Bond Pricing

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1 Summary

Bond: Fixed income security with prearranged payments (coupon C, principal F)
Bonds classified by issuer

- Governments
  - Treasury securities (T-bills, T-bonds)
  - Municipalities
- Corporations
  - Corporate bonds

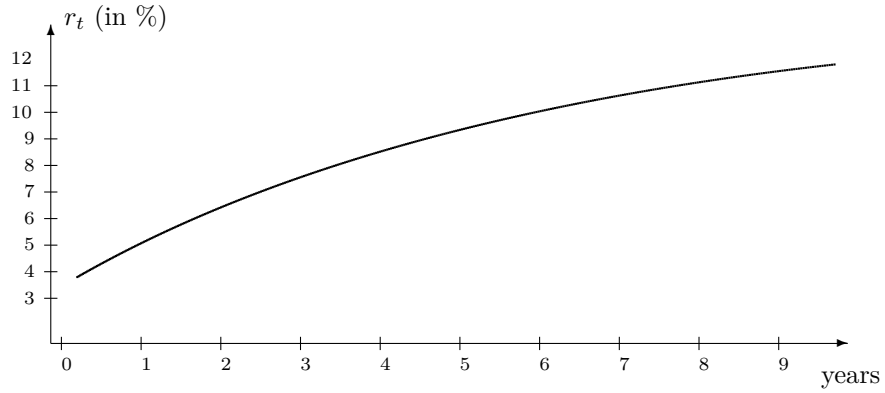
Bonds classified by coupon terms:

- Zero coupon (only repay principal, issued at discount (below face value))
- Fixed coupon
- Variable coupon – Floating rate bond

Bond price B for a fixed interest rate bond with coupon C and principal F. Required interest rate r:

$$B_0 = C\left(\frac{1}{r} - \frac{1}{r(1+r)^T}\right) + \frac{F}{(1+r)^T}$$

Yield to maturity: Internal rate of return of investment of buying a bond and keeping it to maturity.
Empirically – Interests rate varies with maturity – the term structure of interest rates.



To specify link between interest rate and time to maturity:

- Discount factor d_t – current price of zero coupon bond paying one at time t
- Spot rate r_t – current interest rate for payments recieved at time t.

Bond price when accounting for the term structure:

$$B_0 = \sum_{t=1}^{T} d_t C_t + d_T F_T \text{ (with discount factors)}$$

$$B_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t} + \frac{F_T}{(1+r_T)^T} \text{ (with spot rates)}$$

Duration: Average maturity of a bond.
Calculation of duration for a bond with T periods till maturity:

$$D = \frac{1}{P} \sum_{t=1}^{T} tPV(C_t)$$

where P is the current bond price and $PV(\cdot)$ is the present value operator