PROBLEM SET: Bond Pricing

Exercise 1. [2]

A pure discount bond has a face value of 10,000 and matures in six years. The yield-to-maturity of similar bonds is currently 9 percent. Compounding is discrete, annual.

What is the current price of the bond?

1. 1,592.05
2. 4,600.00
3. 5,962.67
4. 7,721.83
5. I choose not to answer.

Exercise 2. [2]

A corporate bond with a face value of kr 1000 has 6 years to maturity and an annual coupon rate of 8% with annual payments. The bond's price today is kr 958. Compounding is discrete, annual.

What is the bond's yield to maturity?

1. 7.46%
2. 7.81%
3. 8.89%
4. 8.93%
5. I choose not to answer.

Exercise 3. *Bond* [2]

A 10-year bond is issued with a face value of €1,000, paying interest of €60 a year. If market yields fall shortly after the bond is issued, what happens to the bond's

1. Coupon Rate?
2. Price?
3. Yield to Maturity?

Exercise 4. [2]

What is the yield to maturity on a 5-year bond with a nominal value of $100, a 10% coupon rate, an annual coupon frequency and a price of 97.856?

Exercise 5. *Dr No's Bond* [4]

Dr No owns a bond, serial number 007, issued by the James Company. The bond pays $100 for each of the next three years, at which time it is retired and pays its face value of $1000.

(a) How much is the James' bond 007 worth to Dr No at an interest rate of 10%?

(b) How valuable is James bond 007 at an interest rate of 5%?

Ms Yes offers Dr No $1,100 for the James bond 007.

(c) Should Dr No say yes or no to Ms Yes if the interest rate is 10%?

(d) What if the interest rate is 5%?

In order to destroy the world, Dr No hires Professor Know to develop a nasty zap beam. In order to lure Professor Know from his cushy-soft university position at Jail university, Dr No will have to pay the professor $100 a year. The nasty zap beam will take three years to develop, at the end of which it can be built for $1000.

(e) If the interest rate is 5%, how much money will Dr No need to finance this dastardly program?

(f) If the interest rate was 10%, would the world be in more danger or less danger from Dr No?

Exercise 6. [2]

A bond is currently priced at $B_0 = 97.5563$. The bond has an annual coupon of 10% (with discrete, annual compounding), a face value of 100, and a time to maturity of 3 years.

1. If the current (annual, discretely compounded) interest rate decreases by one percentage point, what is the new bond price?

Exercise 7. [3]

Suppose you have the following three bonds:

Bond	Coupon	Principal	Maturity
A	50	1000	10 years
B	100	1000	10 years
C	0	1000	10 years

The current term structure of interest rates is as follows:

Year	Spot rate r_t
1	5%
2	6%
3	7%
4	8%
5	9%
6	10%
7	11%
8	12%
9	13%
10	14%

Interest rates are compounded annually.

1. Determine the prices and yield to maturity of these bonds.

Exercise 8. *Bond* [4]

The appropriate discount rate for cash flows received one year from now is 7.5%. The appropriate discount rate for cash flows received two years from now is 11%. The appropriate discount rate for cash flows received three years from now is 14%. Interest rates are compounded annually.

1. What is the price of a two–year bond with a 6% (annual) coupon and a face value of 1000?

2. What is the yield–to–maturity of this bond?

Solutions

PROBLEM SET: Bond Pricing

Exercise 1. [2]

- 5,962.67, (c) is correct

Exercise 2. [2]

t	C_t
0	-958
1	80
2	80
3	80
4	80
5	80
6	1080

IRR = 8.93%

(d) is correct

Exercise 3. *Bond* [2]

1. Nothing
2. Goes up
3. Goes down

Exercise 4. [2]

t	C_t
0	-97.856
1	10
2	10
3	10
4	10
5	110

IRR = 0.105743

Or, in a matrix tool

```
> C
C =
   10   10   10   10  110
> irr([-97.856 C])
ans = 0.10574
```

Exercise 5. *Dr No's Bond* [4]

(a) we calculate todays bond price as the present value of the payments. With an interest rate of 5%:

$$P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{F_T}{(1+r)^T} = \frac{100}{(1.10)} + \frac{100}{(1.10)^2} + \frac{100}{(1.10)^3} + \frac{1000}{(1.10)^3} = 1000$$

(b) With an interest rate of 5%.

$$P_0 = \frac{100}{(1.05)} + \frac{100}{(1.05)^2} + \frac{100}{(1.05)^3} + \frac{1000}{(1.05)^3} = 1136.2$$

(c) Compare the price P_0 calculated above to \$1100. Answer: Yes.

