Interest Rates

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1 Introduction

This lecture is about *interest rates*.

In finance we have to deal with interest rates in many different settings, which lead to a view among many that this is something very complicated.

The problem is that interest rates have many usages, and it is hard to see the simple, underlying structure.

Key points following:

- Economic interpretation of interest rates – it is the *price* for moving consumption between periods.
- Key *usage* of interest rates in corporate finance: Discounting future cash flows.
  - Need to know the rules for discounting.
- Key *problem* with interest rates: There are many of them

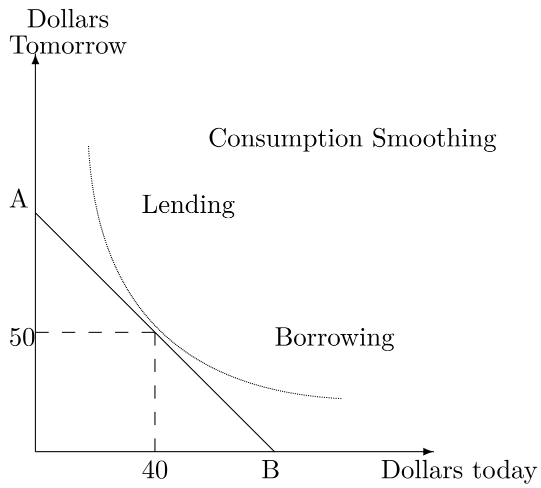
  Interest rates vary with
  - *horizon* of cash flows
  - *risk* of cash flows
  - and change over time.
- Key *skill* to be acquired: Use data from financial markets to find the *relevant* interest rate.

2 The role of interest rates in consumption

Start from the point of view of consumers: For them, capital Markets smoooth consumption

The existence of a well-functioning, competitive capital market opens up borrowing and lending opportunities that remove the obligation of individuals to match their consumption with their incomes.

Let's look at an individual who only lives for two periods. He has an income today of $40, and $50 next period, and can borrow and lend at an interest rate of $r = 25\%$.



Note in this picture that the slope of the "budget constraint" equals $-(1+r)$.

Point A represents the maximum amount of *future* consumption.

$$A = 50 + 40(1+r) = 50 + 40 \cdot 1.25 = 100$$

Point B represent the maximum amount of *current* consumption.

$$B = 40 + \frac{50}{1+r} = 40 + \frac{50}{1.25} = 80$$

With well-functioning, competitive capital markets any possible consumption pattern along the line AB is possible.

Question: Can you illustrate graphically your own current situation? Are you a net borrower or a net lender?

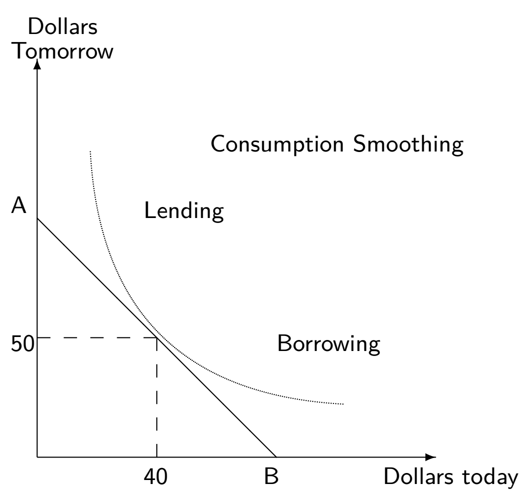
Exercise 1.

Suppose a consumer only lives for two periods, with projected income today of $40, and $50 next period.

1. What financial service can help this consumer moving consumption between periods?
2. What is this consumers maximum consumption today?
3. What is this consumers maximum consumption next period?

Solution to Exercise 1.

1. The ability to borrow and lend at an interest rate r.



2.

Note in this picture that the slope of the "budget constraint" equals $-(1+r)$.
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If for example the interest rate is 25%,

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3 Financial fixed income markets

The financial markets offer a bewildering array of opportinities to transfer money between periods.

From the point of view of consumers:

- Borrowing
  - From a bank - "fixed" interest rate depending on probability of repayment
  - Consumer debt
  - Credit card debt
  - ...
- Lending / Investing
  - Bank placements
  - Fixed income assets
  - ...

From the point of view of corporations:

- Borrowing
  - Bond issuance
  - Bank borrowing
  - ...
- Lending / Investing
  - Bank placements
  - Buy Fixed income assets
  - ...

