

# Financial Derivatives – a survey

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# Lecture overview

- ▶ Intro
- ▶ Example: Currency risk
- ▶ Forward Contracts
- ▶ Options
  - ▶ Payoffs
  - ▶ Pricing - binomial
  - ▶ Pricing - Black Scholes
- ▶ Information in derivatives
- ▶ Capital structure insights

# Introduction

Growth in derivatives contracts — contribution of finance to society.

Hedging of risks that could before not be insured against.

This lecture: corporate use of derivatives.

Derivatives generically defined as: Securities traded in financial markets whose value depend on the price/value of some observable (contractible) financial asset.

## Example: Currency Risk

You are the CFO of a US-based corporation.

It is now the end of February '20, and you have just signed a contract that will pay Euros (EUR) 1 mill one year from now ( $T$ ).  
If you do nothing, you receive next March (in dollars)

$$\text{EUR 1 mill} \times \text{Spot exchange rate } (S_T)$$

The current exchange rate  $S_t$  is 1.09.

The spot exchange rate one year from now  $S_T$  is uncertain.

Can we do something about this uncertainty?

## Example: Currency Risk

- ▶ You can enter a forward contract.

Looking at the CME, you find *Euro FX Futures Quotes*:

Maturity	Forward Price
Jul '20	1.0968
Sep '20	1.1005
Dec '20	1.1056
Mar '21	1.1108

If you enter into a forward contract at the March '21 quote, you commit yourself to exchanging the EUR 1 million at the forward rate in March '21.

## Example: Currency Risk - forward

1. Suppose you enter into a Mar '21 forward contract for 1 mill EUR, with a forward price of 1.1108.
  - 1.1 What if the spot exchange rate in March '21 is 1.05?  
What would your position have been without the forward contract?  
With the forward contract?
  - 1.2 What if the spot exchange rate in March '21 is 1.15?  
What would your position have been without the forward contract?  
With the forward contract?
  - 1.3 Is there a general relation between your position and the spot exchange rate in March '21?

## Example: Currency Risk - forward

One year forward, rate 1.1108:

One year from now, you receive

USD 1,110,800

What if exchange rate next year falls to 1.05?

If you didn't have the commitment of the forward, would be getting

EUR 1 mill  $\times$  1.05 = 1,050,000 USD

instead of the committed

USD 1,110,800

Hedging has produced a (ex post) gain of

EUR 1 mill  $\times$  (1.1108 - 1.05)  
= 60,800 USD

## Example: Currency Risk - forward

What if the exchange rate  $S_T$  increases to 1.15?

Without the forward, would have gotten

USD 1,150,000

but with the forward, gets USD 1,110,800.

Loss of USD  $1,110,800 - 1,150,000 = 39,200$  relative to not using a forward contract.

→ hedging does not necessarily guarantee against losing, but it does give the hedger a *predictable* future cashflow.

With forwards and futures, participate in both directions (losses and gains).

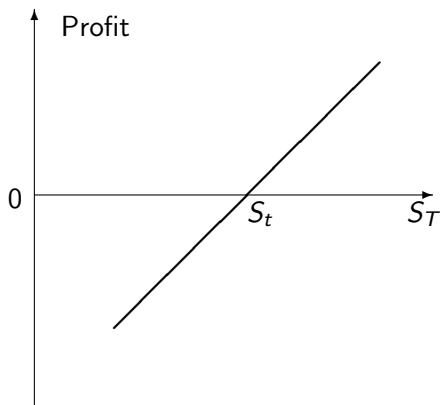


## Example: Currency Risk - forward

Generally, what is your position?

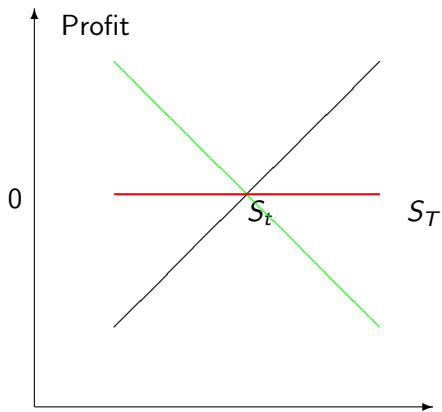
It depends on the exchange rate one year from now ( $S_T$ ).

The position one year from now summarized as.



## Example: Currency Risk - forward

Total position.



## Example: Currency Risk - options

- ▶ Alternative hedging strategy: trade an option.  
At the CME you will also find quotes of FX options EUR/USD with maturity February 2021.

Option Price	Strike quote
37200	1.0900
34100	1.0950
31200	1.1000
28500	1.1050
26000	1.1100

The option price is the USD price for a option for 1 mill EUR with the indicated strike quote.

Consequence of option: you can *choose* to exchange the 1 mill EUR at the indicated strike.

To get the option, pay the premium up front.

## Example: Currency Risk - options

You buy the option with a strike price of 1.1.

1. Suppose the spot exchange rate in March '21 is 1.05.  
What is the value of the option contract?  
What is your total position?
2. Suppose the spot exchange rate in March '21 is 1.15.  
What is the value of the option contract?  
What is your total position?
3. Is there a general relation between your position and the spot exchange rate in March '21?

