SDF based asset pricing

General overview of asset pricing testing.

The purpose of this section is to give an overview of a number of asset pricing models, their testing, and relation to each others. Consider what is typically called the *canonical asset pricing equation*. Most of the models we will look at can be viewed as special cases of this.

$$E_t[m_{t+1} R_{it+1}] = 1 \quad (1)$$

Here $R_{i,t+1}$ is the gross return, and m_t is a random variable. The exact nature of m_t will depend on the nature of our asset pricing model.

$E_t[\cdot]$ is shorthand for the *conditional expectation* given a time t information set. This would be written more correctly as $E[\cdot|\Omega_t]$, where Ω_t is the *market-wide* information set.
You may also want to recall the *Law of iterated expectations*: $E[X] = E\left[E[X|Y]\right]$, for random variable X and Y, which is heavily used in econometric analysis. In the shorthand form used above, this can be written

$$E_t[y_{t+2}] = E_t\left[E_{t+1}[y_{t+2}]\right]$$

This equation is the outcome of a number of models, and m_t has many names, depending on the model. Examples include the intertemporal marginal rate of substitution, a stochastic discount factor, and an equivalent Martingale measure.

Pricing operators

Let me now give a quick reasoning for where this equation is coming from.
The equation in return form is:

$$E_t[m_{t+1}R_{i,t+1}] = 1$$

Since in terms of asset prices P_t

$$R_{i,t+1} = \frac{P_{i,t+1}}{P_{i,t}}$$

We can rewrite

$$E_t\left[m_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\right] = 1$$

implying

$$P_{i,t} = E_t\left[m_{t+1}P_{i,t+1}\right]$$

Let us now map this notation to the the more common asset prcing one.
The *future* payoffs for asset i:

$$P_{i,t+1} = x_i$$

Stack these:

$$\begin{bmatrix} P_{1,t+1} \\ \vdots \\ P_{n,t+1} \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

or

$$\mathbf{P}_{t+1} = \mathbf{x}$$

The interpretation is that $\mathbf{x}$ is the vector of future payoffs.
Further, current payoffs

$$\mathbf{P}_t = \mathbf{q}$$

and the factor

$$m_{t+1} = y$$

We are interested in the price today of the vector $\mathbf{x}$ of future payoffs. This is the *pricing functional* $\pi(\cdot)$ that maps future payoffs into current prices. The prices today of the future payoffs $\mathbf{x}$ is $\mathbf{q}$:

$$\mathbf{q} = \pi(\mathbf{x})$$

Since $\pi(\cdot)$ represent current prices of claims to future payoffs, we can say something about it.
For obvious no-arbitrage reasons, it makes sense to impose *value-additivity*:

$$\pi(\omega_1 x_1 + \omega_2 x_2) = \omega_1 \pi(x_1) + \omega_2 \pi(x_2)$$

and continuity, very small payoffs have small prices.
These are sufficient assumptions to restrict $\pi(\cdot)$ to be a *linear functional* on the space of future payoffs.

$$\mathbf{q} = \pi(\mathbf{x})$$

If $\mathbf{c}$ is a portfolio of assets, linearity implies that

$$\mathbf{c}\mathbf{q} = \pi(\mathbf{c}\mathbf{x})$$

Consider now this linear functional $\pi(\cdot)$.
It can be shown that any pricing functional $\pi(\cdot)$ can be represented by a random variable y as:

$$\mathbf{q} = \pi(\mathbf{x}) = E[y\mathbf{x}]$$

That is, there is some random variable y that can be used to price all payoffs $\mathbf{x}$.
This variable y is the *stochastic discount factor*.
Technical reason: The conditional expectation defines an inner product on the linear space of possible future payoffs.

Present value relationship.