All these various possibilities to transform money between now and some point in the future comes with its own terms of trade.

4 An addition wrinkle: Multiple payment dates during the year

Interest rates are sometimes paid more than once a year.

Exercise 2.

Corporations A and B have both issued bonds with face values of 100 and interest rates of 10%. Both bonds mature in a year. Bond A pays coupon once a year, bond B pays coupon twice a year.

- Which of these bonds have a higher price?
- What would the rate on bond B have to be to make a buyer indifferent between bond A and bond B?

Solution to Exercise 2.

- Consider the cash flows from these two bonds:
  Bond A:
  Coupon 10 after a year
  Bond B:
  Coupon 5 after a half year.
  Coupon 5 after a year.
  One always prefer to get cash as soon as possible.
  With bond B you get more money earlier, it is more valuable.
- You want to find what coupon rate would make you indifferent.
  Let r_2 be the (biannual) interest rate in question. It need to satisfy

$$(1+r_2)(1+r_2) = (1+r_1)$$

$$r_2 = \sqrt{1+0.1} - 1 = 0.489 = 4.89\%$$

  But note that this is an biannual rate, which must be translated to an annual one.
  The stated interest rate on the bond paying interest twice a year would have to be

$$2 * r_2 = 9.78\%$$

Compounding refers to the frequency with which interest is paid. The more frequent the compounding of interest, the higher will be the future value of a current investment of \$1 and the lower will be the present value of a future payment of \$1.

These values are computed according to the following formula:

$$FV_t = PV\left(1 + \frac{r}{m}\right)^{mt}$$

where

- r = Annual (Stated) interest rate.
- m = number of compounding intervals per year.

5 Where do spot interest rates/discount factors come from?

How would you estimate the current interest rate?

Take an example: Treasury securities, for example those issued by the Norwegian government. Figure 1 shows the current bond prices (as of January 20, 2021) for the outstanding debt of the Norwegian state.

The implied *interest rates* are found from the fact that the given *bond price* is the present value of the coupons and face value:

$$B_0 = \sum_{t=1}^{T} \frac{E[C_t]}{(1+r_t)^t} + \frac{E[F_T]}{(1+r_T)^T}$$

Figure 1 Norwegian treasury bond prices, January 20, 2021

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NAVN	UTSTEDER	MARKED	SLUTTDATO	KUPONGRENTE	SISTE	%	DATO/TID
NST474	NO0010572878	XOSL	2021-05-25	3.75%	% 101,731	-0,22%	30 Nov 2020
NST475	NO0010646813	XOSL	2023-05-24	2.0%	% 104,249	-0,04%	02 Dec 2020
NST476	NO0010705536	XOSL	2024-03-14	3.0%	% 108,76	0,03%	20 Nov 2020
NST477	NO0010732555	XOSL	2025-03-13	1.75%	% 105,37	-0,11%	04 Dec 2020
NST478	NO0010757925	XOSL	2026-02-19	1.5%	% 104,715	-0,41%	02 Dec 2020
NST479	NO0010786288	XOSL	2027-02-17	1.75%	% 106,665	0,07%	03 Dec 2020
NST480	NO0010821598	XOSL	2028-04-26	2.0%	% 109,285	-1,15%	30 Nov 2020
NST481	NO0010844079	XOSL	2029-09-06	1.75%	% 108,185	-0,04%	30 Nov 2020
NST482	NO0010875230	XOSL	2030-08-19	1.375%	% 105,025	0,25%	25 Nov 2020

Alternatively, we can think in terms of *discount factors* d_t, where d_t is the current price of a promised payment of one dollar/NOK/... at a future date t

$$B_0 = \sum_{t=1}^{T} d_t E[C_t] + d_T E[F_T]$$

If we rewrite the expression involving spot rates above as

$$B_0 = \sum_{t=1}^{T} E[C_t] \left(\frac{1}{(1+r_t)^t} \right) + E[F_T] \left(\frac{1}{(1+r_T)^T} \right)$$

we see that the price of a zero coupon bond, the discount factor, is

$$d_t = \left(\frac{1}{(1+r_t)^t} \right)$$

Let us take a bit simpler example, showing how the estimation of the discount factors work.

Exercise 3.

A two-year Treasury bond with a face value of 1000 and an annual coupon payment of 8% sells for 982.50. A one-year T bill, with a face value of 100, and no coupons, sells for 90. Compounding is discrete, annual.