## Example: Currency Risk - options

Let us next consider the option case. The option allows you to choose whether to use it at the exchange rate of 1.1.

Suppose exchange rate next year falls to 1.05

With that spot rate, would have gotten

$$\text{EUR 1 mill} \times 1.05 = 1,050,000 \text{ USD}$$

The option gives you the right to exchange at the rate of 1.1, will receive

$$\text{EUR 1 mill} \times 1.1 = 1,100,000 \text{ USD}$$

Having the option results in a gain of

$$1,100,000 - 1,050,000 = 50,000$$

Your total position is 1,100,000.

## Example: Currency Risk - options

On the other hand, suppose the exchange rate  $S_T$  increases to 1.15?

With that spot rate, would get

$$\text{EUR 1 mill} \times 1.15 = 1,150,000 \text{ USD}$$

The option gives you the possibility of translating at an exchange rate of 1.1.

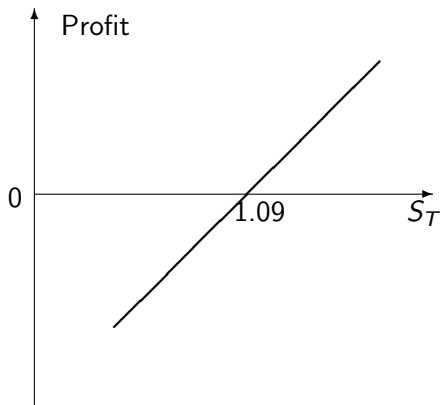
But why should you?

→ The value of the option at the exercise date is zero.

Your total position is 1,150,000.

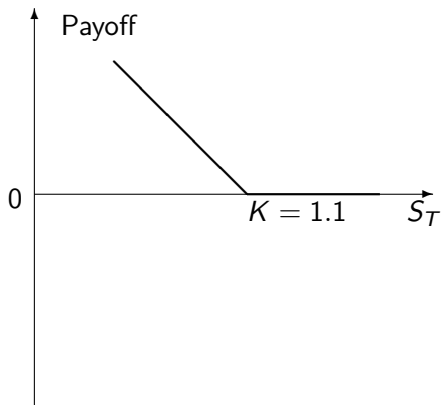
## Example: Currency Risk - options

Options are typically used when you are mainly concerned with unfavorable currency movements (the downside risk).



## Example: Currency Risk - options

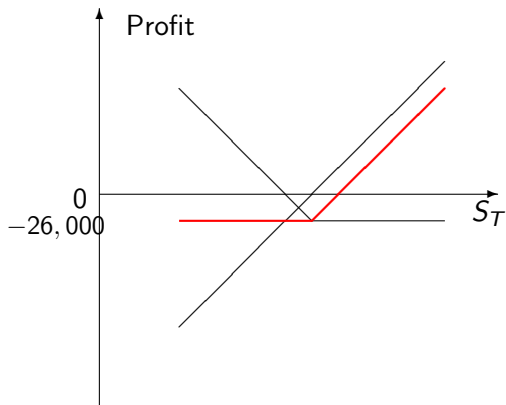
Generally, effect of this option at contract maturity





## Example: Currency Risk - options

Total effect of options: Need to account for the up front *premium*



## Forward/Futures contracts

### Example

You go to the bookstore, look for a textbook. The textbook is out of stock. The clerk puts it on order for you, it will cost you \$50 at delivery.

Congratulations, you have just entered a forward contract.

Define forward contracts: Agreement to buy (sell) given *amounts* underlying at given *price* (forward price) and at given *time* (expiry date).

Two parties:

Long: Party buying commodity in the future (buy forward)

Short: Party selling commodity in the future (sell forward)

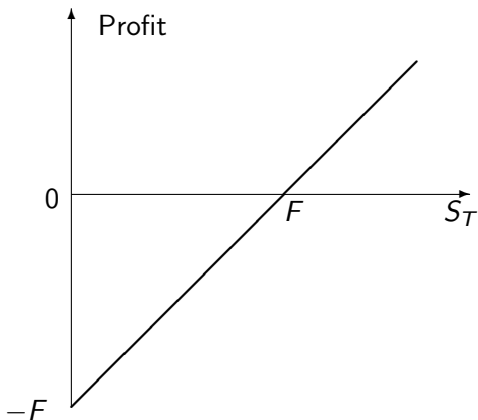
# Forward/Futures contracts

Contract specifies:

- ▶ Amount and other properties of good to be delivered.
- ▶ Forward price ( $F$ )
- ▶ Time of delivery ( $T$ )
- ▶ Where and how delivery is to take place.
- ▶ Each forward contract has both a buyer and a seller  
Zero net supply  
Forwards are pure *risk-sharing* devices.
- ▶ Usually, no money changes hands until the final date.
- ▶ The forward price is set to achieve this.