Let us look at one implication of (1). It can be used as a justification of the **present value model**:

$$
\begin{aligned}
P_t &= E_t[m_{t+1}(d_{t+1} + p_{t+1})] \\
&= E_t[m_{t+1}d_{t+1} + m_{t+1}E_{t+1}[m_{t+2}(d_{t+2} + p_{t+2})]] \\
&= E_t[m_{t+1}d_{t+1} + m_{t+1}m_{t+2}(d_{t+2} + p_{t+2})] \\
&= E_t[m_{t+1}d_{t+1} + m_{t+1}m_{t+2}(d_{t+2} + E_{t+2}[d_{t+3} + p_{t+3}])] \\
&\vdots \\
&= E_t\left[\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i} m_{t+j}\right) d_{t+i}\right]
\end{aligned}
$$

That is, the price of any stream of cash flows is its discounted present value. Note that this assumes that the limit of $\left(\prod_{j=1}^{i} m_{t+j}\right)$ as $i \to \infty$, is finite.

We go through the derivation of the *canonical asset pricing equation* in one special case.
The setting is a general equilibrium model, where we posit the existence of a *representative consumer* who is maximising his (or hers) utility of future consumption.
Let c_t be the consumption in period t. There is only one asset in the economy, with price p_t and paying dividends of d_t in period t. Let q_t be the agents holdings (quantity) of the asset at the beginning of period t. The consumer is assumed to have wage income of w_t.
It should be easy to verify that the agents *budget constraint* is

$$c_t + p_t q_t \leq (p_t + d_t) q_{t-1} + w_t$$

The consumer is assumed to maximise his lifetime expected utility

$$E_0 \left[\sum_{t=1}^{\infty} \beta^t u(c_t) \right]$$

where β is a discount factor.

We will close this model by noting that in equilibrium, the demand of assets is equal to the supply, and we have only one agent, $q_t = q_{t+1} \; \forall \; t$.
The problem we want to solve is then

$$\max_{\{c_t, q_t\}} E_0 \left[\sum_{t=1}^{\infty} \beta^t u(c_t) \right]$$

subject to

$$c_t + p_t q_t \leq (p_t + d_t) q_{t-1} + w_t$$

for $t = 0, 1, 2, \cdots$.
This problem can be solved in a number of ways, the most standard being by dynamic programming. But let us look at what may be the simplest, doing the optimisation directly by forming a Lagrangian:[1]

$$L = E_0 \left[\sum_{t=1}^{\infty} \beta^t u(c_t) - \sum_{t=1}^{\infty} \lambda_t \left(c_t + p_t q_t - (p_t + d_t) q_{t-1} - w_t \right) \right]$$

Take derivatives wrt c_r and q_r we get

$$\begin{aligned} \frac{\partial L}{\partial c_r} &= E_0\left[\beta^r u'(c_r)\right] - \lambda_r &= 0 \\ \frac{\partial L}{\partial q_r} &= -\lambda_r p_r + \lambda_{r+1}(p_{r+1} + d_{r+1}) &= 0 \end{aligned}$$

Use the first equation to substitute in the second, and we get a condition for optimality that will need to hold for any c_t.

$$E_t[\beta^t u'(c_t)p_t] = E_t\left[\beta^{t+1} u'(c_{t+1})(d_{t+1} + p_{t+1})\right]$$

or

$$E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{(p_{t+1} + d_{t+1})}{p_t}\right] = 1$$

This is usually called the *Euler equation* in this type of model.

Beta-pricing relations.

We can also use our fundamental equation to look at *beta-pricing* style relations. Let us first write (1) in standard return form by subtracting 1 from the gross return:

$$r_{it} = R_{it} - 1$$

which gives

$$E_t[m_{t+1} r_{i,t+1}] = 0 \tag{2}$$

Recall the definition of covariance.

$$\text{cov}(X, Y) = E[XY] - E[X]E[Y]$$

Rewrite this for our variables:

$$\text{cov}_{t-1}(m_t, r_{it}) = E_{t-1}[m_t r_t] - E_{t-1}[m_t]E_{t-1}[r_{it}]$$

Solve for $E_{t-1}[r_{it}]$:

$$\begin{aligned} & \text{cov}_{t-1}(m_t, r_{it}) + E_{t-1}[m_t]E_{t-1}[r_{it}] = E_{t-1}[m_t r_{it}] = 0 \\ \Rightarrow \quad & \text{cov}_{t-1}(m_t, r_{it}) + E_{t-1}[m_t]E_{t-1}[r_{it}] = 0 \\ \Rightarrow \quad & -\text{cov}_{t-1}(m_t, r_{it}) = E_{t-1}[m_t]E_{t-1}[r_{it}] \\ \Rightarrow \quad & \frac{-\text{cov}_{t-1}(m_t, r_{it})}{E_{t-1}[m_t]} = E_{t-1}[r_{it}] \end{aligned}$$

or

$$E_{t-1}[r_{it}] = \frac{\text{cov}_{t-1}(-m_t, r_{it})}{E_{t-1}[m_t]}$$

This is a relationship between the return on any asset with its covariance with the pricing variable m_t. In the case of a consumption-based model such as the one studied above, the return on asset i is a function of the asset's covariance with consumption.

