Financial Econometrics
Problem Set
Problem set 4, 2021

Exercise 1. *Five factor model* [8]
Fama and French has recently introduced their five-factor model, which leads to the regression:

$$eR_{i,t} = b^m eR_{m,t} + b^{SMB} SMB_t + b^{HML} HML_t + b^{RMW} RMW_t + b^{CMW} CMW_t + \varepsilon_{i,t}$$

You want to ask whether the two "new" factors RMW and CMW makes much of a difference. You do so in the context of a stochastic discount factor specification. The Stochastic Discount factor approach to asset pricing results in the following expression for pricing any excess return:

$$E[m_t er_{it}] = 0$$

Consider an empirical implementation of this where we write the pricing variable m as a function of a set of prespecified factors f:

$$m_t = 1 + bf_t$$

Compare three different specifications

$$m_t = 1 + b^m eR_{m,t}$$

$$m_t = 1 + b^m eR_{m,t} + b^{SMB} SMB_t + b^{HML} HML_t$$

$$m_t = 1 + b^m eR_{m,t} + b^{SMB} SMB_t + b^{HML} HML_t + b^{RMW} RMW_t + b^{CMW} CMW_t$$

Your test assets are monthly returns for 49 US Industry portfolios, provided by Ken French, with data from 1926 to 2015. Use the equally weighted portfolios.

Does it seem like the two factors RMW_t and CMW_t are important for the crossection of asset returns?

Exercise 2. [7]
Consider the Euler condition from a representative agent formulation.

$$E\left[\beta \frac{U'(c_t)}{U'(c_{t-1})}(1 + \tilde{R}_{it})|Z_{t-1}\right] = 1 \quad \forall\ i, t$$

Where
$\beta = \frac{1}{1+r}$ is the rate of time preference,
c_t is the consumption in period t,
$U(\cdot)$ is the utility function, and
$\tilde{R}_{it}$ is the return on asset i in period t.

Assume the utility function

$$U(c) = \frac{1}{1-B}(c^{1-B} - 1)$$

1. What does B represent?

Restate the Euler condition as

$$E\left[\beta \frac{c_t^{-B}}{c_{t-1}^{-B}}(1 + \tilde{R}_{it})|Z_{t-1}\right] = 1 \quad \forall\ i, t$$

$$E\left[\beta \left(\frac{c_t}{c_{t-1}}\right)^{-B} (1 + \tilde{R}_{it})|Z_{t-1}\right] = 1 \quad \forall\ i, t$$

With data on consumption, this equation is readily estimable. However, without consumption data, what can be used instead to measure consumption MRS? Suppose consumption is a constant fraction k of wealth W.

$$C_{t-1} = kW_{t-1}$$

$$W_t = (1-k)W_{t-1}(1+\tilde{R}_{mt})$$
$$C_t = kW_t = k(1-k)W_{t-1}(1+\tilde{R}_{mt})$$
$$\frac{C_t}{C_{t-1}} = \frac{k(1-k)W_{t-1}(1+\tilde{R}_{mt})}{kW_{t-1}} = (1-k)(1+\tilde{R}_{mt})$$
$$(\frac{C_t}{C_{t-1}})^{-B} = ((1-k)(1+\tilde{R}_{mt}))^{-B}$$

Use this in the equation above to get

$$E[\beta(1+k)^{-B}(1+\tilde{R}_{mt})^{-B}(1+\tilde{R}_{it})|Z_{t-1}] = 1$$

$$E[(1+\tilde{R}_{mt})^{-B}(1+\tilde{R}_{it})|Z_{t-1}] = \frac{1}{\beta}(1+k)^B$$

This hold for any $\tilde{R}_{it}$. Now consider the two *particular* cases: $\tilde{R}_{ti} = \tilde{R}_{mt}$ return on market portfolio and $\tilde{R}_{ti} = R_{ft}$ risk free interest rate

2. Using the above show that we find the following

   $$E[\tilde{x}_t^{-B}(\tilde{x}_t - 1)|Z_{t-1}] = 0$$

   where we have defined

   $$\tilde{x} = \frac{1+\tilde{R}_{mt}}{1+\tilde{R}_{ft}}$$

By taking expectations over information sets Z_{t-1}, get the unconditional relationship

$$E[x_t^{-B}(\tilde{x}_t - 1)] = 0$$

3. Show how the parameter B can be estimated by GMM.
4. Estimate B using GMM on the following stock markeet indices:
   (a) The Global market portfolio from Ken French's data
   (b) The US market portfolio, also from Ken French's data
   (c) The Norwegian market portolio.
5. (Optional) Suppose x is lognormal, calculate the ML estimator of B.

Exercise 3.

The following are currently the crypto assets with currently the largest market cap.

Bitcoin
Ethereum
Cardano
Binance Coin
Tether
Solana
XRP
USD Coin
Polkadot
Dogecoin
Uniswap
Terra
Avalanche
Binance USD
Chainlink
Litecoin
Algorand

It has been argued that crypto assets are useful for their diversification properties. In this exercise you are to investigate the *internal* diversification possibilities in crypto assets.

For the above coins:

1. Get asset prices for as long as possible.
2. Construct a correlation matrix for these returns at monthly, weekly and daily frequency.
3. Investegate the number of different *factors* in these returns.

