The Capital Asset Pricing Model (CAPM)

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1 Summary

Purpose: Formalize a method to account for the *riskiness* in an investment.
Develop under specifi assumptions.

1.1 Mean-Variance preferences

All investors decide their portfolios with preferences that

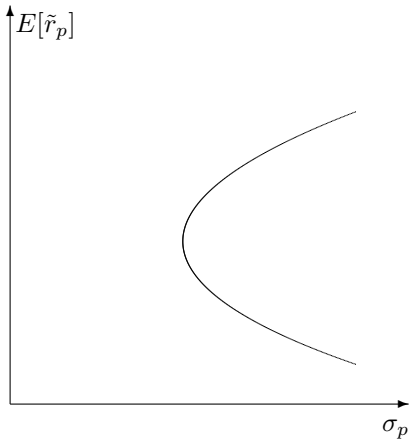
- Prefer higher expected return (mean expected returns)
- Dislike variability of wealth (standard deviation of returns)

A *portfolio* a set of *weights* ω_i for each possible investment asset i.
Portfolio has

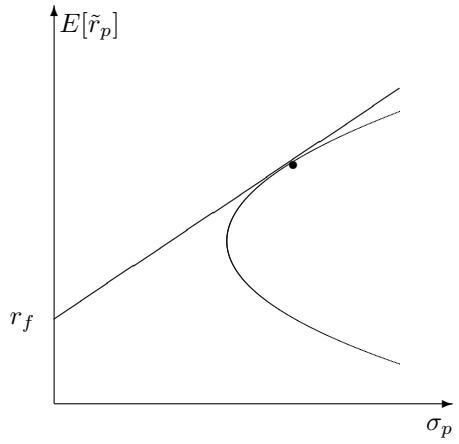
$$\text{Portfolio expected return:} \quad E[r_p] = \sum_{i=1}^{n} \omega_i E[r_i]$$

$$\text{Portfolio variance:} \quad \sigma^2(r_p) = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \text{cov}(r_i, r_j)$$

Implications (graphically): Possible optimal mean variance combinations wih only risky assets,



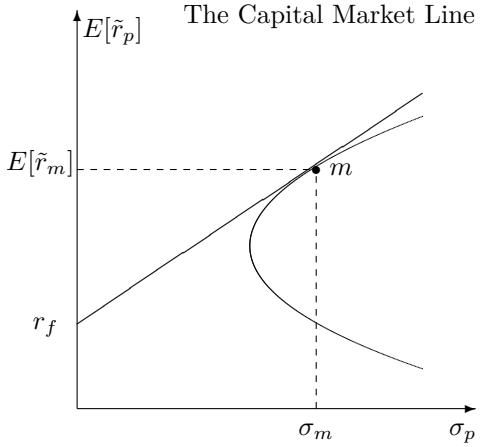
Introduce the possibility of investing/borrowing at the risk free rate r_f, optimal mean/variance portfolios on line between r_f and tangency portfolio on the minimum variance set of risky assets:



1.2 CAPM

Suppose all investors have the same expectations about asset returns, ie. they face the same choice set:

The Capital Market Line



Investors will optimally combine r_f with the portfolio m.
$\rightarrow$ Portfolio m is the *market portfolio.*

This is the CAPM.

Implication: For an individual asset, the only risk that matters is the contribution of risk to the market portfolio.

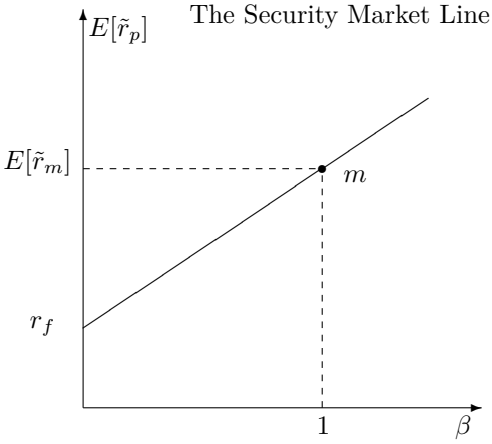
Hence, for an individual asset, its risk is summarized in the assets *beta*:

$$\beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$$

Using the CAPM is then a matter of estimating beta.

Calculation of expected return given beta

$$E[r_i] = r_f + \beta_i \left(E[r_m] - r_f\right)$$



1.3 Portfolios

Linear in both returns and beta.

ω_i fraction of wealth in asset i.

Portfolio beta

$$\beta_p = \sum_i \omega_i \beta_i$$

$$E[r_p] = r_f + \beta_p (E[r_m] - r_f)$$

Portfolio returns

$$E[r_i] = r_f + \beta_i (E[r_m] - r_f)$$

$$E[r_p] = \sum \omega_i E[r_i]$$

1.4 Corporate implication

The CAPM gives the expected return per unit of risk.

This is also the *required return* for corporate investments with that risk.