PROBLEM SET: Capm

Exercise 1.
A company is deciding whether to issue stock to raise money for an investment project which has the same risk as the market and an expected return of 20 percent. The riskfree rate is 10 percent and the expected return on the market is 15 percent. The company should go ahead:

1. Unless the company's old beta is greater than 2.0.
2. Unless the company's old beta is less than 2.0.
3. Independently of the company's beta.
4. Cannot be determined with the information given.
5. I choose not to answer.

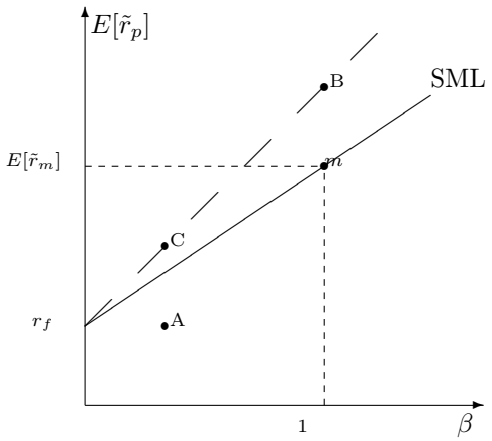
Exercise 2. *XY and Z*
The XY company with an asset beta of 2 is evaluating ways to expand its factory capacity. They have decided to create a fully owned subsidiary to build a new factory. The subsidiary will be called Z company. The present value of the cost of building the factory is 100 million. Z company will issue risk free bonds with a market value of 80 million as partial financing. XY company will provide equity of 20 million to Z company. The expected market return is 10% and the risk free interest rate is 4%. What is the expected return on XY's equity investment in Z company?

- 4%
- 16%
- 20%
- 64%
- I choose not to answer.

Exercise 3.
A portfolio contains equal investments in 10 stocks. Five have a beta of 1.2, the remainder have a beta of 1.4. Which statement is most precise about the value of the portfolio's beta?

1. Exactly equal to 1.3
2. Greater than 1.3 because the portfolio is not completely diversified.
3. Less than 1.3 because diversification reduces beta.
4. Lies between 1.2 and 1.4 depending on the correlation between the stocks.
5. I choose not to answer.

Exercise 4. *SML deviations* [3]
Consider the following illustration of the expected return – beta relation for two stocks, A and B,



Note that the two stocks do not plot on the SML. Stock A plots below and stock B plots above the SML.

1. How would you combine the two in a portfolio?
2. What would be the effects on stock prices?

Exercise 5. *Beta* [5]

Consider the following quotation from a leading investment manager: "The shares of Southern plc have traded close to £12 for most of the past three years. Since Southern's equity has demonstrated very little price movement, it has a low beta. Jungle instruments, on the other hand, has trades as high as £150 and as low as its current £75. Since JI's equity has demonstrated a large amount of price movement, the stock has a very high beta." Do you agree with this analysis? Explain?

Exercise 6. *Interest rate changes* [2]

According to the CAPM, a firm's cost of equity depends on its own systematic risk (β) and on general conditions in financial markets as measured by current interest-rate levels (the risk-free rate). and by the market's attitude toward risk (the "market price of risk", or $E[r_m] - r_f$). What would you expect to happen to a firm's cost of equity if interest rates fall but the expected return on the market portfolio remains unchanged?

Exercise 7. *Mitro* [1]

Calculate the expected return on the stock of Mitro Corporation. The beta of Mitro is estimated to be 1.2, the market risk premium is 8%, and the current risk free rate is 4%

Exercise 8. *Arnold* [4]

After his last holiday with Recall Inc., Arnold the Stock Analyst has suffered a partial loss of memory. He was trying to estimate some stock betas and returns, but he can't remember the procedure. He only has the following data:

Portfolio	Annual Rate of return (nominal)	Annual Rate of return (real)
Common stocks	12.0	8.8
Corporate bonds	5.1	2.1
Treasury bills	3.5	0.4

Average rates of return for the period 1926 to 1985.

Stock	Beta
Digital Eq.	1.21
Exxon	?
General Mills	?
MCI Comm.	?
Compaq	?
Genentech	1.95
Mesa Petroleum	0.68
Holly Sugar	0.62

Betas for selected common stocks. 1981-1986.

Stock	$E[\tilde{r}]$
Digital Eq.	17.66
Exxon	13.46
General Mills	12.29
MCI Comm.	20.27
Compaq	22.03
Genentech	?
Mesa Petroleum	?
Holly Sugar	?

