PROBLEM SET: Discounting

Exercise 1. *Project* [2]

An investment project offers the following pattern of cash flows.

Time (t)	Cash Flow (C_T)
0	-$1000
1	500
2	750
3	250

The appropriate discount rate is 10%.

1. What is the NPV of the investment project?

Exercise 2. *Florida State Lottery (RWJ 4.6)* [2]

You have won the Florida state lottery. Lottery officials offer you the choice of the following alternative pay-outs:

a) $10,000 one year from now.

b) $20,000 five years from now.

1. Which should you choose if the discount rate is
   (a) 0%.
   (b) 10%.
   (c) 20%.
2. Which rate makes the options equally attractive to you?

Exercise 3. *Project* [3]

A project is planned to give the following cash flows for the next 4 years:

$t =$	1	2	3	4
$C_t =$	400	450	400	100

The project requires an initial investment of 1000. The relevant (annually compounded) interest rate is 14%. Disregard taxes.

1. Should you invest in this project?

Exercise 4. [2]

The interest rate with semiannual compounding is $r_n = 10\%$.

1. Determine the equivalent interest rate with continous compounding.

Exercise 5. *Compounding* [2]

A bank quotes you a rate of interest of 14% per annum with quarterly compounding. What is the equivalent annual rate with

1. continuous compounding?
2. annual compounding?

Exercise 6. *Compounding. (BM 3.16)* [4]

For an investment of \$1,000 today, the Tiburon Finance Company is offering to pay you \$1,600 at the end of 8 years.

1. What is the annually compounded rate of interest?
2. What is the continuously compounded rate of interest?

Exercise 7. *Savings* [3]

Suppose you will need 50,000 ten years from now. You plan to make seven equal annual deposits beginning three years from today in an account that yields 11% compounded annually. How large should the annual deposit be?

Exercise 8.

Recall the formula for calculating the present value for a constant annuity

$$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]$$

where the term in brackets is the *annuity factor*.

A *growing annuity* is an annuity where the cash flows increases by a growth rate g each year.

Show that the corresponding annuity factor for a growing annuity is

$$\left[\frac{1}{(r-g)}\right]\left[1 - \left[\frac{(1+g)^T}{(1+r)^T}\right]\right]$$

for a T period annuity, with growth rate g, discount rate r.

Empirical
Solutions
PROBLEM SET: Discounting

Exercise 1. *Project* [2]

1. Calculating the present value

$$PV = \frac{500}{1.10} + \frac{750}{(1.10)^2} + \frac{250}{(1.10)^3} = 455 + 620 + 188 = 1263$$

and then subtracting initial cost gives net present value

$$NPV = PV - C_0 = 1263 - 1000 = 263$$

Exercise 2. *Florida State Lottery (RWJ 4.6)* [2]

1. Calculate PV as:

$$\text{a}):\ PV_a = \frac{10,000}{1+r}$$
$$\text{b}):\ PV_b = \frac{20,000}{(1+r)^5}$$

   (a) Compare:
      i. 0%: $PV_a = 10,000$, $PV_b = 20,000$.
      ii. 10%: $PV_a = 9,090$, $PV_b = 12,418$.
      iii. 20%: $PV_a = 8,333$, $PV_b = 8,037$.

   (b) You are indifferent if they have the same present values: Solve for r in:

$$PV_a = PV_b$$
$$\frac{10,000}{1+r} = \frac{20,000}{(1+r)^5}$$
$$r = 18.921\%$$

Exercise 3. *Project* [3]
Calculate NPV

t	=	0	1	2	3	4
C_t	=	−1000	400	450	400	100

$$NPV = -1000 + \frac{400}{(1+0.14)^1} + \frac{450}{(1+0.14)^2} + \frac{400}{(1+0.14)^3} + \frac{100}{(1+0.14)^4} = 26.3342$$

NPV positive, do the investment.
Exercise 4. [2]
This corresponds to acontinuous compounding of

$$r = n\ln\left(1 + \frac{r_n}{n}\right) = 2\ln\left(1 + \frac{0.1}{2}\right) = 9.78\%$$

Exercise 5. *Compounding* [2]

1. Are given: With quarterly compounding, $r_4 = 14\%$. Want to find the equivalent continuous compounding

$$e^r = \left(1 + \frac{0.14}{4}\right)^4$$

$$r = 4\ln\left(1 + \frac{0.14}{4}\right) = 13.761\%$$

2. Annual compounding

$$(1+r) = \left(1 + \frac{0.14}{4}\right)^4$$

$$r = \left(1 + \frac{0.14}{4}\right)^4 - 1 = 14.752\%$$

Exercise 6. *Compounding. (BM 3.16)* [4]

1. First, look at the case of annual compounding:

$$FV = PV \cdot (1+r)^8$$

$$\Rightarrow \frac{FV}{PV} = (1+r)^8$$

$$\Rightarrow (\frac{FV}{PV})^{-8} = (1+r)$$

$$\Rightarrow (\frac{FV}{PV})^{-8} - 1 = r$$

$$\Rightarrow (\frac{1600}{1000})^{-8} - 1 \approx 0.06$$

$$r \approx 6\%$$

2. Next, look at continuous compounding.

$$FV = PV \cdot e^{rt}$$

$$\Rightarrow \frac{FV}{PV} = e^{rt}$$

$$\Rightarrow \ln(\frac{FV}{PV}) = rt$$

$$\Rightarrow r = \frac{1}{8} \cdot \ln(\frac{FV}{PV})$$

$$r \approx 5.87\%$$

Exercise 7. *Savings* [3]

Remember the annuitiy formula.

$$PV = CA_{n=7,r=0.11}$$

where A is the annuity factor.

$$A = 4.712$$

(Either from annuity tables or by calculation)

Since 50,000 is a FV, find its present value

$$\frac{50,000}{1.11^7} = C4.712$$

$$C = \frac{24,083}{4.712} = 5,111$$

Exercise 8.

Time	1	$\cdots$	T	$T+1$	$\cdots$		Value
Growing Perpetuity from 1 onwards	C	$\cdots$	$C(1+g)$	$C(1+g)^2$	$\cdots$	$=$	$C \cdot [\frac{1}{r-g}]$
$-$Perpetuity from T+1 onwards	0	$\cdots$	0	$C(1+g)^T$	$\cdots$	$=$	$C(1+g)^T \cdot [\frac{1}{(1+r)^T} \frac{1}{r-g}]$
Annuity	C	$\cdots$	C	0	$\cdots$	$=$	$C \cdot \left[\frac{1}{r-g} - \frac{(1+g)^T}{(1+r)^T} \frac{1}{r-g}\right]$