Given these market prices,

1. Find the implied one and two year interest rates.

Solution to Exercise 3.

To find the interest rates, first find the prices d_1 and d_2 of one dollar received respectively one and two years from now. These two discount factors will produce the current prices, and hence it satisfies the following set of equtions.

1. Discount factors (prices):

$$\begin{bmatrix} 982.50 = d_1 80 + d_2 1080 \\ 90 = d_1 100 \end{bmatrix}$$

Solving these equations we find prices d_1 and d_2

$$d_1 = \frac{90}{100} = 0.90$$

$$982.50 = 0.90 \times 80 + d_2 1080$$

$$d_2 = \frac{982.50 - 0.90 \times 80}{1080}$$
$$d_2 = 0.843$$

Summarizing

$$\begin{bmatrix} d_1 = 0.9 \\ d_2 = 0.84 \end{bmatrix}$$

Then, translate from discount factors to interest rates:

$$d_1 = \frac{1}{1 + r_1}$$

$$r_1 = \frac{1}{d_1} - 1 = \frac{1}{0.9} - 1 = 0.11111 \approx 11\%$$

$$r_2 = \frac{1}{\sqrt{0.84}} - 1 = 0.09108945118 \approx 9\%$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \approx \begin{bmatrix} 11\% \\ 9\% \end{bmatrix}$$

For the technically interested (engineering students), these calculations is most compactly done in a matlab-like environment:

```
>> B=[982.50 90]
B =
   982.500    90.000
>> C=[80 1080;100 0]
C =
     80   1080
    100      0
>> d=inv(C)*B'
d =
   0.90000
   0.84306
>> r1=1/d(1)-1
r1 =  0.11111
>> r2=1/sqrt(d(2))-1
r2 =  0.089110
```

6 Term Structure of Interest Rates

The plot of spot interest rates (r_t) against maturity (t) is called the *term structure of interest rates.* The term structure can take a multitude of shapes. Typically, it is rising, but it can also be decreasing, or even “humped.”

Figure 2 shows an example term structure.

The prices d_t (and, hence r_t) are usually estimated from prices of government fixed income securities, such as US Treasury bills and US Treasury bonds.

6.1 The term structure of Norwegian risk free rates

Show two views of Norwegian term structure. Figure 3: Term structures on selected dates.

Figure 4: Time series of three different interest rates: NIBOR(1m), 3 and 10 year treasury rates.