Special case, CAPM – style relations.
In the previous we found the return on any asset i as a function of its covariance with the variable m_t.

$$E_{t-1}[r_{it}] = \frac{\text{cov}_{t-1}(-m_t, r_{it})}{E_{t-1}[m_t]}$$

We now want to show how our familiar asset pricing model the CAPM can be shown to be a special case of this.
Remember that the CAPM specifies a relationship with the market portfolio. Let us first consider the return on any portfolio p, r_{pt}, (not necessarily the market portfolio), with $\text{cov}_{t-1}(r_{pt}, m_t) \neq 0$.

From the definition of covariance,

$$\begin{aligned}\text{cov}_{t-1}(r_{pt}, -m_t) &= E_{t-1}[-m_t r_{pt}] - E_{t-1}[r_{pt}]E_{t-1}[-m_t] \\ &= -0 + E_{t-1}[r_{pt}]E_{t-1}[m_t] \\ &= E_{t-1}[r_{pt}]E_{t-1}[m_t]\end{aligned}$$

Hence

$$E_{t-1}[m_t] = \frac{\text{cov}_{t-1}(-m_t, r_{pt})}{E_{t-1}[r_{pt}]}$$

Now substitute for $E_{t-1}[m_t]$ in the equation for $E_{t-1}[r_{it}]$.

$$E_{t-1}[r_{it}] = \frac{\text{cov}_{t-1}(-m_t, r_{it})}{E_{t-1}[m_t]}$$

giving

$$E_{t-1}[r_{it}] = \frac{\text{cov}_{t-1}(-m_t, r_{it})}{\frac{\text{cov}_{t-1}(-m_t, r_{pt})}{E_{t-1}[r_{pt}]}} = \frac{\text{cov}_{t-1}(-m_t, r_{it})}{\text{cov}_{t-1}(-m_t, r_{pt})} E_{t-1}[r_{pt}]$$

We now have a relationship where the portfolio return appear. Let us next try to get rid of the pricing variable m_t.

Consider replacing m_t with an *estimate*, a function of the returns of the assets in the portfolio. We use a linear regression on the vector

$$R_t = \begin{bmatrix} R_{1,t} \\ \vdots \\ R_{n,t} \end{bmatrix}$$

of individual asset returns

$$m_t = \omega_t R_t + \varepsilon_t$$

By a known result, there is always a vector ω_t such that

$$E_{t-1}[\varepsilon_t' R_t] = 0$$

or equivalently

$$\text{cov}_{t-1}(R_t, \varepsilon_t) = 0$$

In this case we can actually calculate this vector ω_t: We know

$$E_t[m_{t+1} R_{i,t+1}] = 1 \ \ \forall \ i$$

or

$$E_t[m_{t+1} R_{t+1}] = \mathbf{1}$$

Substitute for m_{t+1}:

$$E_t[(\omega_{t+1} R_{t+1}) R_{t+1}] = \mathbf{1}$$

solve for ω_{t+1}:

$$\omega_{t+1} E_t[R_{t+1} R_{t+1}] = \mathbf{1}$$

and

$$\omega_{t+1} = (E_t[R_{t+1} R_{t+1}])^{-1} \mathbf{1}$$

The regression coefficients ω_t of this regression are not guaranteed to sum to one, but we fix that by normalising the weights with the sum: $\mathbf{1}'\omega_t$, where $\mathbf{1}$ is the unit vector. We then have found portfolio weights $\frac{1}{\mathbf{1}'\omega_t}\omega_t$.
Then the return on the portfolio p is

