PROBLEM SET: Bond Portfolios

Exercise 1.

[5]
Consider an equally weighted portfolio of two bonds, A and B. Bond A is a zero coupon bond with 1 year to maturity. Bond B is a zero coupon bond with 3 years to maturity. Both bonds have face values of 100. The current interest rate is 5%.

1. Determine the bond prices.
2. Your portfolio is currently worth 2000. Find the number of each bond invested.
3. Determine the duration of the portfolio.
4. Determine the convexity of your position.

Exercise 2.

Bond Pricing and Interest Rate Sensitivity [4]
A 3 year bond with a face value of $100 makes annual coupon payments of 10%. The current interest rate (with annual compounding) is 9%.

1. Find the bond's current price.
2. Suppose the interest rate changes to 10%, determine the new price of the bond by direct calculation.
3. Instead of direct calculation, use duration to estimate the new price and compare it to the correct price.
4. Use convexity to improve on your estimation using duration.

Exercise 3.

Portfolio Duration [1]
A company invests $1,000 in a five-year zero coupon bond and $4,000 in a ten-year zero-coupon bond.

1. What is the duration of the portfolio?

Exercise 4.

Discount bond [1]
A discount (zero coupon) bond with a principal of 100 has a maturity of 6 years. The term structure of interest rates is flat with a (continously compounded) interest rate of 5%.

1. Determine the duration of the bond.

Solutions

PROBLEM SET: Bond Portfolios

Solution to Exercise 1.

[5]

1. Bond prices Calculate bond prices:
$$P_A = \frac{100}{1.05} = 95.24$$
$$P_B = \frac{100}{1.05^3} = 86.38$$

2. Let n_A be the number of bond A to buy and n_B the number of bond B. Since the fractions are equal, invested 1000 in each bond.
$$n_A = \frac{1000}{95.24} = 10.50$$
$$n_B = \frac{1000}{86.38} = 11.57$$
Want to buy 10.50 A bonds and 11.57 B bonds.

3. Portfolio defined by weights
$$w_A = \frac{1}{2}$$
$$w_B = \frac{1}{2}$$
Since both bonds are zero coupon, duration equals maturity.
$$D_A = 1$$
$$D_B = 3$$
Duration of portfolio then
$$D = w_A D_A + w_B D_B = \frac{1}{2}1 + \frac{1}{2}3 = 2$$

4. Calculating convexity. Bond A:

t	C_t	$PV(C_t)$	$PV(C_t)(t^2+t)$
1	100	95.2	190.5
Sum		95.2	190.5
Bondprice			95.2381
Convexity			1.81

Bond B:

t	C_t	$PV(C_t)$	$PV(C_t)(t^2+t)$
1	0	0.0	0.0
2	0	0.0	0.0
3	100	86.4	1036.6
Sum		86.4	1036.6
Bondprice			86.3838
Convexity			10.88

Convexity of portfolio:
$$\frac{1}{2}1.81 + \frac{1}{2}10.88 = 6.345$$
The portfolio has slightly lower convexity than the obligation.

Solution to Exercise 2.

Bond Pricing and Interest Rate Sensitivity [4]

1. The bond price:

t	$=$	0	1	2	3
C_t	$=$	0	10	10	110

$$NPV = 0 + \frac{10}{(1+0.09)^1} + \frac{10}{(1+0.09)^2} + \frac{110}{(1+0.09)^3} = 102.531$$

2. If the interest rate increases to 10%, the bond will be selling at par, equal to 100, which can be confirmed with direct computation:

$$NPV = 0 + \frac{10}{(1+0.1)^1} + \frac{10}{(1+0.1)^2} + \frac{110}{(1+0.1)^3} = 100$$

3. Calculate the bond's duration:

t	C_t	$PV(C_t)$	$tPV(C_t)$
1	10	9.2	9.2
2	10	8.4	16.8
3	110	84.9	254.8
Sum		102.5	280.8
$Bondprice$			102.531
$Duration$			2.74

Modified duration:

$$D^* = \frac{D}{1+r} = \frac{2.74}{1.09} = 2.51$$

Let us now calculate the change in the bond price

$$\frac{\Delta P}{P} = -D^* \Delta y = -2.51 \cdot 0.01 = -0.0251$$

Which means theat the bond price changes to:

$$P + \delta P = 102.531 + \left(\frac{\Delta P}{P}\right) P = 102.531 - 0.0251 \cdot 102.531 = 99.957$$

4. Calculate the bond's convexity

t	C_t	$PV(C_t)$	$PV(C_t)(t^2+t)$
1	10	9.2	18.3
2	10	8.4	50.5
3	110	84.9	1019.3
Sum		102.5	1088.1
$Bondprice$			102.531
$Convexity$			8.93

Recalculating the change in the bond price using convexity:

$$\frac{\Delta P}{P} = -D^* \Delta y + \frac{1}{2}(\text{Convexity}) = -2.51 \cdot 0.01 + \frac{1}{2} 8.93 (0.01)^2 = -0.0251 + 0.00044 = -0.02465$$

Use this to re-estimate the bond price:

$$P + \delta P = 102.531 \left(1 + \left(\frac{\Delta P}{P}\right) P\right) = 102.531(1 - 0.02465) = 100.0036$$

Solution to Exercise 3.

Portfolio Duration [1]

9 years.

Duration of zero coupon bonds equal maturity.

$$\frac{1000}{5000} 5 + \frac{4000}{5000} 10 = 9$$

Solution to Exercise 4.

Discount bond [1]

For any zero coupon bond duration equals maturity.

Duration = <u>6 years</u>.