The Capital Asset Pricing Model (CAPM)

Introduction

Lecture overview:

- Intuition: What is an asset pricing model? What do we use it for?
- Equilibrium consequence of mean-variance analysis: CAPM
- Using the CAPM: Estimating a cost of capital.

The Capital Asset Pricing Model (CAPM)

How stocks are priced by the market.

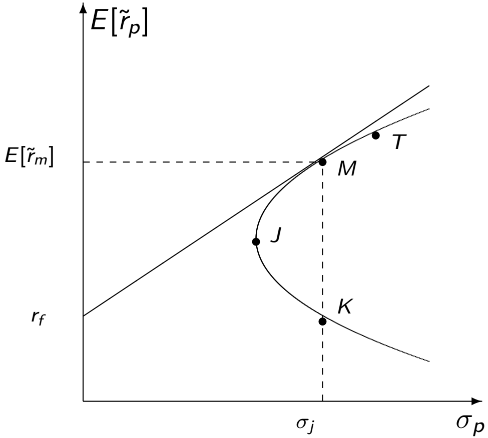
1. How do we measure the risk of an individual stock?
2. How is the expected of required rate of return on a stock related to its risk?

Assumptions of the CAPM

1. Investors choose their portfolios based on expected return and variance.
2. All investors have rational expectations and homogeneous beliefs.
3. Investors trade in perfect capital markets. This means that there are no frictions such as taxes, transaction costs, or restrictions on short sales.

The capital market line (CML)

The portfolio opportunity set available to investors
expected return - standard deviation space.



Capital Market Line

The expected return and standard deviation of any portfolio falling on the CML are:

$$E[\tilde{r}_p] = \omega r_f + (1-\omega)E[\tilde{r}_m]$$

$$\sigma_p = (1-\omega)\sigma_m$$

The equation for the CML is:

$$E[\tilde{r}_p] = r_f + (E[\tilde{r}_m] - r_f)\frac{\sigma_p}{\sigma_m}$$

The market portfolio

The portfolio m, in equilibrium, must be the *market portfolio* consisting of all risky assets in proportion to their relative market values.

The portfolio weights for assets in the market portfolio are:

$$\omega_j = \frac{\text{Market value of asset } j}{\text{Market value of all assets}}$$

Measuring the risk of an individual asset.

The relevant measure of risk for an individual asset is *not* the risk of that asset held in isolation,
It is the contribution of that asset to the risk of your portfolio.
The variance of the market portfolio:

$$\sigma_m^2 = \sum_{j=1}^{N} \sum_{i=1}^{N} \omega_j \omega_i \sigma_{ij} = \sum_{j=1}^{N} \omega_j \sigma_{jm}$$

Where

- ω_j is the weight on asset j in the market portfolio m
- σ_{jm} is the covariance between asset j and the market portfolio.

Measuring the risk of an individual asset.

beta of a security:

$$\beta_j = \frac{\sigma_{jm}}{\sigma_m^2}$$

The variance of the market, σ_m^2, is the same for all securities.
we can measure the risk of a security by

- either its covariance, σ_{im},
- or its beta, β_j.

Since beta is more intuitive, we shall use it as a measure of a security's risk.

Propertios of beta

- The beta of the market portfolio is 1:

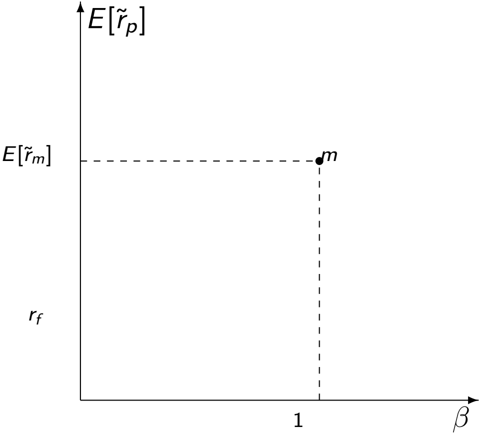
$$\beta_m = \sum_{j=1}^{N} \omega_j \beta_j = \sum_{j=1}^{N} \omega_j \frac{\sigma_{jm}}{\sigma_m^2} = 1$$

- The beta of a portfolio consisting of a fraction ω of the riskless asset and a fraction $(1 - \omega)$ of the market portfolio is:

$$\beta_p = \omega \cdot 0 + (1 - \omega) \cdot 1.0 = (1 - \omega)$$

Relationship beta and expected return

Intuitively, securities with higher betas have more systematic (market) risk
$\rightarrow$ command higher expected rates of return in equilibrium.



