Discounting - Tool number one

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Contents

1 Intro

Lecture topic: How to evaluate the value of cash flows arriving at different times.
Methods covered:

- (Net) Present Value (N)PV
- Internal Rate of Return IRR

Useful special cases (Mechanics of interest rate calculations)

- Perpetuities
- Annuities
- Perpetual
- Compounding
- Compounding frequency

2 Allocating resources over time

3 Introduction to present value

3.1 Relationship between Present value and Future value

Exercise 1.

Suppose you have \$1000 and an opportunity to invest it and earn 10% per year for certain.

1. If you invest \$1000, how much will you have after one year?
2. Suppose after 1 year you reinvest the \$1100 at the same 10% rate of interest. How much will you have after the second year?

Exercise 2.

Suppose you are saving for a trip to the Bahamas in two years and will need \$2000 at that time. The rate at which you can invest is 10%.

1. How much will you need to invest today to have enough money to make your trip two years from now?

3.2 Net Present value (NPV)

Exercise 3.

Suppose you have the opportunity to invest in the development of a new, high–speed personal computer that is expected to revolutionize the computer industry. The cost of the investment is \$1 million. The computer will take exactly one year to develop, at which time you plan to sell the patent to the highest bidder. You currently expect to sell the patent for \$1.25 million. Investment in securities with similar risk are currently expected to return 20%.

1. What is the value of your investment opportunity?
2. Is this a good project?
3. Evaluate the project using the IRR method.

4 The mechanics of discounting.

Exercise 4.

An investment project offers the following pattern of cash flows.

Time (t)	Cash Flow (C_T)
0	-\$1000
1	500
2	750
3	250

The appropriate discount rate is 10%.

What is the NPV of the investment project?

Exercise 5.

A *perpetuity* is a stream of cash flows which pays off a fixed amount C per period. The opportunity cost of capital is r.

1. Show that the present value of the perpetuity can be calculated as

$$PV = C \cdot \frac{1}{r}$$

Exercise 6.

Illinois Power has issued a preferred stock that pays an annual dividend of $2.35. This dividend is fixed for the indefinite future. The opportunity cost of capital is 8%.

1. What is the market value of the preferred stock?

Exercise 7.

An *annuity* is an asset that pays a fixed (constant) amount C each year for a specified finite number of years.

1. Show that the present value of the payments from an annuity that last T years is

$$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]$$

Exercise 8.

You have just won the state lottery and are given the choice of receiving $1 million immediately or $100,000 per year for the next 20 years. The first of these payments is to be paid immediately, with subsequent payments made at one year intervals. The opportunity cost of capital is $r = 8\%$.

1. Which payment alternative should you choose?

Exercise 9.

You are asked to construct a table showing how a current investment of $1 will grow over time as a function of the compounding interval. The interest rate is 10%.

- Show the FV after 1,5, 10, 20 and 50 years,
- Use compounding intervals of annual, biannual, monthly, daily, as well as continous compounding.

Exercise 10.

Your bank offers to lend you $1000 at an annual interest rate of 12% compounded monthly. Your rich uncle offers to lend you $1000 at an annual interest rate of $12\frac{1}{2}\%$ compounded annually.

1. Which is the better deal?

Exercise 11.

You want to borrow $100,000 from your bank to finance the purchase of a new home. The bank has expressed a willingness to lend you the money at an interest rate of 10% compounded monthly.

1. Determine your monthly mortgage payments if you take out a 30-year mortgage.
2. What would your monthly payments be if you take out a 15-year mortgage at the same interest rate?

Exercise 12.

Let r be continuous compounding, and r_n be discrete compounding, where we compound n periods per year.

1. Develop general formulas to convert from either one to the other

Exercise 13.

The interest rate with semiannual compounding is $r_n = 10\%$.

1. Determine the equivalent interest rate with continous compounding.

Exercise 14.

Interest is 8% with continuous compounding.

1. Find the equivalent quarterly interest rate.

Exercise 15.

It is today 29 jun 2012, and you observe the following prices and interest rates for Norwegian State T-bills, traded at the Oslo Stock Exchange.

Ticker	Price (last)	Final Date	Coupon
NST16	99.68	19 sep 2012	-
NST17	99.29	19 dec 2012	-
NST18	99.00	20 mar 2013	-
NST18	98.62	19 jun 2013	-

On 29 jun 2012 the price of a Treasury bill maturing on 19 jun 2013 is 98.62. What is the implicit annualised interest rate in this price?

5 Summary – discounting

1. Method for evaluating current value of stream of future cashflows
   Present Value
2. Subtracting present cost
   Net Present Value
   or: Value added

Future Value

$$FV_t = PV(1+r)^t$$

Present Value

$$PV = \frac{FV_t}{(1+r)^t}$$

Discount factor

$$d_t = 1/(1+r)^t$$

Multiple cash flows – present value additive

$$PV = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t}$$

$$NPV = PV - C_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t} - C_0$$

The internal rate of return: the discount rate that makes the $NPV = 0$.
A *perpetuity*: fixed income for each year for the indefinite future.

$$PV = \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^t}$$

$$PV = \frac{C}{r}$$

A *growing perpetuity*: a fixed income for each year that grows at a constant rate g.

$$PV = \sum_{t=1}^{\infty} \frac{C_1(1+g)^{t-1}}{(1+r)^t}$$

$$PV = \frac{C_1}{r-g}$$

An *annuity*: pays a fixed (constant) amount each year for a specified finite number of years (T).

$$\begin{aligned} PV &= \sum_{t=1}^{T} \frac{C}{(1-r)^t} \\ &= C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right] \end{aligned}$$

The term $A_{r,T} = \left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]$ is called an *annuity factor*.
Compounding refers to the frequency with which interest is paid.

Discrete compounding m times a year

$$FV_t = PV\left(1+\frac{r}{m}\right)^{mt}$$

$$PV = FV_t\left(1+\frac{r}{m}\right)^{-mt}$$

Continous compounding

$$FV_t = PV(e^{rt})$$

$$PV = FV_t(e^{-rt})$$

Translating

$$r = n\ln\left(1+\frac{r_n}{n}\right)$$

$$r_n = n\left(e^{\frac{r}{n}}-1\right)$$

6 Notation

FV future value
PV present value
t time at which cash flows occur
r interest rate used for discounting
C_t cash flow at time t
Abbreviations
PV present value
NPV net present value
IRR internal rate of return