## Forward/Futures - relation to underlying

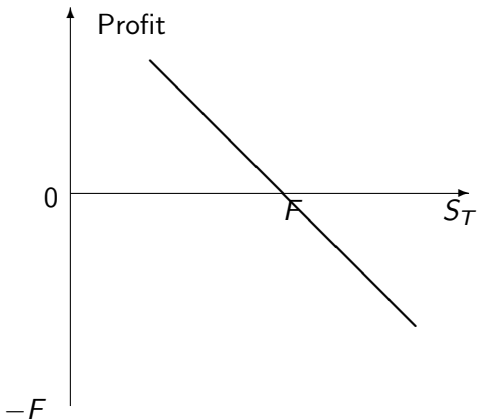
If the forward agreement involves *buying* the asset



Payoff long forward

## Forward/Futures - relation to underlying

If the forward agreement involves *selling* the asset



Payoff short forward

## Pricing of a forward contract

Pricing of a forward contract: application of the no-arbitrage principle.

Key to pricing: realize that one can achieve the same cashflows as those of a forward contract by other means.

## Exercise - Pricing of a forward contract

Consider a forward contract on an underlying asset that provides no income. There are also no restrictions on shortselling of the underlying asset. Let  $S_t$  and  $S_T$  denote the price of the underlying asset at  $t$  and  $T$ , respectively.  $r$  denotes the riskfree rate. Then the (time- $t$ ) forward price  $F_t$  for a contract with deliver date  $T$  has to satisfy

$$F_t = S_t(1 + r)^{(T-t)},$$

i.e., the forward price is the future value of the current price of the underlying.

Use arbitrage arguments to show this.

## Exercise - forward pricing

Need to show that violations of  $F_t = S_t(1+r)^{(T-t)}$  will lead to arbitrage profits (free lunches).

Let us start with the case where

$$F_t > S_t(1+r)^{(T-t)}$$

**Arbitrage strategy for case  $F_t > S_t(1+r)^{(T-t)}$**

Time:	$t$	$T$
Sell forward	0	$F_t - S_T$
Borrow $S_t$	$S_t$	$-S_t(1+r)^{(T-t)}$
Buy underlying	$-S_t$	$S_T$
Total	0	$F_t - S_t(1+r)^{(T-t)} > 0$



## Exercise - forward pricing

On the other hand, if  $F_t < S_t(1+r)^{(T-t)}$ :

**Arbitrage strategy for case  $F_t < S_t(1+r)^{(T-t)}$**

Time:	$t$	$T$
Buy forward	0	$S_T - F_t$
Invest $S$	$-S_t$	$S_t(1+r)^{(T-t)}$
Short underlying	$S_t$	$-S_T$
Total	0	$S_t(1+r)^{(T-t)} - F_t > 0$

To avoid arbitrage we need an exact inequality

$$F_t = S_t(1+r)^{(T-t)}$$

## Options - Definitions

A *Call option* is a right to *buy* a underlying security at a fixed price (exercise price -  $K$ ) in some given time period (expiry).

A *Put option* is the right to *sell* a underlying security at a fixed price (exercise price) in some given time period.

If we *use* the option to buy/sell the asset we *exercise* the option.

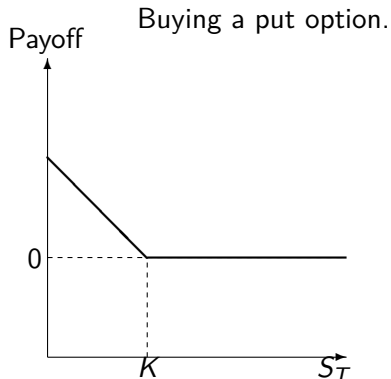
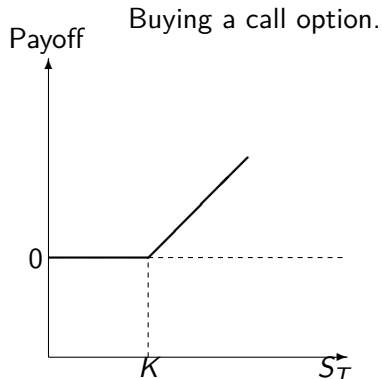
If the option is an *European* option, it can only be exercised at the expiry date.

If the option can be exercised any time up to the expiry date, it is called an *American* option.

## Option payoff at maturity. Position diagrams

Summarize the payoffs at maturity from buying options as:

- ▶ Call option: Payoff =  $\max(0, S_T - X)$ .
- ▶ Put option: Payoff =  $\max(0, X - S_T)$ .



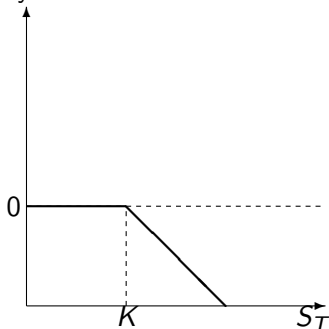
## Option payoff at maturity. Position diagrams

For the seller of options, the payoffs can be summarized as:

- ▶ Call option: Payoff =  $\min(0, S_T - X)$ .
- ▶ Put option: Payoff =  $\min(0, X - S_T)$ .

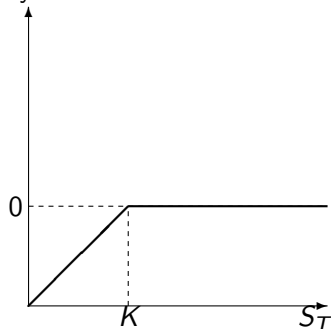
Payoff

Selling a call option.



