Financial Econometrics

Problem Set

Problem set 3, 2021

Exercise 1. *Crypto time series* [3]

We now investigate trading of Bitcoin. We do not go into the workings of this alternative "money", for this exercise we merely investigate the time series properties of a time series of "exchange rates" between cryptocurrencies and USD. Download daily prices of Bitcoin and Ethereum.

- Look at the weekly price observations. Determine the order of integration of the time series.
- Look at the monthly price observations. Determine the order of integration of the time series.

Exercise 2.

You want to investigate whether the volatility of Stock Prices can predict the real economy at a quarterly frequency. Use real GDP for the US (dowloadable from FRED) and the daily S&P 500 (dowloadable from Yahoo Finance). The S&P series starts in 1950, the GDP series in 1947. Use the whole time period.

The S&P 500 series is used to generate an estimate of volatility the following way: Calculate daily returns. A possible volatility estimate for a given quarter is the volatility of that quarter's worth of daily returns. Let $\sigma(\widehat{R_{m,t}})$ be this estimate of volatility.

Set up the estimation

$$dGDP_t = a + b_1 dGDP_{t-1} + b_2 \sigma(\widehat{R_{m,t-1}}) + \varepsilon_t$$

This is now a standard ADL time series estimation. An in-sample test for predictability is to test whether b_2 is significant. Perform this test.

You also want to investigate the stability of the relation.

Split the data into two subperiods, before and after 1973, and do the test separately for these two time periods.

An alternative way of doing the test is to set this up as a VAR system. Test for causality in this VAR formulation.

Exercise 3.

Consider estimating

$$E_t[m_{t+1}\mathbf{R}_{t+1}] = 1$$

where m_t is the stochastic discount factor, and $\mathbf{R}_{t+1}$ is a vector of (gross) returns.

- What is the corresponding specification when estimating this using excess returns?

We parameterize $m_t = c + \mathbf{b}\mathbf{f}_t$ where $\mathbf{f}_t$ a set of pre-defined factors. When we estimate this using excess returns we usually impose $c = 1$ to force m away from zero.

An alternative way of doing this may be to use information about additional assets.

In particular, what if we have a *risk free* asset, with a per-period return R_{ft}?

- What restriction does the existence of a risk free asset put on $E[m]$?
- How could this be used together with a crossection of excess returns to avoid specifying $c = 1$?