The relative positions in the riskfree asset ($\beta = 0$) and the market portfolio ($\beta = 1$) in expected return - beta space

The Security Market Line

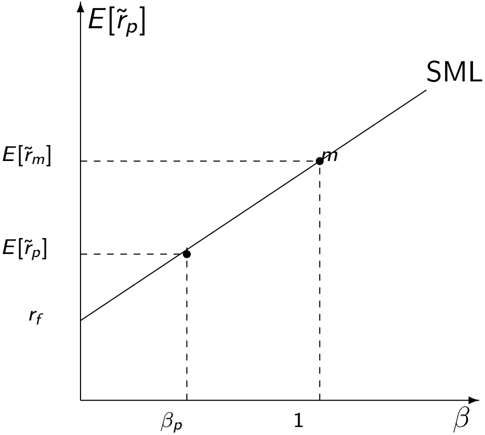
Combining the risk-free asset with the market portfolio:
ω: the weight on the risk free asset.

$$E[\tilde{r}_p] = \omega r_f + (1 - \omega)E[\tilde{r}_m]$$

and

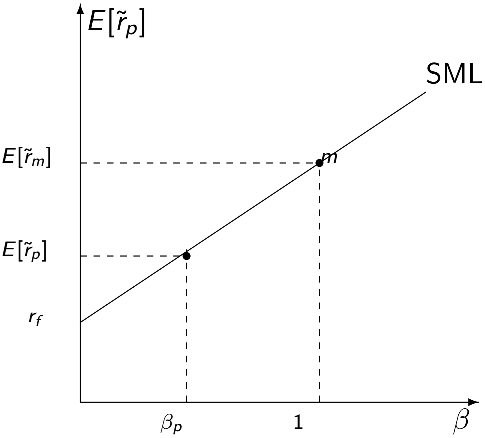
$$\beta_p = \omega \times 0 + (1 - \omega) \times 1 = (1 - \omega)$$

The Security Market Line (SML)



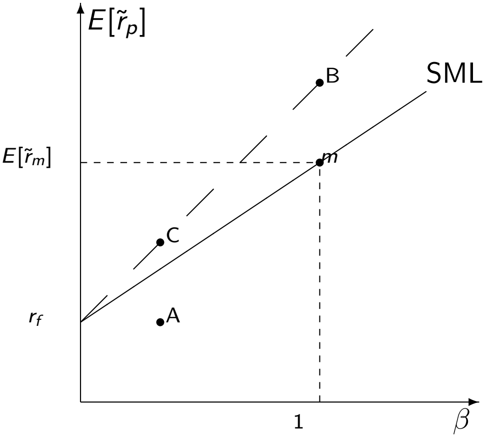
So we see that combinations of the riskfree asset and the market portfolio must fall along the security market line SML.

But what about an arbitrary asset or portfolio? Where do they plot in expected return - beta space? Well, to get markets to clear (i.e. demand equals supply), they too must plot along the SML.



Exercise

The expected return – beta relation for two stocks, A and B,



Note that the two stocks do not plot on the SML. Stock A plots below and stock B plots above the SML.

1. How would you combine the two in a portfolio?
2. What would be the effects on stock prices?

Exercise solution

Since the average beta is 1, if there exist a stock that plots *below* the SML, there must also exist at least one stock that plots *above* the SML.