The current T-bill rate is 7.5%.

1. Arnold needs your help to fill in the question marks. Also explain him how you found the numbers.

Exercise 9. *Fund* [2]

Suppose you are the manager of an investment fund in a CAPM world. Ignore taxes. Given the following forecast:

$$E[\tilde{r}_m] = 16\%$$
$$\sigma(r_m) = 0.20$$
$$r_f = 8\%$$

1. Would you recommend investment in a security j with the following characteristics: $E[\tilde{r}_j] = 12\%$ and $\text{cov}(\tilde{r}_j, \tilde{r}_m) = 0.01$?
2. Suppose next period it turns out that this security j has had a return of only 5%. How would you explain this, given that $E[\tilde{r}_j] = 12\%$?

Exercise 10. *UVM (BM 8.11)* [2]

The stock of United Venetian Merchants (UVM) has a beta of 1.0 and a very high unique risk.

1. If the expected return on the market is 20%, what is the required return of UVM?

Exercise 11. [3]

You are given the following information about three stocks that are in your portfolio. In addition, you know that the market portfolio has an expected return of 13% and a standard deviation of 18%. The risk free rate is 5%.

Stock	Beta	Weight in portfolio
A	1.1	20%
B	0.8	50%
C	1.0	30%

1. What is the expected return on your portfolio?

Exercise 12. [3]

The expected rate of return on the market portfolio is 14% and the risk-free rate is 6%.

1. Find the β for a portfolio that has an expected rate of return equal to 10%.

Exercise 13. *Widgets* [6]

The following 3 firms are the only firms currently operating in the widget industry.

	Debt		Equity	
Firm	β_D	Market value	β_E	Market value
A	0	100	1.0	200
B	0.05	75	1.5	125
C	0.10	50	1.5	50

We also know that

$$r_f = 8\%$$
$$E[\tilde{r}_m] = 16\%.$$

Disregard taxes.

1. Find expected returns on debt and equity for A, B and C.
2. Find expected returns for each firm A, B and C.
3. Find the *asset beta* for the widget industry.
4. Is the expected return for the widget industry higher than the market return? Explain how you can conclude this.

Empirical

Solutions

PROBLEM SET: Capm

Exercise 1.

As it has the same risk as the market, and earns more than the market, should go ahead independent of the market.

Exercise 2. *XY and Z*

Asset return on Z company:

$$r = r_f + \beta_Z(E[r_m] - r_f) = 0.04 + 2(0.1 - 0.04) = 16\%$$

$$r_E 0.2 + 0.8 r_D = r$$
$$r_E = \frac{0.16 - 0.8 \cdot 0.04}{0.2} = 64\%$$

(d) is correct

Exercise 3.

Exactly equal to 1.3

Exercise 4. *SML deviations* [3]

Since the average beta is 1, if there exist a stock that plots *below* the SML, there must also exist at least one stock that plots *above* the SML. Now ask yourself whether you would hold stock A at all. You would do better if you instead dropped stock A from your portfolio and replaced it with asset C (a combination of the riskfree asset and stock B). By including asset C in your portfolio, you could increase the expected return on your portfolio without increasing the risk. This would create excess demand for stock B and excess supply for stock A. Consequently, in equilibrium, all assets and portfolios must plot on the SML.

Exercise 5. *Beta* [5]

The investment manager is talking through his hat. He is talking about unsystematic risk, which has little to do with beta.

Exercise 6. *Interest rate changes* [2]

We have

$$r_e = r_f + \beta(E[r_m] - r_f)$$

r_f falls, pushing the r_e down. However, because r_f decreases, and $E[r_m]$ remains constant, $(E[r_m] - r_f)$ increases. To see the total effect clearly, rewrite the above as

$$r_e = r_f(1 - \beta) + \beta(E[r_m])$$

It is easy to see that the overall effect on r_e depends on the value of beta. For firms with $\beta < 1$, the cost of capital decreases, whereas firms with $\beta > 1$ will have to provide increased yields. Basically, the time value of money has decreased, whereas the market premium demanded for risk has increased. For firms with a low risk, the former dominates, and vice versa.