Figure 2 Example Term Structure of Interest Rates

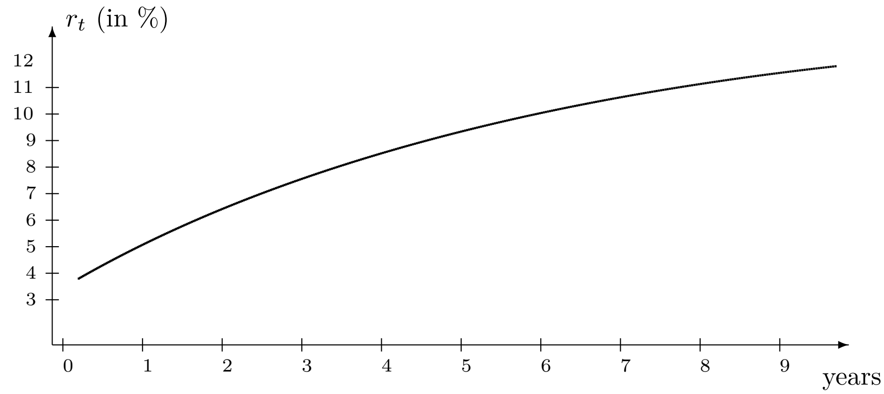
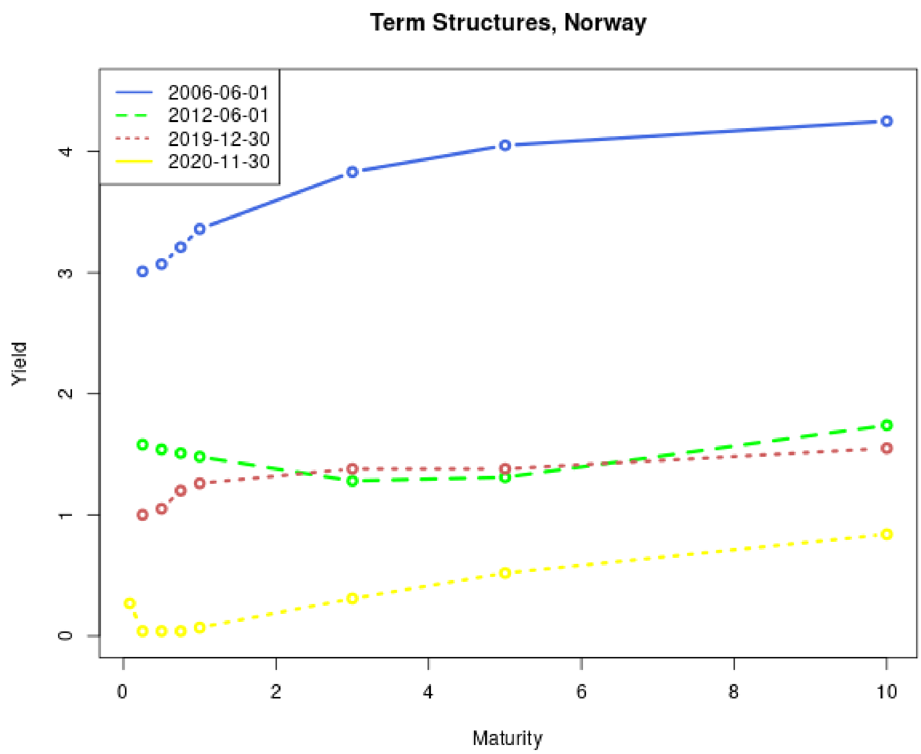
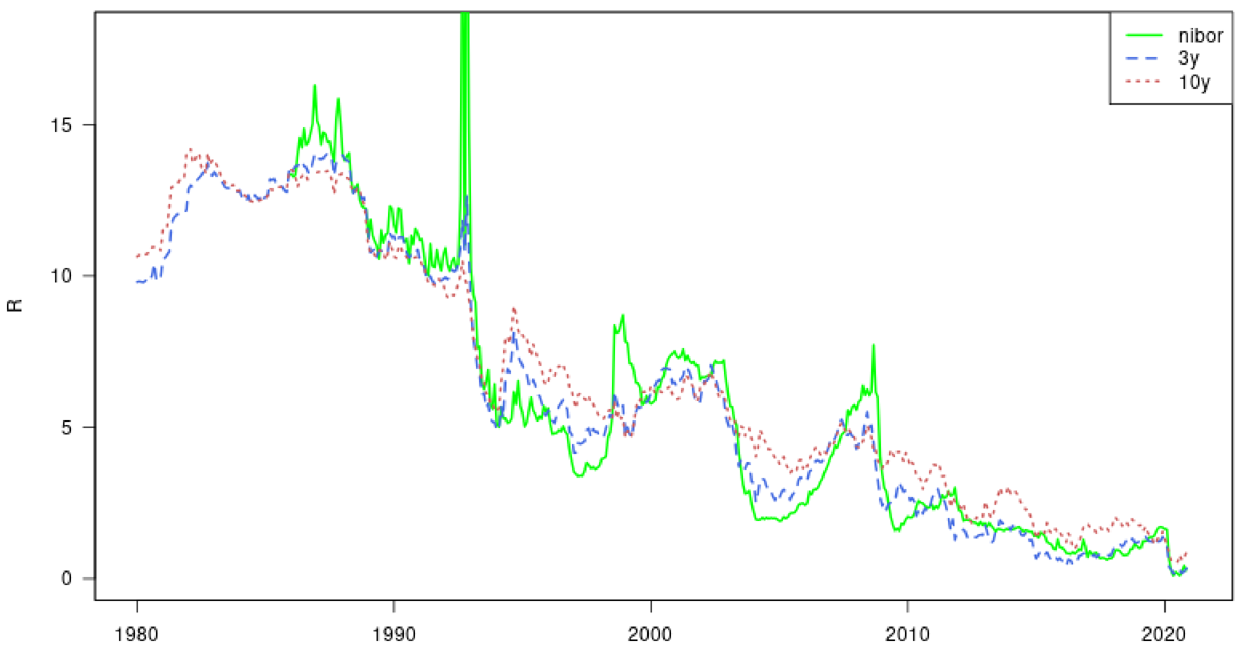


Figure 3 Selected term structure, Norwegian risk free rates



The figure plots interest rates for NIBOR(1 month) and treasury rates for 3 and 6 months and 1, 5 and 10 year rates. NIBOR data from Norges Bank and Oslo Børs. Treasury data from Norges Bank. Term structures for three different dates: 1 jun 2006, 1 jun 2012 and 30 dec 2019.