Payoff

Selling a put option.

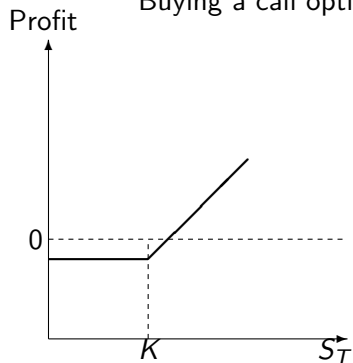


## Option profit. Profit diagrams

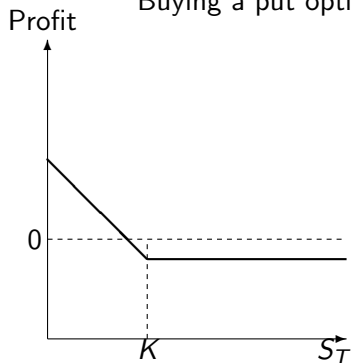
Long options

- ▶ Call option: Profit =  $\max(0, S_T - X) - C_0$ .
- ▶ Put option: Profit =  $\max(0, X - S_T) - P_0$ .

Buying a call option.



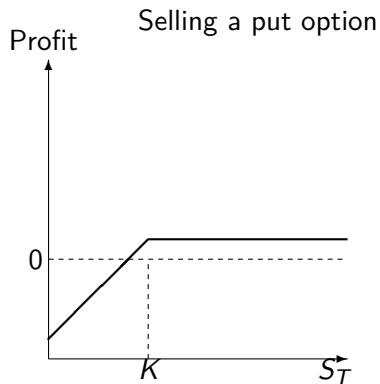
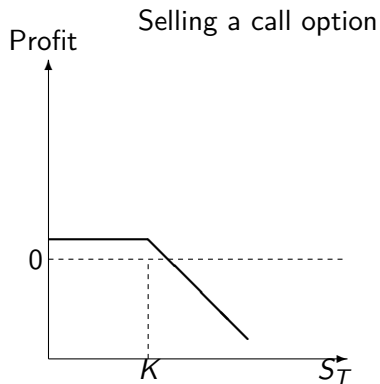
Buying a put option.



# Option profit. Profit diagrams

## Short options

- ▶ Call option: Payoff =  $C_0 + \min(0, S_T - X)$ .
- ▶ Put option: Payoff =  $P_0 + \min(0, X - S_T)$ .



## On the pricing of options

Option, or “Contingent Claim”

A recent innovation in Finance.

Indeed, the existence of a simple formula for the price of an option (The Black-Scholes formula) was one of the reasons for the quick growth in these markets.

The CBOE (Chicago Board of Options Exchange) started trading options on common stock in 1973.

In the same year two important papers describing option pricing formulas were published: Black and Scholes (1973) and Merton (1973)

## On the pricing of options

Pricing of options (and other derivatives) relies on a *no-arbitrage* argument, which we can summarise as

If two portfolios or assets have the same payoffs tomorrow, they must have the same price today.

The challenge:

*contingent* feature of options

This is different from e.g. pricing a forward.

To price a forward contract: find portfolio which replicate the payoffs from the forward using a position in the underlying security and riskfree borrowing/lending.

This replicating portfolio only needs to be set up once



## On the pricing of options

An options contract can also be replicated using the underlying security and riskfree borrowing/lending

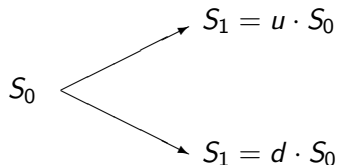
However: The replicating portfolio needs to be changed as time passes (and the price of the underlying security changes).

Will show how to price options in a simple setting, the *binomial* framework.

In practical use, we use a more complex algorithm, the Black Scholes formula.

## Binomial option pricing - setup

Suppose stock price next period can take on only two values,



it can increase to  $u \cdot S_0$  or decrease to  $d \cdot S_0$ .

Example:

$$S_0 = 20$$

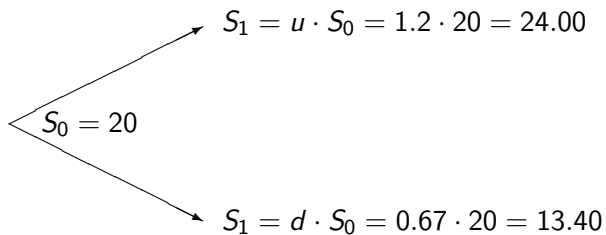
$$u = 1.2$$

$$d = 0.67$$

$$r_f = 10\%$$

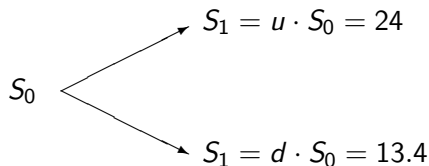
## Binomial option pricing - setup

Possible prices after one period.