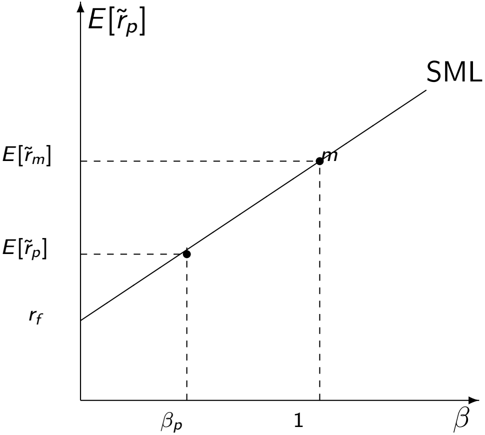
Now ask yourself whether you would hold stock A at all. You would do better if you instead dropped stock A from your portfolio and replaced it with asset C (a combination of the riskfree asset and stock B). By including asset C in your portfolio, you could increase the expected return on your portfolio without increasing the risk. This would create excess demand for stock B and excess supply for stock A. Consequently, in equilibrium, all assets and portfolios must plot on the SML.

Security market line

Summmarizing, the SML is:

$$E[\tilde{r}_j] = r_f + (E[\tilde{r}_m] - r_f)\beta_j$$

where $(E[\tilde{r}_m] - r_f)$ is called the *market risk premium* (MRP).



Exercise

You are given the following information

Portfolio	Average annual Rate of return (nominal)	Average annual Rate of return (real)	Average risk premium (Extra return versus T-bills)
Common stocks	12.0	8.8	8.4
Corporate bonds	5.1	2.1	1.7
Government bonds	4.4	1.4	1.0
Treasury bills	3.5	0.4	0.0
Average rates of return for the period 1926 to 1985.			

Exercise

Betas for selected common stocks.

Stock	Beta
AT&T	0.81
Digital Eq.	1.21
Bristol Myers	0.91
Exxon	0.71
General Mills	0.57
MCI Comm.	1.52
Compaq	1.73
Genentech	1.95
Mesa Petroleum	0.68
Holly Sugar	0.62

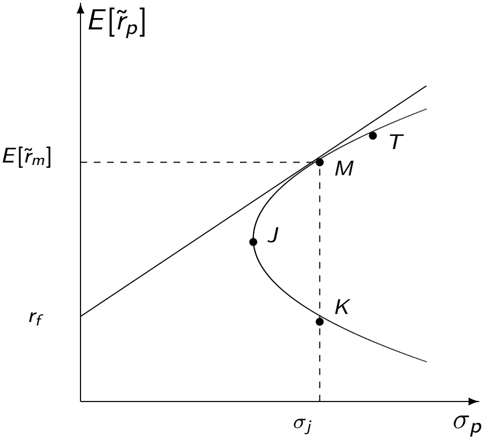
Use the information to compute the expected rates of return on these ten stocks. Assume that the current T-bill rate is 7.5%.

Exercise solution

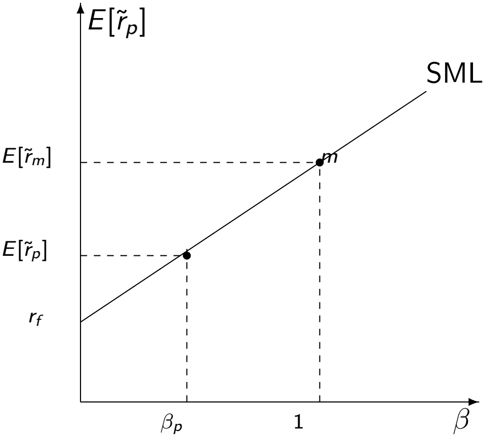
Market risk premium $= (E[\tilde{r}_m] - r_f) = 8.4\%$.
Riskfree interest rate $= r_f = 7.5\%$.

Stock	$E[\tilde{r}] = r_f + \beta(E[\tilde{r}_m] - r_f)$
AT&T	14.30
Digital Eq.	17.66
Bristol Myers	15.14
Exxon	13.46
General Mills	12.29
MCI Comm.	20.27
Compaq	22.03
Genentech	23.88
Mesa Petroleum	13.21
Holly Sugar	12.71

Relationship between the CML and the SML.