Figure 4 Norway, time series of term structure



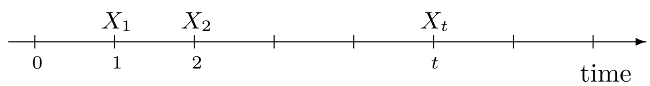
Interest rate on 1 month NIBOR and 3 and 10 year Norwegian Treasuries. Treasury data from Norges Bank. NIBOR data from Norges Bank and Oslo Børs.

7 The term structure as prices of future payments.

A basic unit of account in finance is a set of future cash flows. There are two important properties of these cash flows. One, the *amounts*. Two, the *dates* at which the amounts are paid.

For example, a US Government Treasury bill is a promise to pay USD 1000 at a certain future date. This is the typical example of a *risk free* security, one with no uncertainty as to both the amount of and timing of cash flow.

We concentrate on the valuation of a sequence of *certain* future cash flows. We use the symbol X_t for the amount X to be paid at a future date t, and we want to value a set of future cash flows:

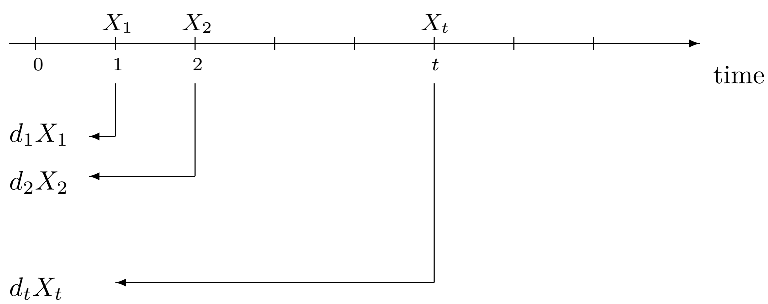


Given the set of dated cash flows, we define its Present Value (PV) as its value today.

Evaluating the PV is simplified by using the axiom of value additivity, since we can then split the problem into summing the values of the individual dated cash flows. The problem is then reduced to finding the value today of a cash flow X_t at some future date t. To do this we use the set of *prices* d_t today of receiving one dollar at time t in the future.

The PV of the entire stream is then:

$$PV = \sum_{t=1}^{T} d_t X_t.$$



Example: A US Government Treasury bill pays USD 1000 263 days from now. Today's value of one dollar received 263 days from now is USD 0.945. The Present Value of the Treasury Bill is

$$PV = 0.945 \times 1000 = 945.$$

This set of *prices* d_t today of receiving one dollar at time t in the future are important objects in finance. They are typically *estimated* from actual prices in financial markets.

Since most people are impatient, and would put more value today on receiving a dollar tomorrow than one year from now, you would expect the following property to hold:

$$d_1 > d_2 > d_3 > \dots$$

If $d_1 \leq 1$ this can also be shown to be an implication of the no free lunch assumption, which is left as an exercise.

8 Valuing Fixed Income Securities

A fixed income security is a security that offers a predetermined sequence of future payments. The typical fixed income security is a bond.

Example

A US Government Bond (T Bond) with maturity 10 years and stated interest 7% is a promise to pay interest of 3.5% of the principal twice a year for 10 years, and repay the principal after 10 years.

Valuing bonds should by now be straightforward. We need to find the present value of the promised sequence of payments, using either prices P_t or interest rates r_t.

9 Determinants of interest rates

Three ideas:

- *Level* of interest rates, primarily linked to inflation.
- *Time Preferences.*

  Savings possibilities

  - Invest at a fixed rate for two years.
  - Invest at a fixed rate for one year, and then see what the interest rate is for the next year.

  Shape of the term structure driven by such considerations.

  Primarily: Expectation about macroeconomic evolution.
- *Risk.*

  Safest (in nominal terms): Treasury securities.

  Riskier: Corporate debt.