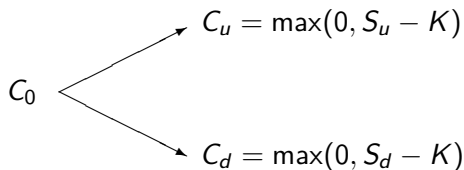
# Binomial option pricing - call option payoffs

Stock price movement



Payoff at time 1 of a call option with exercise price  $K = 20$  maturing at time 1:

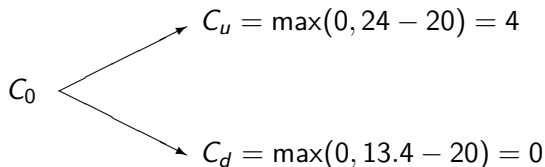
$$C_1 = \max(0, S_1 - K)$$



## Binomial option pricing - call option payoffs

$$\begin{aligned}C_u &= \max(0, S_1 - K) \\ &= \max(0, u \cdot S_0 - K) \\ &= \max(0, 24 - 20) = 4\end{aligned}$$

$$\begin{aligned}C_d &= \max(0, S_1 - K) \\ &= \max(0, d \cdot S_0 - K) \\ &= \max(0, 13.40 - 20) = 0\end{aligned}$$



## Binomial option pricing - constructing a hedge portfolio

To construct arbitrage portfolio, need a portfolio with known payoff next period (corresponding to pricing of forward contract)

Ask: If we buy one stock, how many call options do we need to buy/sell to make the payoff next period riskless?

Let  $m$  be the number of calls.

The payoff from a strategy of buying 1 stock and  $m$  call options should be the equal no matter what happens to the stock price

$$mC_u + S_u = mC_d + S_d$$

Solve for  $m$ :

$$\Rightarrow mC_u + uS_0 = mC_d + dS_0$$

$$\Rightarrow m(C_u - C_d) = (d - u)S_0$$

$$\Rightarrow m = \frac{(d - u)S_0}{(C_u - C_d)}$$

## Binomial option pricing - constructing a hedge portfolio

In our example

$$\begin{aligned}m &= \frac{(0.67 - 1.2)20}{4 - 0} \\ &= -0.1325 \cdot 20 \\ &= -2.65\end{aligned}$$

Check: Does buying 1 stock and selling 2.65 call options give the same payoffs?

State:	<i>u</i>	<i>d</i>
Buy stock:	24	13.40
Sell 2.65 Call:	$-2.65 \cdot 4 = -10.60$	0
Total payoff:	13.40	13.40

## Binomial option pricing - arbitrage argument

The strategy of buying 1 stock and buying -2.65 calls will next period have a known payoff.

It therefore has a certain (known) return.

This return has to be equal to risk free rate of return,  $r_f$ , to avoid arbitrage.

The cost of the portfolio is  $S_0 + mC_0$ .

The one period return is  $(S_0 + m(C_0))(1 + r_f)$ .

Use this to find  $C_0$ .



## Binomial option pricing - arbitrage argument

In our example, the cost is  $20 - 2.65C_0$ , giving a return on the portfolio of

$$\frac{13.40}{20 - 2.65C_0} - 1$$

This return has to be equal to the riskfree rate  $r_f = 10\%$ .  
Solve for  $C_0$ :

$$\frac{13.40}{20 - 2.65C_0} = 1 + r_f$$

$$20 - 2.65C_0 = \frac{13.40}{1.1}$$

$$C_0 = \frac{20 - \frac{13.40}{1.1}}{2.65} = 2.95$$

The price of the call is  $C_0 = \underline{2.95}$ .

## Binomial option pricing - arbitrage argument

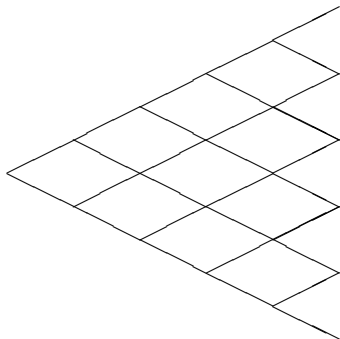
Control: With the calculated call price ( $C_0 = 2.95$ ),  
is the portfolio return equal to the risk free rate

$$\begin{aligned}\text{Return} &= \frac{13.40}{20 - 2.65 \cdot 2.95} - 1 \\ &= 1.0999 - 1 \approx 10\%\end{aligned}$$

## From binomial to a realistic price assumption

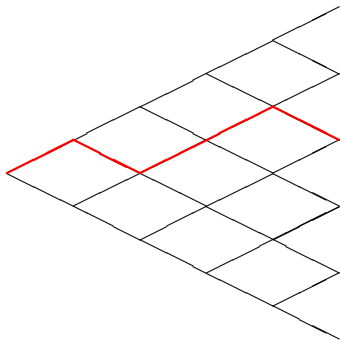
One or two dates in a binomial model: Seem unrealistic.

Improve realism: Increase number of nodes, keep time to maturity constant.