$$R_{pt} = \frac{1}{\mathbf{1}'\omega_t}\omega_t' R_t$$

Also note that we can rewrite m_t as

$$m_t = \mathbf{1}'\omega_t R_{pt} + \varepsilon_t$$

Hence

$$\begin{aligned}
\text{cov}_{t-1}(-m_t, r_{it}) &= \text{cov}_{t-1}(-(\omega_t' R_t + \varepsilon), r_{it}) \\
&= \text{cov}_{t-1}(-\omega_t' R_t, r_{it}) + \text{cov}_{t-1}(\varepsilon_t, r_{it}) \\
&= -\mathbf{1}'\omega_t \text{cov}_{t-1}(R_{pt}, r_{it}) + 0 \\
&= -\mathbf{1}'\omega_t \text{cov}_{t-1}(R_{pt}, r_{it})
\end{aligned}$$

Use this to get

$$
\begin{aligned}
E_{t-1}[r_{it}] &= \frac{\text{cov}_{t-1}(-m_t, r_{it})}{\text{cov}_{t-1}(-m_t, r_{pt})} E_{t-1}[r_{pt}] \\
&= \frac{-\mathbf{1}'\omega_t \text{cov}_{t-1}(R_{pt}, r_{i,})}{-\mathbf{1}'\omega_t \text{cov}_{t-1}(R_{pt}, r_{pt})} E_{t-1}[r_{pt}] \\
&= \frac{\text{cov}_{t-1}(r_{pt}, r_{it})}{\text{cov}_{t-1}(r_{pt}, r_{pt})} E_{t-1}[r_{pt}] \\
&= \frac{\text{cov}_{t-1}(r_{pt}, r_{it})}{\text{var}_{t-1}(r_{pt})} E_{t-1}[r_{pt}]
\end{aligned}
$$

Finally, let us posit the existence of some asset z with return R_{zt}, and with $\text{cov}_{t-1}(R_{zt}, R_{pt}) = 0$. (usually called the "zero-beta" asset.)
We can then write

$$E_{t-1}[r_{it} - r_{zt}] = \frac{\text{cov}_{t-1}(r_{pt}, r_{it})}{\text{var}_{t-1}(r_{pt})} E_{t-1}[r_{pt} - r_{zt}]$$

If there is a risk free rate r_{ft}, by definition it has $\text{cov}_{t-1}(r_{pt}, r_{ft}) = 0$, and we get the CAPM in its usual form

$$E_{t-1}[r_{it}] - r_{ft} = \frac{\text{cov}_{t-1}(r_{pt}, r_{it})}{\text{var}_{t-1}(r_{pt})}(E_{t-1}[r_{pt}] - r_{ft})$$

Note that this is a *conditional* version of the CAPM, it holds given the current information set.

Factor models, APT

By some more work, we can also get an APT-style relation in asset returns,

$$E_t[R_{i,t+1}] = \lambda_{0,t} + \sum_{j=1}^{K} b_{ijt} \frac{\text{cov}_t(F_{j,t+1}, -m_{t+1})}{E_t[m_{t+1}]}$$

as a special case of our generic relation.

The problem with the APT is that it is a relationship that holds for some "factors," but we do not know what the factors are.

There are two main methods used in estimation of the APT.

1. Estimate the factors from the data, using one of
   1.1 Factor analysis.
   1.2 Principal components analysis.
2. Prespecify the factors as economic variables we believe may influence asset returns.

The above shows how a large number of the models we know can be viewed as special cases of a relation

$$E_t[r_{t+1} m_{t+1}] = 0$$

Note that this formula is in the form of the *conditional* expectation.
The ability to use *conditioning information* in a meaningful way is one of the major breakthroughs in current research in empirical asset pricing.
In this class we will see how it is done in particular models, and how recent research differs from the classical tests.
Let me note a couple of ways to use conditioning information

- Use of variables in the information set as instruments in the estimation.

- Try to model the conditional expectations directly (latent variables)

Characterising m_t directly

Usually, we do estimation in the context of particular *asset pricing model*.
In the context of the equation

$$E_t[m_{t+1}R_{i,t+1}] = 1$$

this means putting some structure on m_t. Examples: consumption based asset pricing model, where $m_t = \frac{u'(c_{t+1})}{u'(c_t)}$,
CAPM a relationship with a *reference portfolio*.
Alternatively: write $m_t = f($ "factor"$)$ (in the factor analysis spirit),
Example

$$m_t = 1 + ber_{m,t}$$

Another way to ask what factors influence the cross section of assets.

