

## PROBLEM SET: Bond Pricing

### Exercise 1. [2]

A bond has a face value of 1,000. The bond pays 8% semi-annual coupon and has two years left to maturity. The semi-annually compounded yield to maturity on similar bonds is currently 12%.

What is the price of the bond?

- (a) 795.52
- (b) 826.44
- (c) 930.70
- (d) 947.93
- (e) I choose not to answer.

### Exercise 2. *Dr No's Bond* [4]

Dr No owns a bond, serial number 007, issued by the James Company. The bond pays \$100 for each of the next three years, at which time it is retired and pays its face value of \$1000.

- (a) How much is the James' bond 007 worth to Dr No at an interest rate of 10%?
- (b) How valuable is James bond 007 at an interest rate of 5%?

Ms Yes offers Dr No \$1,100 for the James bond 007.

- (c) Should Dr No say yes or no to Ms Yes if the interest rate is 10%?
- (d) What if the interest rate is 5%?

In order to destroy the world, Dr No hires Professor Know to develop a nasty zap beam. In order to lure Professor Know from his cushy-soft university position at Jail university, Dr No will have to pay the professor \$100 a year. The nasty zap beam will take three years to develop, at the end of which it can be built for \$1000.

- (e) If the interest rate is 5%, how much money will Dr No need to finance this dastardly program?
- (f) If the interest rate was 10%, would the world be in more danger or less danger from Dr No?

### Exercise 3. *Bond* [2]

A 10-year bond is issued with a face value of \$1,000, paying interest of \$60 a year. If market yields increase shortly after the T-bond is issued, what happens to the bond's

1. Coupon Rate?
2. Price?
3. Yield to Maturity?

### Exercise 4. [2]

A bond is currently priced at  $B_0 = 97.5563$ . The bond has an annual coupon of 10% (with discrete, annual compounding), a face value of 100, and a time to maturity of 3 years.

1. If the current (annual, discretely compounded) interest rate decreases by one percentage point, what is the new bond price?

**Exercise 5.** [2]

What is the price of a 5-year bond with a nominal value of \$100, a yield to maturity of 7% (with annual compounding frequency), a 10% coupon rate and an annual coupon frequency.

**Exercise 6.** [2]

What is the yield to maturity on a 5-year bond with a nominal value of \$100, a 10% coupon rate, an annual coupon frequency and a price of 97.856?

## Empirical

## Solutions

### PROBLEM SET: Bond Pricing

#### Exercise 1. [2]

$t$	=	0	1	2	3	4
$C_t$	=	0	40	40	40	1040

$$NPV = 0 + \frac{40}{(1+0.06)^1} + \frac{40}{(1+0.06)^2} + \frac{40}{(1+0.06)^3} + \frac{1040}{(1+0.06)^4} = 930.698$$

(c) is correct.

#### Exercise 2. *Dr No's Bond* [4]

(a) we calculate today's bond price as the present value of the payments. With an interest rate of 5%:

$$P_0 = \sum_{t=1}^T \frac{C_t}{(1+r)^t} + \frac{F_T}{(1+r)^T} = \frac{100}{(1.10)} + \frac{100}{(1.10)^2} + \frac{100}{(1.10)^3} + \frac{1000}{(1.10)^3} = 1000$$

(b) With an interest rate of 5%.

$$P_0 = \frac{100}{(1.05)} + \frac{100}{(1.05)^2} + \frac{100}{(1.05)^3} + \frac{1000}{(1.05)^3} = 1136.2$$

(c) Compare the price  $P_0$  calculated above to \$1100. Answer: Yes.

(d) No.

(e) Notice that the cash flows are the same as on the bond, so the PV of the payments is the same is the same, but now we have to think of this PV as the *cost today* of starting the program: Cost of financing the program = 1136.2.

(f) If the interest rate is 5%, the cost of financing is 1000. Dr No needs to raise less money today to finance his program. The world is in *more* danger.

#### Exercise 3. *Bond* [2]

1. Nothing
2. Goes down
3. Goes up

#### Exercise 4. [2]

Calculating the YTM:

$t$	=	0	1	2	3
$C_t$	=	-97.5563	10	10	110

$$IRR = 0.110016$$

If the interest rate falls to 10%, the bond is a par.  $B = 100$ .

#### Exercise 5. [2]

$t$	$C_t$
1	10
2	10
3	10
4	10
5	110

$$B_0 = \frac{10}{(1+0.07)^1} + \frac{10}{(1+0.07)^2} + \frac{10}{(1+0.07)^3} + \frac{10}{(1+0.07)^4} + \frac{110}{(1+0.07)^5} = 112.301$$

Or, done in a matrix tool

```
> C=[10 10 10 10 110]
C =
    10    10    10    10   110
> t=[1 2 3 4 5]
t =
    1    2    3    4    5
> y=0.07
y = 0.070000
> B0=C*(1/(1+y)).^t'
B0 = 112.30
```

**Exercise 6.** [2]

$t$	$C_t$
0	-97.856
1	10
2	10
3	10
4	10
5	110

$$\text{IRR} = 0.105743$$

Or, in a matrix tool

```
> C
C =
    10    10    10    10   110
> irr([-97.856 C])
ans = 0.10574
```