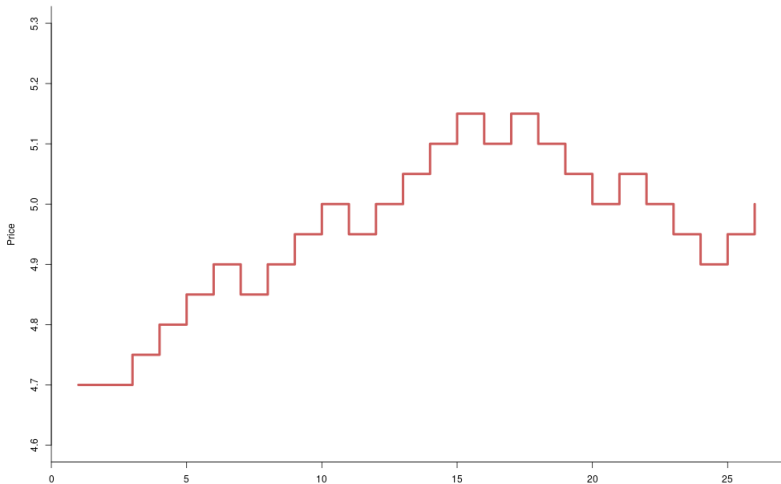
## From binomial to a realistic price assumption

Ex post *outcome*: Only one path in this tree



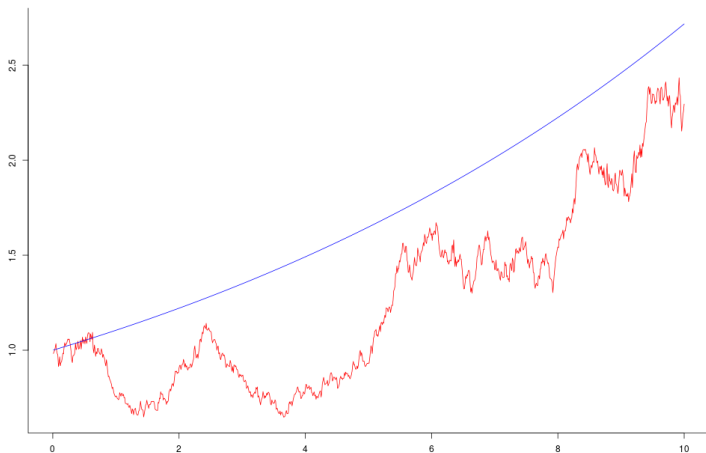
# From binomial to a realistic price assumption

Plot history of outcomes in a binomial setting:



## From binomial to a realistic price assumption

Increase the number of nodes indefinitely, price path:



The limit is **Geometric Brownian Motion**.

# The Black Scholes formula

The Black Scholes formula for a call option is

$$c = S \cdot N(d_1) - K \cdot e^{-r(T-t)} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + r(T-t)}{\sigma\sqrt{T-t}} + \frac{1}{2}\sigma\sqrt{T-t} = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$N(\cdot)$  = The cumulative normal distribution

The price of a put option is

$$p = Ke^{-r(T-t)} N(-d_2) - SN(-d_1)$$

## Exercise - Black Scholes calculation

Consider 3 month options with exercise prices of  $K = 45$ . The variance of the underlying security is  $\sigma^2 = 0.20$ . The risk free interest rate is  $r = 6\%$ . The current price of the underlying security is  $S = 30$ .

1. Determine the Black Scholes prices for call and put options.
2. Check that your calculations satisfy put call parity.



## Exercise - Black Scholes calculation - solution

The Black Scholes formula for a call option is

$$c = S \cdot N(d_1) - K \cdot e^{-r(T-t)} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + r(T-t)}{\sigma\sqrt{T-t}} + \frac{1}{2}\sigma\sqrt{T-t} = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$N(\cdot)$  = The cumulative normal distribution

The price of a put option is

$$p = Ke^{-r(T-t)} N(-d_2) - SN(-d_1)$$

## Exercise - Black Scholes calculation - solution

All except the volatility is given, the volatility is the square root of the variance

$$\sigma = \sqrt{0.20} = 0.447$$

Call

$$C_{BS}(S = 30, K = 45, r = 0.06, \sigma = 0.447214, (T - t) = 0.25)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_1 = \frac{\ln\left(\frac{30}{45}\right) + (0.06 + \frac{1}{2}0.20)\frac{3}{12}}{0.447\sqrt{\frac{3}{12}}}$$

$$d_1 = -1.63441$$

$$N(d_1) = 0.051$$

$$d_2 = -1.85802$$

$$N(d_2) = 0.032$$

$$C_{BS} = 0.133$$

## Exercise - Black Scholes calculation - solution

Put prices

$$N(-z) = 1 - N(z)$$

$$\begin{aligned} p &= Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \\ &= 45e^{-0.06\frac{3}{12}}(1 - 0.032) - 30(1 - 0.051) = 14.46 \end{aligned}$$

Check this using put-call parity

$$c - p = S - Ke^{-r(T-t)}$$

$$\begin{aligned} p &= c - S + Ke^{-r(T-t)} \\ &= 0.133 - 30 + 45e^{-0.06\frac{3}{12}} = 14.46 \end{aligned}$$

## Using the Black Scholes pricing formula.

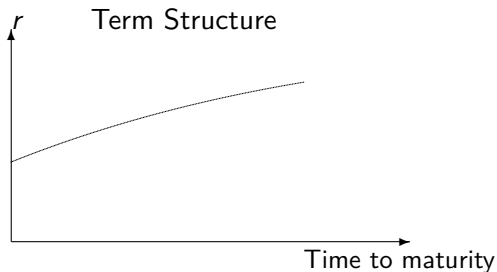
Where do the inputs to the Black Scholes model come from?

