Forecasting

A lecture on forecasting.

Forecasting

What is forecasting?

The estabishment of a *probability* statement about the future value of an economic variable.

Let x_t be the variable of interest.

Want: the conditional distribution of x_{t+1} based on a set of data observed *prior* to date $t+1$.

The choice of data: up to the econometrician.

Typical choices:

- Past values of the same variable $(x_t, x_{t-1}, \ldots)$,
- Past values of other variables $(\mathbf{y}_t, \mathbf{y}_{t=1}, \ldots)$.

Need to assume the existence of the distribution (is this a meaningful question?)

Of particular interest: The *point estimate* of the variable x_{t+1}.

Estimated as: The conditional expectation, conditioned on all available information at time t.

Choice of forecasting model - theory

Theoretical basis for a choice of a forecasting model:
Can not be divorced from the question of *what is the forecast used for*?
The relevant theoretical framework for choosing a forecasting model is *decision theory*: The forecast is used to change the economic decisions of some decision maker.
Choose the forecasting model to maximize the utility of this decision maker.
See Granger and Machina [2006].
See also Geweke and Whiteman [2006] for the Bayesian perspective.
Keep in mind that this kind of thinking is ideal.
However, we will not deal with this theoretical basis

Choice of forecasting model - common practice

In practice the modelling choices tend to much more *ad hoc*.

1. Specify a parsimonious (which typically will be linear) specification in the variables of interest.
2. Evaluate the specified model using common econometric techniques

Our approach:
Illustrate the econometric techniques used to evaluate forecasting models.

- In sample testing of model fit.
  - Nested models
  - Non-nested models
- In sample forecast evaluation.
- Out of sample forecast evaluation.

Illustrative Example

Use a simple forecasting exercise (from Næs et al. [2011]) to illustrate the issues.

Let us first go through this abstractly before getting to specifics of the model

Model

x_t: variable we want to forecast

y_t: some other variable believed to contain information relevant for forecasting x_t.

Specify (guess) the linear ADL(1,1) model

$$x_{t+1} = a + b_x x_t + b_y y_t + \varepsilon_{t+1}$$

In sample testing of model fit.

$$x_{t+1} = a + b_x x_t + b_y y_t + \varepsilon_{t+1}$$

A typical question to ask in this context: Does y_t have information about the future x_{t+1} (or – is it useful for forecasting).
Answering this: a standard test of whether b_y is significant.
So we can use standard methods here.

In sample testing of model fit (ctd).

Have two competing explanatory variables: y_{t-1} and z_{t-1}.
Want to compare the two specifications:

$$x_{t+1} = a + b_x x_t + b_y y_t + \varepsilon_{t+1}$$

$$x_{t+1} = a + b_x x_t + b_z z_t + \varepsilon_{t+1}$$

Is this a non-nested test?
In sample: not really, we can write a model that nests both specifications

$$x_{t+1} = a + b_x x_t + b_y y_t + b_z z_t + \varepsilon_{t+1}$$

and test restrictions on the coefficients b_y and b_z.
A better example of non-nested specifications is AR specifications vs MA specifications.

In sample forecast evaluation

Forecast evaluation: comparison of a forecast with the realized value.
The better the forecast, the closer the forecast is to the realization.
Compare two forecasting models, one with, and one without y as explanatory variable:

$$\hat{x}^a_{t+1} = a + b_x x_t + \varepsilon_{t+1}$$

$$\hat{x}^b_{t+1} = a + b_x x_t + b_y y_t + \varepsilon_{t+1}$$

Forecast evaluation is based on the difference between the forecasts and realizations

Forecast evaluation: Calculate forecast errors

$$e^a = x_{t+1} - \hat{x}^a_{t+1}$$

$$e^b = x_{t+1} - \hat{x}^b_{t+1}$$

To compare: Need a metric to aggregate this over many forecasts.
Common example: Mean Squared Error (But there are many alternatives)

$$MSE = \sum_t (x_{t+1} - \hat{x}_{t+1})^2$$

Note that in sample the MSE is the same as the squared sum of residuals.
An equivalent way of comparing two specifications like the above (in sample):
Difference in R^2 (often presented as the increase in R^2 when adding one explanatory variable).

Out of sample forecast evaluation

In the above tests, the whole data series is used to construct parameter estimates.

