The mechanics of discounting, interest rate calculations

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1 Summary

1. Method for evaluating current value of stream of future cashflows

   Present Value

2. Subtracting present cost

   Net Present Value

   or: Value added

Future Value

$$FV_t = PV(1+r)^t$$

Present Value

$$PV = \frac{FV_t}{(1+r)^t}$$

Discount factor

$$d_t = 1/(1+r)^t$$

The internal rate of return: the discount rate that makes the $NPV = 0$.

Multiple cash flows – present value additive

$$PV = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t}$$

$$NPV = PV - C_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t} - C_0$$

A *perpetuity*: fixed income for each year for the indefinite future.

$$PV = \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^t}$$

$$PV = \frac{C}{r}$$

A *growing perpetuity*: a fixed income for each year that grows at a constant rate g.

$$PV = \sum_{t=1}^{\infty} \frac{C_1(1+g)^{t-1}}{(1+r)^t}$$

$$PV = \frac{C_1}{r-g}$$

An *annuity*: pays a fixed (constant) amount each year for a specified finite number of years (T).

$$
\begin{aligned}
PV &= \sum_{t=1}^{T} \frac{C}{(1-r)^t} \\
&= C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]
\end{aligned}
$$

The term $A_{r,T} = \left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]$ is called an *annuity factor*.

Compounding refers to the frequency with which interest is paid.

Discrete compounding m times a year

$$FV_t = PV\left(1 + \frac{r}{m}\right)^{mt}$$

$$PV = FV_t\left(1 + \frac{r}{m}\right)^{-mt}$$

Continous compounding

$$FV_t = PV(e^{rt})$$

$$PV = FV_t(e^{-rt})$$

Translating

$$r = n \ln\left(1 + \frac{r_n}{n}\right)$$

$$r_n = n\left(e^{\frac{r}{n}} - 1\right)$$