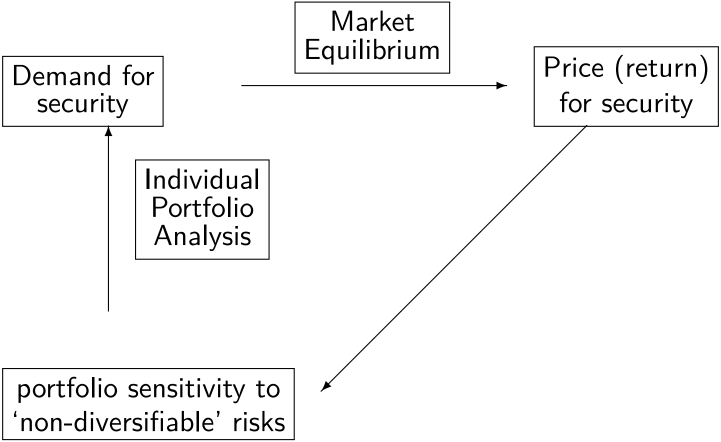
Relationship between the CML and the SML.



Relationship between the CML and the SML.

1. All stocks and portfolios plot along the SML, only efficient portfolios plot along the CML.
2. All efficient portfolios plotting on the CML can be obtained by combining the riskfree asset with the market portfolio.
3. All efficient portfolios plotting on the CML are perfectly positively correlated with the market portfolio.
4. All stocks (and other risky assets) lie on the interior of the portfolios opportunity set. They do *not* plot on the CML.
5. Betas combine linearly and measure the systematic (market) risk of an asset. The beta of the market portfolio is 1.0.

Basic lesson of the CAPM



CAPM and combinations of assets (portfolios)

Riskiness/cost of capital of *portfolios*, combinations of assets/projects.
Useful: CAPM is linear in betas.
Collection of projects with betas β_i:
Riskiness of the portfolio (the beta of the portfolio β_p):

$$\beta_p = \sum_i \omega_i \beta_i$$

Alternatively: Use returns directly.
Use CAPM to find return for each individual asset, and then use the individual asset returns to calculate portfolio returns:

$$E[r_i] = r_f + \beta_i (E[r_m] - r_f)$$

$$E[r_p] = \sum \omega_i E[r_i]$$

Exercise

You are given the following information about your portfolio

Stock	beta β_i	weight ω_i
A	1.25	0.25
B	1.00	0.50
C	0.75	0.25

Let $E[\tilde{r}_m] = 10\%$ and $r_f = 5\%$.

1. How do we calculate the expected return on this portfolio?

Exercise solution

Calculate expected returns for each stock using individual betas, and then find the portfolio returns.

$$
\begin{aligned}
E[\tilde{r}_i] &= r_f + (E[r_m] - r_f)\beta_i \\
E[\tilde{r}_A] &= 0.05 + (0.10 - 0.05)1.25 \\
&= 11.25\% \\
E[\tilde{r}_B] &= 0.05 + (0.10 - 0.05)1.0 \\
&= 10\% \\
E[\tilde{r}_C] &= 0.05 + (0.10 - 0.05)0.75 \\
&= 8.75\%
\end{aligned}
$$

Expected portfolio return

$$
\begin{aligned}
E[\tilde{r}_p] &= \sum_{i=1}^{3} \omega_i E[\tilde{r}_i] \\
&= \frac{1}{4}11.25\% + \frac{1}{2}10\% + \frac{1}{4}8.75\% \\
&= 10\%
\end{aligned}
$$

Exercise solution

Find the beta for the portfolio, and then use this beta to find expected portfolio return.

$$
\begin{aligned}
\beta_p &= \sum_{i=1}^{3} \omega_i \beta_i \\
&= 0.25 \cdot 1.25 + 0.5 \cdot 1.0 + 0.25 \cdot 0.75 \\
&= 1
\end{aligned}
$$

Expected portfolio return

$$E[\tilde{r}_p] = r_f + (E[r_m] - r_f)\beta_p = 10\%$$

Exercise

Suppose you are the manager of an investment fund in a CAPM world. Ignore taxes. Given the following forecast:

$E[\tilde{r}_m] = 16\%$

$\sigma(r_m) = 0.20$

$r_f = 8\%$

1. Would you recommend investment in a security j with the following characteristics: $E[\tilde{r}_j] = 12\%$ and $\text{cov}(\tilde{r}_j, \tilde{r}_m) = 0.01$?
2. Suppose next period it turns out that this security j has had a return of only 5%. How would you explain this, given that $E[\tilde{r}_j] = 12\%$?