Implication: You use *future* information to estimate parameters.

Is this a fair method to evaluate forecasts?

The *out of sample* method is an attempt to get around such critiques. Here, one is careful to only use *past information* to construct any forecast.

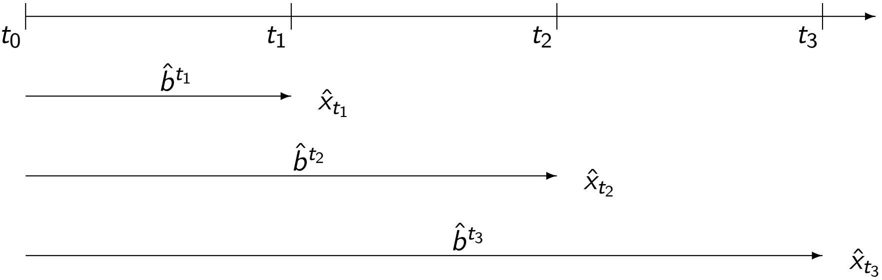
Out of sample forecast evaluation ctd

In the comparison of two linear specifications

$$\hat{x}^{a}_{t+1} = a + b_x x_t + \varepsilon_{t+1}$$

$$\hat{x}^{b}_{t+1} = a + b_x x_t + b_y y_t + \varepsilon_{t+1}$$

One adopts a rolling forward procedure



Each forecast is a function of a different parameter $\hat{b}^{t_i}$, $i = 1, 2, \ldots$, a parameter that is re-estimated based on information up to time t_i.

Out of sample forecast evaluation ctd

One again constructs the difference between the forecast and realization, and aggregate.
Then evaluate the forecast error.
One can also compare mean squared differences, but here it is more common to use a *out-of-sample forecast evaluation criterion*.
Mention some common choices in the more recent literature.

Out of sample forecast evaluation ctd

1

The Diebold and Mariano [1995] is calculated as follows: Suppose we have a candidate predictor i and a competing predictor k. We want to test the null hypothesis of equal predictive accuracy that $E[\bar{d}] = 0 \; \forall \; t$, where $\bar{d} = P^{-1} \cdot \sum_t (\varepsilon^2_{k,t+1} - \varepsilon^2_{i,t+1})$, P is the number of rolling out-of-sample forecasts, and $\varepsilon^2_{k,t+1}$ and $\varepsilon^2_{i,t+1}$ are the squared forecast errors from the two models. The DM statistic is calculated as

$$DM = \frac{\bar{d}}{(\sigma^2_{\bar{d}}/P)^{1/2}}, \tag{1}$$

Out of sample forecast evaluation ctd

2

The encompassing test (ENC-NEW) is proposed by Clark and McCracken [2001] The ENC-NEW test asks whether the restricted model (the model that does not include y) encompasses the unrestricted model that includes y. The ENC-NEW test statistic is given as

$$\text{ENC-NEW} = (P - h + 1) \cdot \frac{P^{-1} \sum_t \left[\varepsilon_{r,t+1}^2 - \varepsilon_{r,t+1} \cdot \varepsilon_{u,t+1} \right]}{MSE_u}, \quad (2)$$

where P is the number of out-of-sample forecasts, $\varepsilon_{r,t+1}$ denotes the rolling out-of-sample errors from the restricted (baseline) model that excludes y, $\varepsilon_{u,t+1}$ is the rolling out-of-sample forecast errors from the unrestricted model that includes y, and MSE_u denotes the mean squared error of the unrestricted model that includes y.

Out of sample forecast evaluation ctd

3

An F-type test for equal MSE between two nested models proposed by McCracken [2007], termed MSE-F. This test is given by

$$\text{MSE-F} = (P - h + 1) \cdot \frac{MSE_r - MSE_u}{MSE_u}, \tag{3}$$

where MSE_r is the mean squared forecast error from the restricted model that excludes y, and MSE_u is the mean squared forecast error of the unrestricted model that includes y. Both the ENC-NEW and MSE-F statistics are nonstandard and one need to use the bootstrapped critical values provided by Clark and McCracken [2001].

These are illustrative that out-of-sample evaluation tend to get more involved than in-sample evaluation, but are clearly closer to the actual *use* of forecasts.

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