Exercise

The Stochastic Discount factor approach to asset pricing results in the follwing expression for pricing any excess return:

$$E[m_t er_{it}] = 0$$

Consider an empirical implementation of this where we write the pricing variable m as a function of a set of prespecified factors f:

$$m_t = 1 + bf_t$$

Consider the case of the one factor model $f = 1 + ber_m$, where the only explanatory factor is the return on a broad based market index.

Implement this approach on the set of 5 size sorted portfolios provided by Ken French. Use data 1926–2012.

Is the market a relevant pricing factor?

Exercise Solution

Reading data

```
source("read_size_portfolios.R")
source("read_pricing_factors.R")
eRi <- FFSize5EW - RF
data  <- merge(eRi,RMRF,all=FALSE)
summary(data)
eRi    <- as.matrix(data[,1:5])
eRm   <- as.vector(data[,6])
```

Exercise Solution

The specification of the GMM estimation:

```
X      <- cbind(eRi,eRm)
g1      <- function (parms,X) {
  b <- parms[1];
  f <- as.vector(X[,6])
  m <- 1 + b * f
  e <- m * X[,1:5]
  return (e);
}
```

Exercise Solution

Running the GMM analysis

```
t0   <- c(0.1);
res <-  gmm(g1,X,t0,method="Brent",lower=-10,upper=10)
summary(res)
```

Exercise Solution

```
> summary(data)
     Index           Lo20               Qnt2                Qnt3
 Min.   :1926   Min.   :-32.010   Min.   :-31.9600   Min.   :-31.
 1st Qu.:1948   1st Qu.: -3.110   1st Qu.: -2.8650   1st Qu.: -2.
 Median :1970   Median :  0.960   Median :  1.1700   Median :  1.
 Mean   :1970   Mean   :  1.373   Mean   :  0.9704   Mean   :  0.
 3rd Qu.:1991   3rd Qu.:  4.695   3rd Qu.:  4.5350   3rd Qu.:  4.
 Max.   :2013   Max.   :110.670   Max.   : 81.1900   Max.   : 56.
      Qnt4              Hi20              RMRF
 Min.   :-29.760   Min.   :-30.100   Min.   :-28.980
 1st Qu.: -2.470   1st Qu.: -2.195   1st Qu.: -2.105
 Median :  1.160   Median :  0.930   Median :  1.010
 Mean   :  0.787   Mean   :  0.655   Mean   :  0.628
 3rd Qu.:  4.125   3rd Qu.:  3.640   3rd Qu.:  3.655
 Max.   : 50.010   Max.   : 41.790   Max.   : 37.770
```

Exercise Solution

```
Call:
gmm(g = g1, x = X, t0 = t0, method = "Brent", lower = -10, upper

Method:  twoStep
Kernel:  Quadratic Spectral(with bw =  3.56894 )

Coefficients:
          Estimate    Std. Error  t value     Pr(>|t|)
Theta[1]  -0.0199775   0.0060763  -3.2877763   0.0010098

J-Test: degrees of freedom is 4
                 J-test     P-value
Test E(g)=0:     14.606060   0.005592
```

Exercise Solution

	Model 1
Theta[1]	-0.02^{***}
	(0.01)
Criterion function	1411.21
Num. obs.	1035

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Exercise

The Stochastic Discount factor approach to asset pricing results in the follwing expression for pricing any excess return:

$$E[m_t er_{it}] = 0$$

Consider an empirical implementation of this where we write the pricing variable m as a function of a set of prespecified factors f:

$$m_t = 1 + bf_t$$

Consider the case of the three factor model $f = 1 + b_1 er_m + b_2 SMB + b_3 HML$, where the explanatory factors are the return on a broad based market index, and the two Fama French factors SMB and HML.
Implement this approach on the set of 5 size sorted portfolios provided by Ken French. Use data 1926–2012.
Which are the relevant pricing factors?