Price of “underlying”:  $S$ : Look up current market prices at the exchange.

Exercise price  $K$  and Time to maturity  $T - t$  are given in the option contract.

Interest rate: Take from Treasury data.

Problem: term structure of interest rate, there is no one interest rate.



Solution: Take matching maturity interest rate.

# Using the Black Scholes pricing formula

## **Volatility**

Standard deviation ( $\sigma$ ).

Two methods for finding estimates of volatility.

- ▶ Historical volatility.
- ▶ Implied volatility

## Using the Black Scholes pricing formula.

**Historical volatility.** Given a sequence of e.g.  $n$  daily observations of the underlying,

$$\{S_n, S_{n-1}, S_{n-2}, \dots, S_1\}$$

Calculate sample standard deviation the standard way

$$\bar{S} = \frac{1}{n-1} \sum_{t=2}^n (\ln S_t - \ln S_{t-1})$$

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{t=2}^n (\ln S_t - \ln S_{t-1} - \bar{S})^2$$

Adjust to get annualized volatility

$$\sigma^2 = 260 \cdot \hat{\sigma}^2$$

## Exercise - Implied volatility

Consider an option contract where the current price  $S = 100$ , the exercise price is  $K = 100$ , and time to maturity  $T - t$  is one year. The risk free interest rate is 5%.

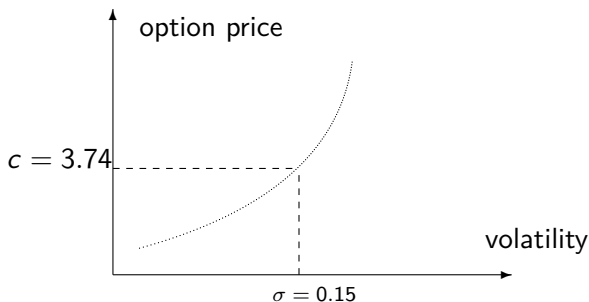
1. Suppose you observe yesterday's call price to be  $C = 14.97$ . What is the volatility implied in this price?

## Exercise - Implied volatility - solution

Given  $C = 14.97$ , can find the implied volatility by solving the equation

$$C_{obs} = 14.97 = C(S = 100, K = 100, \sigma, (T - t) = 1, r = 0.05)$$

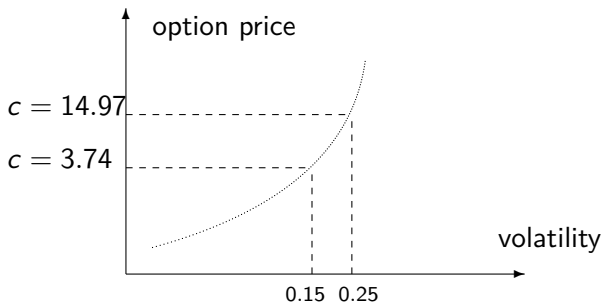
Can not solve analytically,  
Instead, numerical search. Try, e.g.  $\sigma = 0.15$





## Exercise - Implied volatility - solution

Try higher  $\sigma = 0.25$

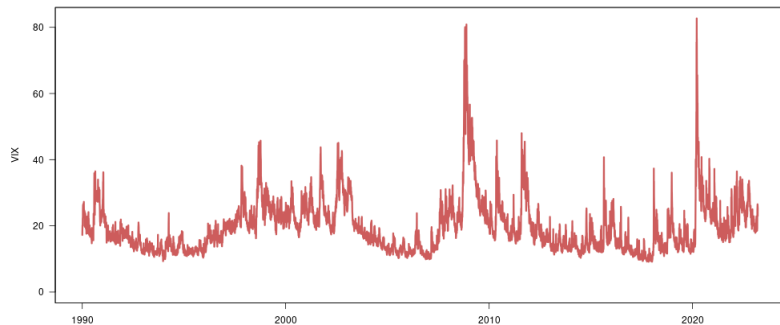


Find

$$\sigma_{implied} = 0.25$$

# Information in derivatives

## The VIX index



## Insight in capital structure from option theory

Simplified view of debt and equity.

1. Debt: pure discount bonds where the firm is to pay  $F$  dollars at the maturity date  $T$ .
2. In the event that the firm does not make the promised payment (“default”) then the firm goes bankrupt, its assets are turned over to the bondholders, and each bondholder will receive his pro rata share of the “reorganized” firm.  
The original equity holders receive nothing.

Let  $V_t$  denote the market value of the firm at time  $t$ .

# Insight in capital structure from option theory

## Payoff at maturity.

On the maturity date of debt:

- ▶ Possibility 1: Value of the firm exceeds the face value of debt:  
 $V_T > F$ .  
Debt is paid in full  
The value of debt:  $F$   
The value of equity:  $V_T - F$ .
- ▶ Possibility 2: The value of the firm is less than the face value:  
 $V_T < F$ )  
The firm will default  
The value of debt:  $V_T$   
The value of equity: 0.