Exercise 7. *Mitro* [1]

$$CAPMreturn = 4 + 1.2 \cdot 8 = 13.6\%$$

Exercise 8. *Arnold* [4]

1. Use

$$E[\tilde{r}] = r_f + \beta \cdot (E[\tilde{r}_m] - r_f)$$

and

$$\beta = \frac{E[\tilde{r}] - r_f}{E[\tilde{r}_m] - r_f}$$

$$E[r_m] = 15.9\%$$

Stock	Beta
Digital Eq.	1.21
Exxon	0.71
General Mills	0.57
MCI Comm.	1.52
Compaq	1.73
Genentech	1.95
Mesa Petroleum	0.68
Holly Sugar	0.62

Stock	$E[r]$
Digital Eq.	17.66
Exxon	13.46
General Mills	12.29
MCI Comm.	20.27
Compaq	22.03
Genentech	23.88
Mesa Petroleum	13.21
Holly Sugar	12.71

Exercise 9. *Fund* [2]

1.

$$\beta_j = \frac{\text{cov}(\tilde{r}_j, \tilde{r}_m)}{\text{var}(\tilde{r}_m)} = \frac{0.01}{(0.20)^2} = \frac{0.01}{0.04} = 0.25$$

$$\begin{aligned} r_j &= r_f + (E[r_m] - r_f)\beta_j \\ &= 0.08 + (0.16 - 0.08)0.25 = 0.10 = 10\% \end{aligned}$$

Since the expected return of 12% is higher than the required return, should invest.

2. The after – the – fact returns may be different from the expected returns. You might just had a bad outcome, the CAPM might still be true.

Exercise 10. *UVM (BM 8.11)* [2]

$$\beta = 1.0$$

$$E[\tilde{r}_m] = 20\%$$

The required return is 20%. Since the stock has a beta of 1, it has the same return as the market portfolio, and its large unique (and therefore unsystematic) risk is irrelevant.

Exercise 11. [3]

1. Calculate the beta of the portfolio as

$$\beta_p = 0.20 \cdot 1.1 + 0.5 \cdot 0.8 + 0.3 \cdot 1.0 = 0.92$$

Therefore, the return on the portfolio is

$$\begin{aligned} E[r_p] &= r_f + (E[r_m] - r_f)\beta_p \\ &= 0.05 + (0.13 - 0.05) \cdot 0.92 \\ &= 12.36\% \end{aligned}$$

Exercise 12. [3]

1. SML:

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f)$$

Using

$$r_f = 6$$
$$E[r_m] = 14$$

want

$$10 = 6 + \beta_i \cdot 8$$

therefore

$$\beta_i = \frac{10 - 6}{8} = \frac{1}{2}$$

Exercise 13. *Widgets* [6]

1. Expected returns on debt and equity for A, B, and C:

$$E[\tilde{r}_D] = r_f + (E[r_m] - r_f)\beta_D$$
$$A: \ r_D = 0.08 + (0.16 - 0.08) \cdot 0 = 8\%$$
$$B: \ r_D = 0.08 + (0.16 - 0.08) \cdot 0.05 = 8.4\%$$
$$C: \ r_D = 0.08 + (0.16 - 0.08) \cdot 0.1 = 8.8\%$$

and

$$E[\tilde{r}_E] = r_f + (E[r_m] - r_f)\beta_E$$
$$A: \ r_E = 0.08 + (0.16 - 0.08) \cdot 1.0 = 16\%$$
$$B: \ r_E = 0.08 + (0.16 - 0.08) \cdot 1.5 = 20\%$$
$$C: \ r_E = 0.08 + (0.16 - 0.08) \cdot 1.5 = 20\%$$

2. Expected return for each firm A, B and C:

$$E[r_A] = \frac{D}{V} r_D + \frac{E}{V} r_E = 13.3\%$$
$$E[r_B] = 15.6\%$$
$$E[r_C] = 14.4\%$$

3. Asset beta for the industry

	Debt		
Firm	β_D	Market value	r_D
A	0	100	8%
B	0.05	75	8.4%
C	0.10	50	8.8%
Industry		225	

	Equity		
Firm	β_E	Mkt value	r_E
A	1.0	200	16%
B	1.5	125	20%
C	1.5	50	20%
Industry		375	

	Total		
Firm	r^*	β^*	Mkt value
A	13.3%	0.67	300
B	15.6%	0.96	200
C	14.4%	0.8	100
Industry			600

$$\beta^* = \frac{300}{600}0.66 + \frac{200}{600}0.96 + \frac{100}{600}0.8 = 0.78$$

4. Depends on whether the asset beta for industry is smaller or larger than one. In this case $\beta^* = 0.78 < 1$, the return is lower than the market.