Exercise Solution

Organizing the data

```
library(gmm)
source("read_size_portfolios.R")
source("read_pricing_factors.R")

eRi <- FFSize5EW - RF
data  <- merge(eRi,RMRF,SMB,HML,all=FALSE)
summary(data)
eRi    <- as.matrix(data[,1:5])
eRm   <- as.vector(data$RMRF)
smb <- as.vector(data$SMB)
hml <- as.vector(data$HML)
```

Exercise Solution

The GMM specification

```
X     <- cbind(eRi,eRm,smb,hml)
g3     <- function (parms,X) {
  b1 <- parms[1];
  b2 <- parms[2];
  b3 <- parms[3];
  erm <- as.vector(X[,6])
  smb <- as.vector(X[,7])
  hml <- as.vector(X[,8])
  m <- 1 + b1 * erm + b2*smb + b3*hml
  e <- m * X[,1:5]
  return (e);
}
```

Exercise Solution

```
     Index              Lo20                Qnt2                Qnt3
 Min.   :1926   Min.   :-32.010   Min.   :-31.9600   Min.   :-31.3100
 1st Qu.:1948   1st Qu.: -3.110   1st Qu.: -2.8650   1st Qu.: -2.5550
 Median :1970   Median :  0.960   Median :  1.1700   Median :  1.2000
 Mean   :1970   Mean   :  1.373   Mean   :  0.9704   Mean   :  0.8869
 3rd Qu.:1991   3rd Qu.:  4.695   3rd Qu.:  4.5350   3rd Qu.:  4.4050
 Max.   :2013   Max.   :110.670   Max.   : 81.1900   Max.   : 56.8400
      Qnt4               Hi20               RMRF               SMB
 Min.   :-29.760   Min.   :-30.100   Min.   :-28.980   Min.   :-16.3900
 1st Qu.: -2.470   1st Qu.: -2.195   1st Qu.: -2.105   1st Qu.: -1.5200
 Median :  1.160   Median :  0.930   Median :  1.010   Median :  0.0500
 Mean   :  0.787   Mean   :  0.655   Mean   :  0.628   Mean   :  0.2352
 3rd Qu.:  4.125   3rd Qu.:  3.640   3rd Qu.:  3.655   3rd Qu.:  1.7750
 Max.   : 50.010   Max.   : 41.790   Max.   : 37.770   Max.   : 39.0400
      HML
  Min.   :-13.450
  1st Qu.: -1.295
  Median :  0.220
  Mean   :  0.382
  3rd Qu.:  1.745
  Max.   : 35.480
```

Exercise Solution

Running the GMM analysis

```
> t0   <- c(1.0,0,0);
gmm(g = g3, x = X, t0 = t0)

Method:  twoStep
Kernel:  Quadratic Spectral(with bw =  4.83032 )

Coefficients:
          Estimate    Std. Error  t value     Pr(>|t|)
Theta[1]  -0.0133913   0.0086600  -1.5463407   0.1220223
Theta[2]   0.0131771   0.0145060   0.9083904   0.3636720
Theta[3]  -0.0778065   0.0264944  -2.9367123   0.0033171

J-Test: degrees of freedom is 2
                J-test   P-value
Test E(g)=0:    3.51806  0.17221
```

Exercise Solution

	Model 1
Theta[1]	-0.01
	(0.01)
Theta[2]	0.01
	(0.01)
Theta[3]	-0.08^{***}
	(0.03)
Criterion function	339.91
Num. obs.	1035

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Exercise

Using the moment condition

$$E[m_t er_{it}] = 0$$

where

$$m_t = 1 + bf_t$$

Using data for 1980-2012, apply this to the one factor model $f = 1 + b_1 er_m$ and apply it to the set of ten size portfolios at the OSE.

Is er_m a significant determinant for the crossection?

Does it seem sufficient?

