

Financial Econometrics

Problem Set

Exercise 1. *Linear Equations.* [3]

Consider the system of linear equations

$$x_1 + 3x_2 = 0$$

$$x_1 + x_2 = 1$$

$$2x_1 + 4x_2 = 1$$

1. Write this system in matrix form.
2. Determine if the system is well-defined.
3. If it has a solution, find the solution.

Exercise 2. [4]

Consider

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

1. Compute \mathbf{ABC} , \mathbf{CAB} , \mathbf{BCA} , $\mathbf{CB'A'}$ and $\mathbf{C'B'A'}$.
2. Verify that $(\mathbf{ABC})' = \mathbf{C'B'A'}$.
3. Find the inverses of these matrices. Verify that $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}(\mathbf{AB})^{-1}$.
4. Verify that $\text{tr}(\mathbf{BCA}) = \text{tr}(\mathbf{ABC}) = \text{tr}(\mathbf{CAB})$.
5. Show that $\mathbf{A}(\mathbf{A'A})^{-1}\mathbf{A}'$ and $\mathbf{B}(\mathbf{B'B})^{-1}\mathbf{B}'$ are both idempotent. What are the ranks of these two matrices?

Exercise 3.

Consider the linear equation

$$3x_1 + 4x_2 = 5$$

$$4x_1 + 6x_2 = 8$$

Solve this system of equations using a matrix tool.

Exercise 4.

Calculate the sum $1 + 2 + 3 + \dots + 99 + 100$.

Exercise 5.

Define the following variables:

Real numbers $r = 1$, $s = 10$, $u = 2$.

Vectors

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Calculate

$$r\mathbf{a} + s\mathbf{b}$$

$$s\mathbf{A} + u\mathbf{B}$$

$$B'$$

Calculate the determinants of the three matrices

Which of the three matrices are invertible?

Calculate

$$B^{-1}$$

Solve the linear system

$$\mathbf{B}\mathbf{x} = \mathbf{a}$$