Bond Pricing

Bernt Arne Ødegaard

30 August 2023

Contents

1 What is a Bond?

A bond, which is the classical example of a *fixed income security*, is a (traded) asset with a predetermined stream of future cash payments.

1.1 Classifying bonds

Relevant dimensions: Issuers, maturity, contractual features.

1.2 Key players

Issuers of Debt Securities

Governments and their agencies
Corporations
Commercial Banks
States and municipalities
Special purpose vehicles
Foreign institutions

Financial Intermediaries

Primary dealers
Other dealers
Investment banks
Credit rating agencies
Credit and liquidity enhancers

Institutional and Retail investors

Governments
Pension funds
Insurance companies
Mutual funds
Commercial banks
Foreign institutions
Households

2 Bond Pricing

2.1 Pricing the most standard bond - fixed coupon, terminal payment of principal

$$B_0 = \sum_{t=1}^{T} \frac{E[C_t]}{(1+r)^t} + \frac{E[F_T]}{(1+r)^T}$$

where

- $E[C_t]$ = The expected coupon payment in period t.
- $E[F_T]$ = The expected face value payment in period T.
- r the relevant cost of capital

Special case: Fixed coupon C

$$B_0 = E[C]\left(\frac{1}{r} - \frac{1}{r(1+r)^T}\right) + \frac{E[F_T]}{(1+r)^T}$$

Exercise 1.

A Treasury bond has a coupon rate of 9%, a face value of \$1000 and matures 10 years from today. For a treasury bond the interest on the bond is paid in semi-annual installments. The current riskless interest rate is 12% (compounded semi-annually).

1. Suppose you purchase the Treasury bond described above and immediately thereafter the riskless interest rate falls to 8%. (compounded semi-annually). What would be the new market price of the bond?

2. What is your best estimate of what the price would be if the riskless interest rate was 9% (compounded semi-annually)?

2.2 Yield to maturity.

YTM is equal to the interest rate y that solves the following equation (for a fixed coupon bond with coupon C)

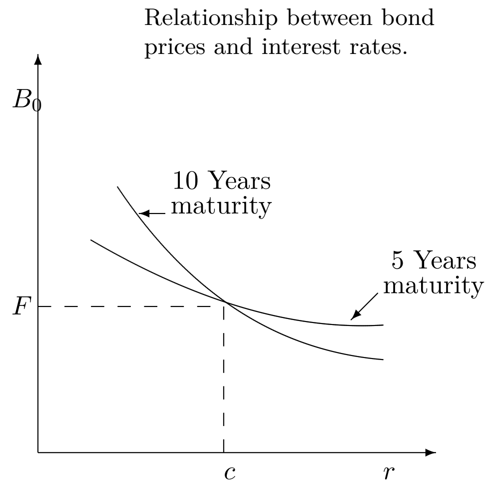
$$P_0 \quad = \quad C\left(\frac{1}{y} - \frac{1}{y(1+y)^T}\right) + \frac{F_T}{(1+y)^T}$$

Exercise 2.

A \$100, 10 year bond was issued 7 years ago at a 10% annual interest rate. The current interest rate is 9%. The current price of the bond is 100.917. Use annual, discrete compounding.

1. Calculate the bonds yield to maturity.

3 Interest rate sensitivity



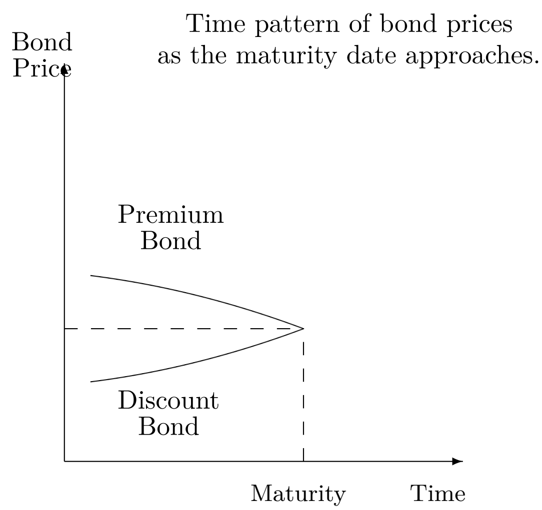
The slope of the curve is

$$
\begin{aligned}
&\text{Slope} \\
&= -\frac{1}{1+r}\left[\sum_{t=1}^{T}\frac{t\cdot C_t}{(1+r)^t}+\frac{T\cdot F_T}{(1+r)^T}\right] \\
&= -\frac{1}{1+r}\left[\sum_{t=1}^{T} t\cdot PV(C_t)+T\cdot PV(F_T)\right]
\end{aligned}
$$

