Financial Econometrics

Problem Set

Exercise 1. [3]

Consider the following prices of zero-coupon bonds with nominal value of \$1,000 and maturites $T = 1, 2, 3, 4$: $P_{0,1} = 968.52$. $P_{0,2} = 929.02$. $P_{0,3} = 915.15$. $P_{0,4} = 905.95$. Disounting in discrete, annual.

1. Compute the spot rates implied by these prices, and plot the yield curve.
2. Use the information in the above bond prices to find the price of a coupon bond with maturity 4 years from now, annual coupon rate $c = 7\%$ and face value \$1000. Is the bond selling a premium, par or discount?

Exercise 2. *Bond Price* [2]

Consider a 5-year bond with a nominal value of \$100, a yield to maturity of 7% (with annual compounding frequency) and a 10% coupon rate (with annual coupon frequency).

1. Determine the bond price.

Exercise 3. [3]

A 10%, two year bond is traded at a price of 90. The current one year spot rate is $r(0,1) = 12\%$ (with discrete, annual compounding). The bond has a face value of 100.

1. Determine the duration and convexity of the bond, using both the full term structure and the Macaulay style calculations.

Exercise 4. [3]

In their book, Copeland and Weston gives the following exercise:

> Interplanetary starship captain José Ching has been pondering the investment of his recent pilot's bonus of 1000 stenglers. His choice is restricted to two securities: Galactic Steel, selling for 20 stenglers per share, and Nova Nutrients, at 10 stenglers per share. The future state of his solar system is uncertain. If there is a war with a nearby group of asteroids, Captain Ching expects Galactic Steel to be worth 36 stenglers per share. However, if peace prevails, Galactic Steel will be worth only 4 stenglers per share. Nova Nutrients should sell at a future price of 6 stenglers per share in either eventuality.

Your Mission, should you accept is, is to answer the following question:

1. If you have the possibility to transfer wealth across period as money (or stenglers, as it were), would you ever expect to observe the above prices in equilibrium?

Exercise 5. *Bonds* [4]

You observe the three risk free bonds A, B and C:

		Cashflow in period		
Bond	Price	1	2	3
A	95	100	0	0
B	90	10	110	0
C	85	10	10	110

1. What is the current value of receiving one dollar at time 3?
2. What are the interest rates (with annual compounding) implied in these prices?

Another risk free bond D is traded, with the following cash flows:

time:	1	2	3
D	20	20	520

2. What is the current price of bond D?

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Solutions

Exercise 1. [3]

First calculate discount factors (zero coupon prices)

```
> P=[968.52 929.02 915.15 905.95]
P =
  968.52  929.02  915.15  905.95
> C=[1000 1000 1000 1000]
C =
  1000  1000  1000  1000
> d=P./C
d =
  0.96852  0.92902  0.91515  0.90595
```

To check that this gives the right answer, consider the repricing of the bonds with the calculated discount factors `d`.

```
> d.*C
ans =
  968.52  929.02  915.15  905.95
```

Now calculate spot rates

```
> r=(1.0./d).^(1./t)-1
r =
  0.032503  0.037498  0.029997  0.025000
```

Price the bond

```
> Cflow=[70 70 70 1070]
Cflow =
    70    70    70  1070
> B = Cflow * (1./(1+r).^t)'
B = 1166.3
octave>  B = Cflow * d'
B = 1166.3
```

The bond is a premium bond, which is obvious given that the highest spot rate is 3.25%, while the bond coupon is 7%.

Exercise 2. *Bond Price* [2]

```
>> t=1:5
t =
  1  2  3  4  5
> y=0.07
y = 0.070000
>> C=[10 10 10 10 110]
C =
   10   10   10   10  110
>> B= C*(1/(1+y)).^t'
B= = 112.30
```

The bond price is 112.30

Exercise 3. [3]

First need to find the two year spot rate

$$90 = 10d_1 + 110d_2$$
$$d_1 = \frac{1}{1+0.12} = 0.89286$$
$$d_2 = \frac{90 - 10d_1}{110} = 0.73701$$
$$r_2 = 0.16483$$

Here are some of the calculation in a matrix tool

```
> C=[10 110]
C =
   10  110
> r(1)=0.12
r =   0.12000
> d(1)=1/(1+r(1))
d =   0.89286
> d(2)=(90-10*d(1))/110
d =
  0.89286  0.73701
> d=d'
d =
  0.89286
  0.73701
> BondPrice=C*d
BondPrice = 90
> r(2)=d(2)^(-1/2)-1
r =
  0.12000  0.16483
> y = irr([-BondPrice C],0)
y = 0.16249x
> checkprice=C(1)/(1+y)+C(2)/(1+y)^2
checkprice = 90
```

We calculate duration using the two definitions

```
> Duration=1/BondPrice * ( 1*d(1)*C(1) + 2*d(2)*C(2))
Duration = 1.9008
> Duration=1/BondPrice * ( 1*C(1)/(1+y) + 2*C(2)/((1+y)^2) )
Duration = 1.9044
```

Using the term structure we find duration as

$$D = 1.9008$$

using the Macaulay definition we find

$$D = 1.9044$$

Thus, not a major difference.

We also calculate the convexity for the two definitions

```
> Cx=1/BondPrice * 1/(1+y)^2 * ( (1+1)*d(1)*C(1) + (2+2^2)*d(2)*C(2))
Cx = 4.1463
> Cx=1/BondPrice * 1/(1+y)^2 * ( (1+1)*C(1)/(1+y) + (2+2^2)*C(2)/(1+y)^2)
Cx = 4.1570
```

Again, not a major difference with the two methods of calculating

Exercise 4. [3]

1. The answer is no, which is most easily seen by calculation the state prices from the above data:

```
> X=[36 4; 6 6]
X =

  36   4
   6   6

> p=[20 10]'
p =

  20  10

> q = inv(X)*p'
q =

  0.41667
  1.25000
```

   The state price for one state is above one.

   If you can transfer stenglers across periods, you should be selling the state security at 1.25 and paying pack 1 if the state occurs. Sounds like a good deal.

Exercise 5. *Bonds* [4]

1. One way to think of this is using state prices. Want to calculate the price of receiving one dollar for certain in period (state) t.

   If

```
B =
  100     0     0
   10   110     0
   10    10   110
```

   is the payoffs, and

```
P =
  95
  90
  85
```

   is the current prices of the bonds, find the current state prices d (discount factors) from the relation:

$$P = Bd$$

$$d = B^{-1}P$$

```
d = inv(B)*P
  0.95000
  0.73182
  0.61983
```

The current value of receiving one dollar at time 3 is thus 0.61983.

2. To find the interest rates we solve the expressions

$$d_t = \frac{1}{(1+r)^t}$$

wrt r. Doing everything at once:

```
>> t=[1;2;3];
>> r=d.^(-1./t)-1
r =
  0.052632
  0.168957
  0.172847
```

The interest rates are increasing with time. To check that these are correct, recalculate

```
>> (1./(1+r)).^t
ans =
  0.95000
  0.73182
  0.61983
```

3. To price bond D we use the "discount factors" d:

```
> D =
   20   20  520
> d =
  0.95000  0.73182  0.61983
D*d' = 355.95
```

Bond D has a price equal to 355.95.