
Term Structure of Interest Rates

The *Term structure of interest rates*: The relationship between the spot rate and the date of the cash flow.

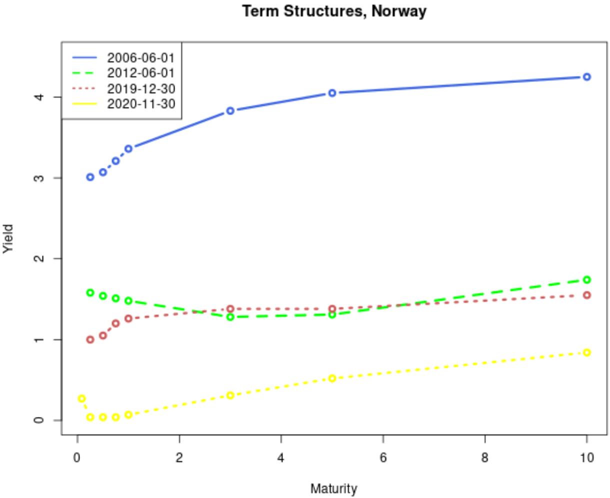
Alternatively, the relationship between the yield on a pure discount bond and the date it matures.

Typically present term structure as a plot of spot interest rates (r_t) against maturity (t).

Example Term Structure of Interest Rates

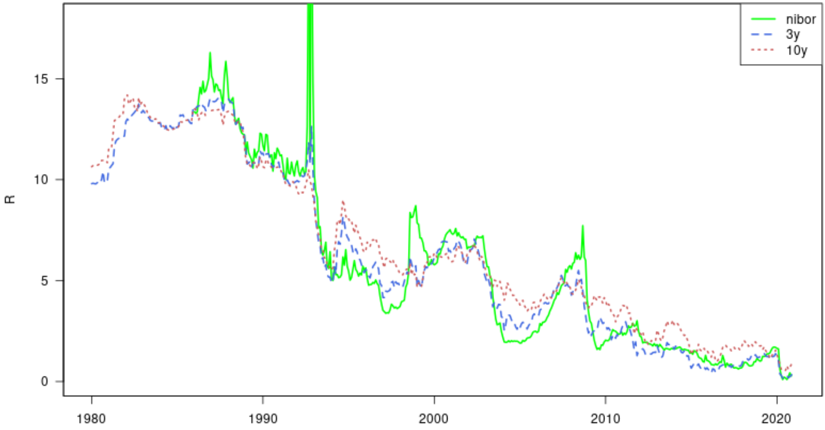


The term structure of Norwegian risk free rates



The figure plots interest rates for NIBOR(1 month) and treasury rates for 3 and 6 months and 1, 5 and 10 year rates. NIBOR data from Norges Bank and Oslo Børs. Treasury data from Norges Bank. Term structures for three different dates: 1 jun 2006, 1 jun 2012 and 30 dec 2019.

The time series of term structures of Norwegian risk free rates



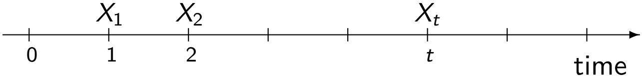
Interest rate on 1 month NIBOR and 3 and 10 year Norwegian Treasuries. Treasury

The term structure as prices of future payments

Important Properties of future cash flows

- *amounts* (X_t)
- *dates* (t) at which the amounts are paid.

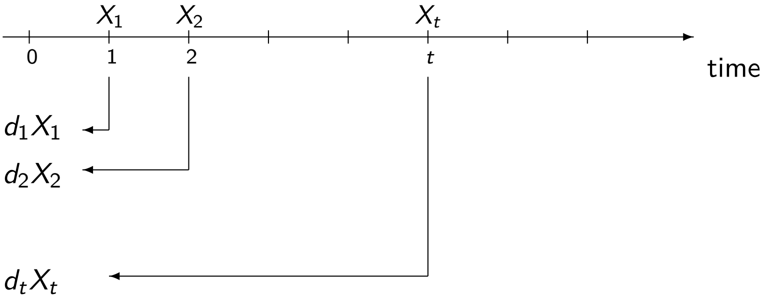
For example, a US Government Treasury bill is a promise to pay USD 1000 at a certain future date.