Exercise solution

$$\beta_j = \frac{\text{cov}(\tilde{r}_j, \tilde{r}_m)}{\text{var}(\tilde{r}_m)} = \frac{0.01}{(0.20)^2} = \frac{0.01}{0.04} = 0.25$$

$$\begin{aligned} r_j &= r_f + (E[r_m] - r_f)\beta_j \\ &= 0.08 + (0.16 - 0.08)0.25 = 0.10 = 10\% \end{aligned}$$

Since the expected return of 12% is higher than the required return, should invest.

Exercise solution

Question: Suppose next period it turns out that this security j has had a return of only 5%. How would you explain this, given that $E[\tilde{r}_j] = 12\%$?
Answer: The after – the – fact returns may be different from the expected returns. You might just had a bad outcome, the CAPM might still be true.

Performance evaluation with CAPM

Use of CAPM for evaluating equity investments:
Is the return from owning a stock the stocks *risk-compensated required return*?

Exercise

A stock has a beta of 0.9. A security analyst who specializes in studying this stock expects its return to be 13%. Suppose the risk free rate is 8% and the market risk premium is 6%.

1. Is the analyst pessimistic or optimistic about this stock relative to the markets expectation?

Exercise solution

The market expects

$$\begin{aligned} E[r] &= r_f + (E[r_m] - r_f)\beta \\ &= 0.08 + 0.06 \cdot 0.9 \\ &= 13.4\% \end{aligned}$$

The analyst is pessimistic, since his expectation of 13% is lower than the 13.4% expected return for a stock with $\beta = 0.9$.

Use of betas in practice

There is a variety of problems with the estimation of betas.

1. Betas may change over time.
   - Due to estimation errors.
   - Due to changes in the underlying risk characteristics of the firm.
2. Determining the appropriate time period to do the estimation.
3. If you want to estimate the beta for a new investment project, you would have no past information about how the returns on the new project covary with the market.

Estimating beta from market data

Find beta from historical data for stock in question

Exercise

You want to estimate the beta of the company Norsk Hydro (NHY), traded at the Oslo Stock Exchange. Use the last five years of observations of the stock and the market (an Oslo Stock Exchange Index).

1. Use daily returns to estimate the beta.
2. Do you get a different estimate if you use monthly returns?

Exercise Solution

Solution

The first step in this exercise is to gather returns data necessary to do the calculations.

This example uses data download from the Oslo Stock Exchange, five years worth of data of historical data ending in 2012.

Note that the stock prices are adjusted for corporate events, such as the SEO in july of 2010.

The beta calculation is to find

$$\beta_{NHY} = \frac{\text{cov}(r_m, r_{NHY})}{\text{var}(r_m)}$$

where r_m is the return on the index, and r_{NHY} is the return on NHY.

Exercise Solution

We therefore first transform prices to returns using

$$r_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

and then use the excel functions for covariance (COVAR) and variance (VAR) to do the calculation
Doing so we find the beta of NHY as

$$\beta_{NHY} = \frac{\text{cov}(r_{NHY}, r_m)}{\text{var}(r_m)} \approx= 1.21$$

This calculation is illustrated in the spreadsheet `nhy_beta.xls` (available on the course page).

Exercise Solution

To implement the second part of the question, do the calculation from monthly returns, one need to first find monthly returns.
While this can be done in a spreadsheet, it is by no means straightforward to pick end-of-month prices.
It is much easier to do these calculations as regressions in R, as illustrated next.
Note that the data for doing this is, as well as the code for the R examples, is available on the course homepage
First, using daily returns, running the regression.