$$
\begin{aligned}
&\text{Percentage change in bond price} \\
&= -\frac{1}{1+r}\left[\sum_{t=1}^{T}\frac{t\cdot PV(C_t)}{P_0}+\frac{T\cdot PV(F_T)}{P_0}\right]
\end{aligned}
$$

The expression in brackets is called the *duration* of the bond.
It can be interpreted as the *weighted average maturity.*

3.1 Interest rate sensitivity depend on time to maturity



4 Bond Duration

$$\begin{aligned} &\text{Percentage change in bond price} \\ = \quad & -\frac{1}{1+r}\left[\sum_{t=1}^{T}\frac{t\cdot PV(C_t)}{P_0}+\frac{T\cdot PV(F_T)}{P_0}\right] \end{aligned}$$

The expression in brackets is called the *duration* of the bond. It can be interpreted as the *weighted average maturity.*

Exercise 3.

Suppose you are trying to determine the interest rate sensitivity of two bonds. Bond 1 is a 12% coupon bond with a 7-year maturity and a $1000 principal. Bond 2 is a 'zero-coupon' bond that pays $1000 after 7 years. The current interest rate is 12%.

1. Determine the duration of each bond.
2. If the interest rate increases 100 basis points (100 basis points = 1%), what will be the capital loss on each bond?

Exercise 4.

Consider the pricing of a bond with a flat term structure, where C_t is the cash flow in period t.

$$P_0=\sum_{t=1}^{\infty}\frac{C_t}{(1+r)^t}=\sum_{t=1}^{\infty}C_t\left(\frac{1}{1+r}\right)^t$$

1. Determine the first derivative of the price with respect to the interest rate.
2. Find the duration part of this expression.

Exercise 5.

The term structure is flat with annual compounding. Consider the pricing of a perpetual bond. Let C be the per period cash flow

$$B_0=\sum_{t=1}^{\infty}\frac{C}{(1+r)^t}=\frac{C}{r}$$

1. Determine the first derivative of the price with respect to the interest rate.
2. Find the duration of the bond.

5 Convexity

Recall duration – first derivative.

$$\frac{\Delta P}{P}\approx -D^*\Delta y$$

Only an approximation, to be more accurate also account for second order effects.

Convexity: curvature of the relationship between bond prices and interest rates.

Modify above as

$$\frac{\Delta P}{P}=-D^*\Delta y+\frac{1}{2}\times\text{Convexity}\times(\Delta y)^2$$

Calculate convexity of a bond with T periods left as:

$$\text{Convexity}=\frac{1}{P(1+r)^2}\sum_{t=1}^{T}(t+t^2)PV(C_t)$$

where C_t is the cash flow at time t and $PV(\cdot)$ is the present value operator.

Exercise 6.

A 3 year bond with a face value of $100 makes annual coupon payments of 10%. The current interest rate (with annual compounding) is 9%.

1. Find the bond's current price.
2. Suppose the interest rate changes to 10%, determine the new price of the bond by direct calculation.
3. Instead of direct calculation, use duration to estimate the new price and compare it to the correct price.
4. Use convexity to improve on your estimation using duration.

6 Bond Pricing - term structure of interest rates

Exercise 7.

A bond promises the following sequence of payments:

t	=	1	2	3	4
Cashflow X_t	=	10	10	10	110

The interest rates r_t and prices d_t of future risk free cash flows are as follows

t	=	1	2	3	4
r_t	=	5.3%	5.4%	5.6%	5.7%
d_t	=	0.95	0.9	0.85	0.80

Interest rates are compounded annually.

1. Calculate the bond's price

References

Jonathan Berk and Peter DeMarzo. *Corporate Finance*. Pearson, fifth edition, 2020.