9.1 Real and nominal interest rates

Bonds are nominal claims and inflation erodes the purchasing power associated with the payments they make. The realised *real* return on say a one–year T-bill over a given year is computed as follows.

$$r = \frac{(1 + r_n)}{1 + i} - 1$$

where r is the *real* return, r_n is the *nominal* return and i is the *inflation rate*. Rearranging the terms and canceling gives:

$$r_n = r + i + ir$$

Typically, the last term is small and often ignored, leaving

$$r_n \approx r + i$$

Thus, if investors expect inflation to be high, we would expect interest rates to be high as well.

The *Fisher Hypothesis* asserts that nominal interest will be set high enough to compensate investors for expected inflation.

$$r_n = r + E[i]$$

where r is the real rate of interest demanded by investors.

The Fisher hypothesis is similar to the Expectations hypothesis in that it ignores the *risk* that inflation may be different from what investors expect. Inflation imposes additional risk on the holders of nominal securities, and, therefore, we would expect them to require compensation for bearing that risk. Thus, we would expect nominal interest rates to be set even higher than the Fisher hypothesis suggest. In particular, we would expect

$$r_n = r + E[i] + \text{inflation risk premium}$$

9.2 Interest and Inflation in Norway

Figure 5 Norway, long term treasury interest rate and inflation

Interest rate on 10 year Norwegian Treasuries. Inflation is annual inflation. Both measured at monthly frequencies. Treasury data from Norges Bank. Inflation from SSB.

9.3 Interest rates and risk

Interest rates depend on risk. The more risky the future cash flows, the higher necessary expected return.

For example:

Least risky: Treasury securities – in nominal terms risk free, the government can always print more money.
More risky: Corporate Bonds – If the company goes bankrup, payment to corporate bonds reduced to what is left of values in the firm.

The difference in interest between the typical corporate bond and the treasury bond called the "default premium".

10 Summary – interest rates

Interest rates: The *price* for moving consumption between periods.

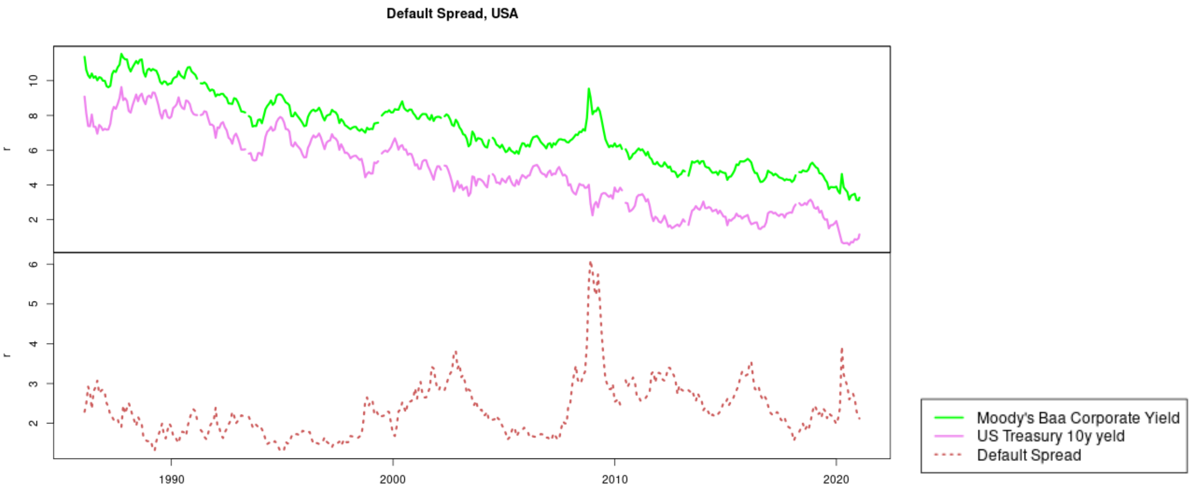
Discounting:

- Present value/ Future value

$$PV = \frac{FV}{(1+r)}$$

- The *frequency* of adding interest.

Figure 6 US Default premium: Moody Baa vs 10y treasuries



- Interest rates differ by the *holding period* (term structure of interest rates)
- Interest rates differ by the *riskiness* of issuer
  - Treasury securities – riskless
  - Corporate bonds – risky
  - Default premium: Difference risky/riskless borrowing

What is the current interest rate?

- Inferred from market prices.

Determinants of the level of interest rates

- Inflation
- Macroeconomic policy

Opportunity cost of capital

- What would you earn on a *comparable* investment?