Exercise Solution

Reading the data

```
# read files pulled from OSE homepage
library(zoo)
OSEBXPric <- read.zoo("../data/osebx.csv",
                   header=TRUE,format="%d.%m.%y",sep=",",sk
# pick first column, the closing value
OSEBXPric <- OSEBXPric[,1]
names(OSEBXPric)[1] <- "OSEBXClose"
NHYPric   <- read.zoo("../data/nhy.csv",
                   header=TRUE,format="%d.%m.%y",sep="\t",s
NHYPric   <- NHYPric[,1]
names(NHYPric)[1] <- "NHYClose"
```

Exercise Solution

Running the regression on daily data:

```
> OSEBXdRet <- (OSEBXPric-lag(OSEBXPric))/lag(OSEBXPric)
> NHYdRet <- (NHYPric - lag(NHYPric))/lag(NHYPric)
> beta <- cov(OSEBXdRet,NHYdRet) / var(OSEBXdRet)
> print(beta)
[1] 1.216354
> lm(NHYdRet~OSEBXdRet)

Call:
lm(formula = NHYdRet ~ OSEBXdRet)

Coefficients:
(Intercept)    OSEBXdRet
  0.0008635    1.2163540
```

Exercise Solution

More detailed output

```
lm(formula = NHYdRet ~ OSEBXdRet)

Residuals:
      Min        1Q    Median        3Q       Max
-0.104060 -0.008742 -0.000554  0.009200  0.171459

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0008635  0.0005197   1.662   0.0969 .
OSEBXdRet   1.2163540  0.0248864  48.876   <2e-16 ***

Residual standard error: 0.01843 on 1256 degrees of freedom
Multiple R-squared:  0.6554,Adjusted R-squared:  0.6551
F-statistic:  2389 on 1 and 1256 DF,  p-value: < 2.2e-16
```

Exercise Solution

Then doing the regression on monthly data

```
> # now find monthly returns, first monthly prices
> OSEBXMpric <- OSEBXPric[endpoints(OSEBXPric,on="months")]
> NHYMPric <- NHYPric[endpoints(NHYPric,on="months")]
> OSEBXMrets <- (OSEBXMpric-lag(OSEBXMpric))/lag(OSEBXMpric
> NHYMrets <- (NHYMPric - lag(NHYMPric))/lag(NHYMPric)
> lm (NHYMrets ~ OSEBXMrets)

Call:
lm(formula = NHYMrets ~ OSEBXMrets)

Coefficients:
(Intercept)   OSEBXMrets
    0.01642      1.28031
```

Exercise Solution

More detailed output

```
lm(formula = NHYMrets ~ OSEBXMrets)

Residuals:
      Min        1Q    Median        3Q       Max
-0.117934 -0.037916 -0.005473  0.034074  0.118008

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.016415   0.007252   2.263   0.0274 *
OSEBXMrets  1.280306   0.081641  15.682   <2e-16 ***

Residual standard error: 0.05607 on 58 degrees of freedom
Multiple R-squared:  0.8092,Adjusted R-squared:  0.8059
F-statistic: 245.9 on 1 and 58 DF,  p-value: < 2.2e-16
```

Exercise Solution

Frequency	Beta estimate
Daily	1.216
Monthly	1.280

Industry betas/comparables

Estimating the beta for a new project or for a division.
Alternative method

- Comparable company
- Industry betas
  Use a beta estimate for the industry to which the project belongs.
  These industry betas are often more reliable than the estimates for individual firms and should provide a relatively good measure of the project or division's market risk.

Use of betas in practice

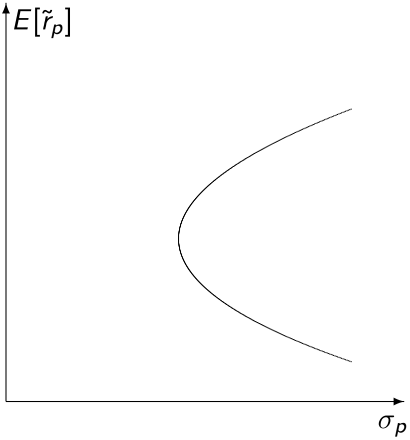
Industry	Beta
Electronic components	1.49
Crude petroleum and natural gas	1.07
Retail department stores.	.95
Chemicals	.89
Food	.84
Trucking	.83
Paper and allied products	.82
Airlines	.75
Steel	.66
Railroads	.61
Telephone companies	.50
Electric utilities	.46

Lecture Summary - CAPM

- Formalize a method to account for the *riskiness* in an investment.
- Assume mean-variance preferences:
  - Prefer higher expected return (mean expected returns)
  - Dislike variability of wealth (standard deviation of returns)