Exercise Solutio

Reading the data

```
# estimate m=1+b*f in crossection
library(zoo)
library(texreg)
Rets <- read.zoo("../../data/equity_size_portfolios_monthly
                 header=TRUE,sep=";",format="%Y%m%d")
Rf <- read.zoo("../../data/NIBOR_monthly.txt",
               header=TRUE,sep=";",format="%Y%m%d")
Rm <- read.zoo("../../data/market_portfolios_monthly.txt",
               header=TRUE,sep=";",format="%Y%m%d")
eRmew <- Rm$EW - lag(Rf,-1)
eR  <- Rets - lag(Rf,-1)
# take intersection to align data
data  <- merge(eR,eRmew,all=FALSE)
er  <- as.matrix(data[,1:10])
erm <- as.vector(data[,11])
```

Exercise Solutio

The GMM specification of the moment conditions

```
X <- cbind(er,erm)
g <- function (parms,X) {
  b <- parms[1];
  f <- as.vector(X[,11])
  m <- 1 + b * f
  e <- m * X[,1:10]
  return (e);
}
```

Exercise Solution

```
gmm(g = g, x = X, t0 = t0, method = "Brent", lower = -10, u

Method:  twoStep
Kernel:  Quadratic Spectral(with bw =  3.60168 )

Coefficients:
          Estimate     Std. Error   t value      Pr(>|t|)
Theta[1]  -4.3783e+00   1.0679e+00  -4.1000e+00   4.1321e-0

J-Test: degrees of freedom is 9
                J-test      P-value
Test E(g)=0:    3.2419e+01  1.6849e-04
```

Exercise Solution

To answer the two questions:

- the p value of the coefficient is used to answer the first, the market is a significant determinant.
- the p value of the J test is used to answer the second. Since we reject the J test, we do not find the one factor to be sufficient.

Exercise Solution

	Model 1
Theta[1]	-4.38^{***}
	(1.07)
Criterion function	8207.37
Num. obs.	395

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Exercise

Using the moment condition

$$E[m_t er_{it}] = 0$$

where

$$m_t = 1 + bf_t$$

Using data for 1980-2012, apply this to the three factor model $f = 1 + b_1 er_m + b_2 SMB + b_3 HML$ and apply it to the set of ten size portfolios at the OSE.
Are the three factors significant determinants for the crossection?
Do they seem sufficient?

Exercise Solution

```
Rets <- read.zoo("../../data/equity_size_portfolios_monthly
                 header=TRUE,sep=";",format="%Y%m%d")
Rf <- read.zoo("../../data/NIBOR_monthly.txt",
               header=TRUE,sep=";",format="%Y%m%d")
Rm <- read.zoo("../../data/market_portfolios_monthly.txt",
               header=TRUE,sep=";",format="%Y%m%d")
FF <- read.zoo("../../data/pricing_factors_monthly.txt",
               header=TRUE,sep=";",format="%Y%m%d")
eRmew <- Rm$EW - lag(Rf,-1)
eR  <- Rets - lag(Rf,-1)
data  <- merge(eR,eRmew,na.omit(FF$SMB),na.omit(FF$HML),al
er  <- as.matrix(data[,1:10])
erm <-as.matrix(data[,11])
SMB <- as.matrix(data[,12])
HML <- as.matrix(data[,13])
```

Exercise Solution

Doing the GMM

```
X <- cbind(er,erm,SMB,HML)
g <- function (parms,X) {
  b1 <- parms[1]
  b2 <- parms[2]
  b3 <- parms[3]
  m <- 1 + b1 * X[,11] + b2 * X[,12] + b3 * X[,13]
  e <- m * X[,1:10]
  return (e);
}
```

Exercise Solution

```
> t0 <- c(-1,-1,-1)

Call:
gmm(g = g, x = X, t0 = t0)

Method:  twoStep
Kernel:  Quadratic Spectral(with bw =  3.24629 )

Coefficients:
          Estimate    Std. Error  t value     Pr(>|t|)
Theta[1]  -3.6594378   1.1744252  -3.1159395   0.0018336
Theta[2]  -4.5064637   1.4152014  -3.1843268   0.0014509
Theta[3]  -7.4386994   2.9039257  -2.5616011   0.0104191

J-Test: degrees of freedom is 7
                J-test      P-value
Test E(g)=0:    22.8291039   0.0018254
```

Exercise Solution

Here see that all three pricing factors are significant, so they are influencing the crossection.
We also reject that the model is sufficient, the J statistic is significant.

	Model 1
Theta[1]	-3.66^{***}
	(1.17)
Theta[2]	-4.51^{***}
	(1.42)
Theta[3]	-7.44^{**}
	(2.90)
Criterion function	6039.45
Num. obs.	378

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$