## Insight in capital structure from option theory

### **Payoff at maturity** (summarized)

$$\text{Value of debt} = \min(F, V_T)$$

$$= F - \max(0, F - V_T)$$

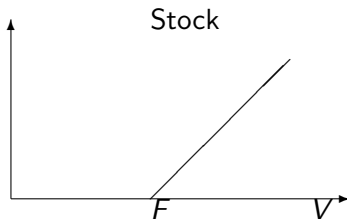
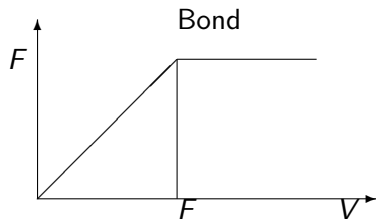
$$\text{Value of equity} = V_T - \min(F, V_T)$$

$$= V_T - F + \max(0, F - V_T)$$

$$= \max(0, V_T - F)$$

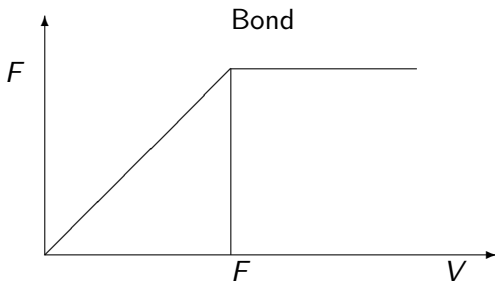
# Insight in capital structure from option theory

Plot payoff to bond and equity:



# Insight in capital structure from option theory

**What kind of option is a bond?**

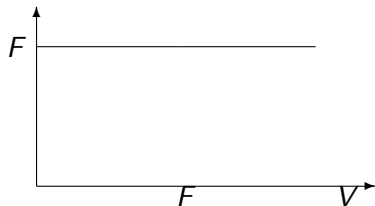


# Insight in capital structure from option theory

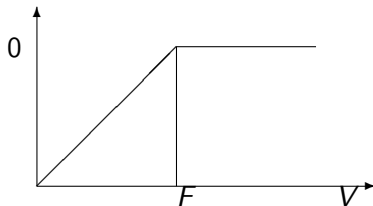
**What kind of option is a bond?**

Possibility 1: Risk free bond + short put.

Long risk free bond



Short put

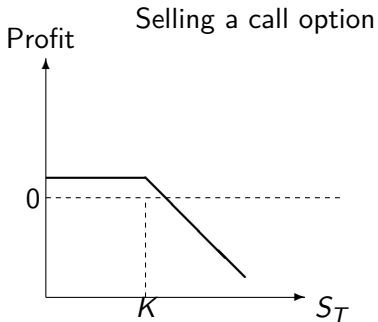
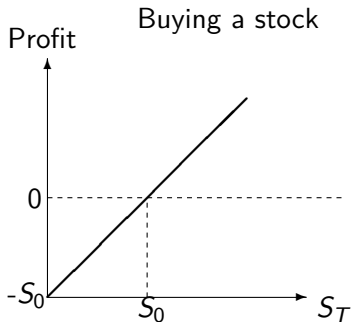




# Insight in capital structure from option theory

## What kind of option is a bond?

Possibility 2: Hold stock + short call.

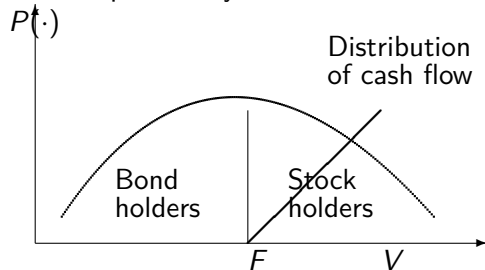


## Insight in capital structure from option theory

Claim: Equity owners may want to take on project increasing the risk of the company, even if they are negative NPV.

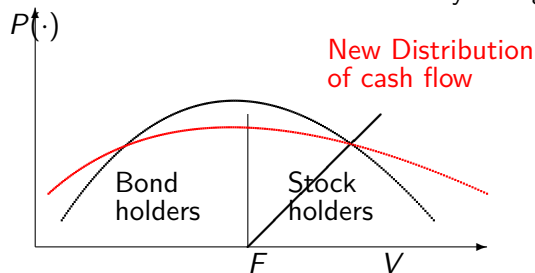
To see:

Current probability distribution of the firm value:



## Insight in capital structure from option theory

By taking on a negative NPV, high risk project, the distribution of the firm value may change as:



## Insight in capital structure from option theory

What option property can give us this intuition:

Option value (both put and call) is increasing in the volatility of the underlying.

- ▶ Option position of equity: Long call option.  
→ increase in volatility, increase in value of equity.
- ▶ Option position of debt: Short option leg (either put or call).  
→ increase in volatility, decrease in value of debt.

## Summary – derivatives

Tools for *hedging* of company risks.

Sources of risk: Exposure

Main tools:

- ▶ Forward Contracts
  - ▶ Fixing future value.
- ▶ Options
  - ▶ Putting a *floor* on future value
- ▶ Pricing
  - ▶ Tool: Arbitrage portfolio
  - ▶ Challenge: Constructing risk free “match” of futures/options
- ▶ Capital structure insights

Jonathan Berk and Peter DeMarzo. *Corporate Finance*. Pearson, fifth edition, 2020.