(d) No.

(e) Notice that the cash flows are the same as on the bond, so the PV of the payments is the same is the same, but now we have to think of this PV as the *cost today* of starting the program: Cost of financing the program = 1136.2.

(f) If the interest rate is 5%, the cost of financing is 1000. Dr No needs to raise less money today to finance his program. The world is in *more* danger.

Exercise 6. [2]

Calculating the YTM:

t	=	0	1	2	3
C_t	=	−97.5563	10	10	110

IRR = 0.110016

If the interest rate falls to 10%, the bond is a par. $B = 100$.

Exercise 7. [3]

1. To determine the bond prices, we discount the coupon and principal payments using the appropriate spot rates. That is,

$$P = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t} + \frac{F_T}{(1+r_T)^T}$$

The values of our three bonds are:

Bond	Price
A	574.56
B	879.37
C	269.74

The calculation is, more detailed

$$P_A = \frac{50}{(1+0.05)} + \frac{50}{(1+0.06)^2} + \frac{50}{(1+0.07)^3} \cdots \frac{1050}{(1+0.14)^{10}} = 574.65$$

$$P_C = \frac{10000}{(1+0.14)^{10}} = 269.74$$

The yield to maturity (YTM) is determined by finding the interest rate y that equates the discounted value of the coupon and principal payments to the current price of the bond.

$$P = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{F_T}{(1+y)^T}$$

For our three bonds, the YTM's are

Bond	YTM
A	12.8%
B	12.1%
C	14.0%

So the YTM is a solution to

$$BondPrice_A = 574.65 = \frac{50}{(1+YTM_A)} + \frac{50}{(1+YTM_A)^2} + \frac{50}{(1+YTM_A)^3} \cdots \frac{1050}{(1+YTM_A)^{10}}$$

Illustrating some of the calculations in this example in matlab

```
>> r=(5:14)'/100o
r =
   0.050000
   0.060000
   0.070000
   0.080000
   0.090000
   0.100000
   0.110000
   0.120000
   0.130000
   0.140000

>> t=1:10

t =
    1    2    3    4    5    6    7    8    9   10

>>  r = r'
>>  d = (1./(1.+r).^t')
d =
   0.95238
   0.89000
   0.81630
   0.73503
   0.64993
   0.56447
   0.48166
   0.40388
   0.33288
   0.26974

>>  CflowsA = [50*ones(9,1)',1050]
CflowsA =
     50     50     50     50     50     50     50     50     50   1050

>> BondPriceA = CflowsA*d
BondPriceA =  574.56

>>  CflowsB = [100*ones(9,1)',1100]
```

```
CflowsB =
    100     100     100     100     100     100     100     100     100    1100

>> BondPriceB = CflowsB*d
BondPriceB =  879.37

>>  CflowsC = [zeros(9,1)',1000]
CflowsC =
      0      0      0      0      0      0      0      0      0    1000

>> BondPriceC =  CflowsC * d
BondPriceC =  269.74

>> irr_A = irr (CflowsA, 574.56)*100
irr_A = 12.768
>> irr_B = irr (CflowsB, BondPriceB)*100
irr_B = 12.148
>> irr_C = irr (CflowsC, BondPriceC)*100
irr_C = 14.000
>>
```

Exercise 8. *Bond* [4]

1. Price
$$P_0 = \frac{60}{1.075} + \frac{1060}{(1.11)^2} = 916.13$$

2. Yield to maturity is found by calculating the IRR of the following cashflows

t	$=$	0	1	2
C_t	$=$	-916.13	60	1060

$$\text{IRR} = 0.108917$$

The YTM is 10.89%.