Exercise 4. *Risk contribution* [4]

In this exercise we will revisit the analysis of NBIM in Dahlquist and Ødegaard (2018). One of the analyses done in the report was a calculation of the *contribution to total risk* of the portfolio. The concept of total portfolio risk one uses is typically the variance/standard deviation of the portfolio. Consider the variance of a portfolio (using obvious vector/matrix notation):

$$\sigma_p^2 = w^\top \Sigma w.$$

Risk contributions are defined as

- Marginal contribution to risk (MCTR):

$$\begin{aligned} \frac{\partial \sigma_p}{\partial w} &= \frac{\partial (w^\top \Sigma w)^{1/2}}{\partial w} \\ &= \frac{1}{2}(w^\top \Sigma w)^{-1/2} 2 \Sigma w \\ &= \frac{\Sigma w}{\sigma_p}. \end{aligned}$$

- Marginal contribution to risk for asset i (MCTR$_i$):

$$\frac{\partial \sigma_p}{\partial w_i} = \frac{(\Sigma w)_i}{\sigma_p} = \frac{\sigma_{ip}}{\sigma_p} = \beta_{ip}\sigma_p.$$

- Percentage contribution to risk for asset i (PCTR$_i$):

$$\frac{w_i \frac{\partial \sigma_p}{\partial w_i}}{\sigma_p} = w_i \frac{\sigma_{ip}}{\sigma_p^2} = w_i \beta_{ip}.$$

Note that (a) the weighted sum of the marginal contributions for all assets equals the portfolio standard deviation, (b) the sum of the percentage contributions for all assets is one, and (c) the weighted sum of the betas is one. (These results follow from the sum of the weights being one.)

One can refer to a portfolio having an equity weights of, say, 60%, but the risk contribution being, say, 90%.

To compute risk contributions for the NBIM portfolio we can use the historical variance-covariance matrix of actual or benchmark returns (using equity and fixed income) for Σ and the benchmark weights for w. In this problem you should only consider equity and fixed income. Calculate two covariance matrices between equity and fixed income returns: $\mathbf{V}^{bench}$ – estimated using benchmark returns 1998-2017:06, and $\mathbf{V}^{fund}$ – estimated using fund returns 1998-2017:06. Do the estimation using returns denominated in USD.

1. Report the estimated covariances and correlations between bonds and equities.
2. Use current benchmark weights, $w^{bonds} = 0.4$ and $w^{equity} = 0.6$. Estimate the percentage contribution to risk of the equity and fixed income parts of the Fund's portfolio.
3. Note that the Fund has changed weights several times. Let us look at that.

(a) Up till 2007 the weights were 40/60 equity/fixed income. Recalculate the percentage contribution to risk for this period.

(b) The equity/bond weights are currently planned to move to 70/30 equity/fixed income. Estimate the consequences of this for the percentage contribution to risk.

Exercise 5.

Among the anlyses in Dahlquist and Ødegaard (2018) is the regressions reported in Table 10 of their report, one of which is an alpha regression concerning the equity part of the portfolio:

$$R_t - R_t^b = a + b_{MKT}\texttt{MKT}_t + b_{SMB}\texttt{SMB}_t + b_{HML}\texttt{HML}_t + b_{RMW}\texttt{RMW}_t + b_{CMA}\texttt{CMA}_t + \varepsilon_t,$$

(for definitions look at the report.)

Your task is to do a similar alpha estimation for

- The period 1998–2020.
- The period 2014–2020.

How does the alpha estimate look?

Note that the number in the report are in the fund's currency basket. For simplicity, for this problem you can use data in USD. It may be informative to do the analysis for the period used in the report, to illustrate the differences between using USD and the currency basket, before doing the above two specifications.

Exercise 6. *Rolling Factor Exposures* [5]

We consider NBIM's monthly returns. Let R_p be the returns of the equity part of the fund. Let R_b be the return on the corresponding benchmark. For simplicity work with the USD returns.

In the report there is a discussion of time variation in factor exposure, which is only mentioned, without showing results. The obvious way to illustrate it is to calculate a "rolling beta" estimation. In this case, consider the regression

$$R_{p,t} - R_{b,t} = \alpha_p + (r_{m,t} - r_{f,t})\beta_{p,m} + b_p^{smb}SMB_t + b_p^{hml}HML_t + \varepsilon_{pt}$$

Estimate this on the first five years, and "roll forward" using this five year window, i.e. estimate this regression monthly using a five year window.

Plot the resulting time series of estimates of the factor coefficients. Discuss.

Exercise 7. *Adding factors* [6]

We consider NBIM's monthly returns. Let R_p be the returns of the equity part of the fund. Let R_b be the return on the corresponding benchmark. For simplicity work with the USD returns.

Consider the analysis in Dahlquist and Ødegaard (2018). In the report we ask whether a specific model is reasonable by adding a number of alternative "factors" and evaluating the added explanatory power for the "new" variable. In this problem we will do a similar exercise

Consider the three factor regression

$$R_{p,t} - R_{b,t} = a + b_p^m RMRF_t + b_p^{SMB}SMB_t + b_p^{HML}HML_t + \varepsilon_t$$

where $RMRF_t$, SMB_t and HML_t are the three Fama-French factors. (Global versions).

For each of the following factors, you will calculate two statistics

- The increase in R^2 when adding the factor to the three-factor model
- The partial correlation of the additional factor.

The additional factors you will consider are

- The global RMW factor (From Ken French's Homepage)

- The global CMA factor (From Ken French's Homepage)
- The global Momentum factor (From Ken French's Homepage)
- The change in the VIX (from FRED)
- The change in the TED spread (from FRED)

Hints: Both the VIX and the TED spread are daily series. Look at the change between end-of-month observations.

References

Magnus Dahlquist and Bernt Arne Ødegaard. A review of norges bank's active management of the government pension fund global. Technical report, January 2018. URL https://www.regjeringen.no/no/aktuelt/ekspertrapporter-om-spu/id2585465/. Report to Norwegian Ministry of Finance.