The term structure as prices of future payments

To value, need current *price* d_t of receiving one dollar at time t in the future.

$$PV = \sum_{t=1}^{T} d_t X_t.$$



Price d_t - discount factor.

Example A US Government Treasury bill pays USD 1000 263 days from now.
Suppose today's value of one dollar received 263 days from now is USD 0.945.
The Present Value of the Treasury Bill is

$$PV = 0.945 \times 1000 = 945.$$

Valuing Fixed Income Securities

A fixed income security is a security that offers a predetermined sequence of future payments. The typical fixed income security is a bond.

Example

A US Government Bond (T Bond) with maturity 10 years and stated interest 7% is a promise to pay interest of 3.5% of the principal twice a year for 10 years, and repay the principal after 10 years.

Valuing bonds: the present value of the promised sequence of payments, using either prices P_t or interest rates r_t.

Determinants of interest rates

Three ideas:

- *Level* of interest rates
- *Time Preferences*.
- *Risk*.

Real and nominal interest rates

The realised *real* return over a given year:

$$r = \frac{(1 + r_n)}{1 + i} - 1$$

where

- r is the *real* return,
- r_n is the *nominal* return and
- i is the *inflation rate*.

Rearranging:

$$r_n = r + i + ir$$

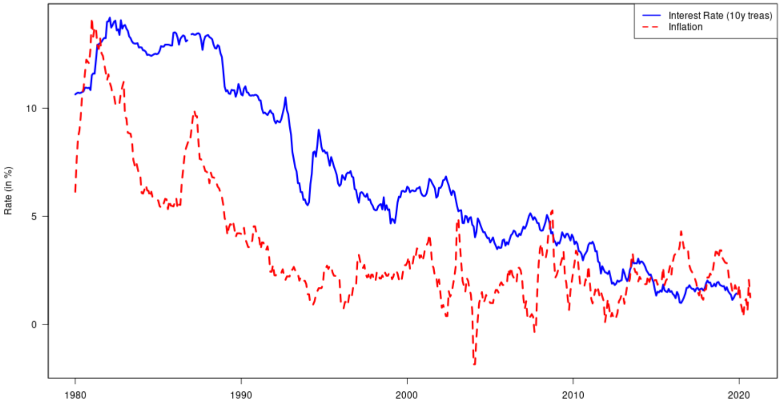
Typically, the last term is small, can be ignored

$$r_n \approx r + i$$

If investors expect inflation to be high, we would expect interest rates to be high as well.

Interest and Inflation in Norway

Norway, long term treasury interest rate and inflation



Interest rate on 10 year Norwegian Treasuries. Inflation is annual inflation. Both measured at monthly frequencies. Treasury data from Norges Bank. Inflation from SSB.

Interest rates and risk

Interest rates increases with risk.
Example:
Least risky: Treasury securities
More risky: Corporate Bonds
The difference in interest between the typical corporate bond and a treasury bond – “default premium”.

US Default premium: Moody Baa vs 10y treasuries

Summary – interest rates

Interest rates: The *price* for moving consumption between periods.
Discounting:

- Present value/ Future value
- The *frequency* of adding interest.
- Interest rates differ by the *holding period* (term structure of interest rates)
- Interest rates differ by the *riskiness* of issuer
  - Treasury securities – riskless
  - Corporate bonds – risky
  - Default premium: Difference risky/riskless borrowing

Summary – interest rates

What is the current interest rate?

- Inferred from market prices.

Determinants of the level of interest rates

- Inflation
- Macroeconomic policy

Opportunity cost of capital

- What would you earn on a *comparable* investment?