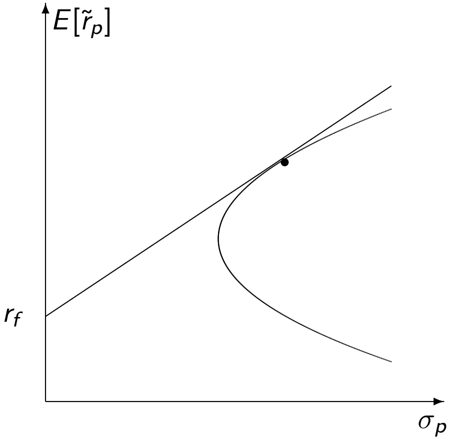
Lecture Summary - CAPM

A *portfolio* a set of *weights* ω_i for each possible investment asset i. Portfolio has expected return $E[r_p]$ and variance $\sigma^2(r_p)$ Possible optimal mean variance combinations wih only risky assets,



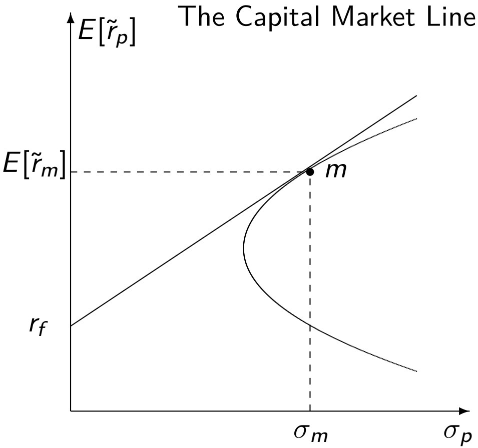
Lecture Summary - CAPM

Possibility of investing/borrowing at the risk free rate r_f, optimal mean/variance portfolios on line between r_f and tangency portfolio on the minimum variance set of risky assets:



Lecture Summary - CAPM

Suppose all investors have the same expectations about asset returns, ie. they face the same choice set:



Investors will optimally combine r_f with the portfolio m.
→ Portfolio m is the *market portfolio*. This is the CAPM.

Lecture Summary - CAPM

Implication: For an individual asset, the only risk that matters is the contribution of risk to the market portfolio.
Hence, for an individual asset, its risk is summarized in the assets *beta*:

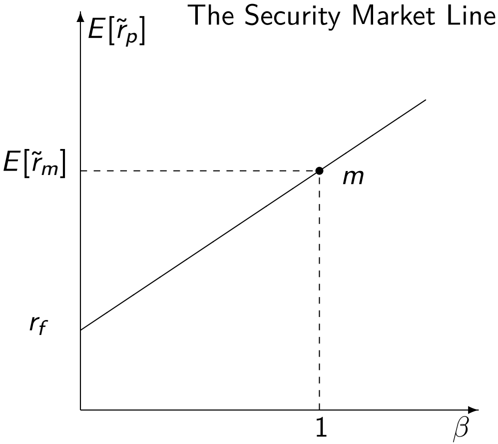
$$\beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$$

Using the CAPM is then a matter of estimating beta.
Calculation of expected return given beta

$$E[r_i] = r_f + \beta_i \left(E[r_m] - r_f\right)$$

Lecture Summary - CAPM

$$E[r_i] = r_f + \beta_i \left(E[r_m] - r_f\right)$$



Lecture Summary - CAPM

Portfolios Linear in both returns and beta.
ω_i fraction of wealth in asset i.
Portfolio beta

$$\beta_p = \sum_i \omega_i \beta_i$$

$$E[r_p] = r_f + \beta_p(E[r_m] - r_f)$$

Portfolio returns

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f)$$

$$E[r_p] = \sum \omega_i E[r_i]$$

Lecture Summary - CAPM

Corporate implication

The CAPM gives the expected return per unit of risk.
This is also the *required return* for corporate investments with that risk.
Company risk weighted average of risk of

- debt
- equity

The *asset beta* is given by:

$$\beta^* = \beta_D \cdot \frac{D}{V} + \beta_E \cdot \frac{E}{V}$$

where $\beta_D =$ Beta of the firm's debt.
$D =$ Market value of the firm's debt.
$E =$ Market value of the firm's equity.
$V = D + E$