

A lecture on pictures, plotting, in financial econometrics

1. Intro - why we want to think in terms of pictures.
2. The term structure of interest rates - how to easiest show its evolution over time.
3. Looking at the term structure of Norwegian interest rates.

What is the “term structure”?

Interest rate - the price we are willing to be paid to get money *later* rather than *now*.

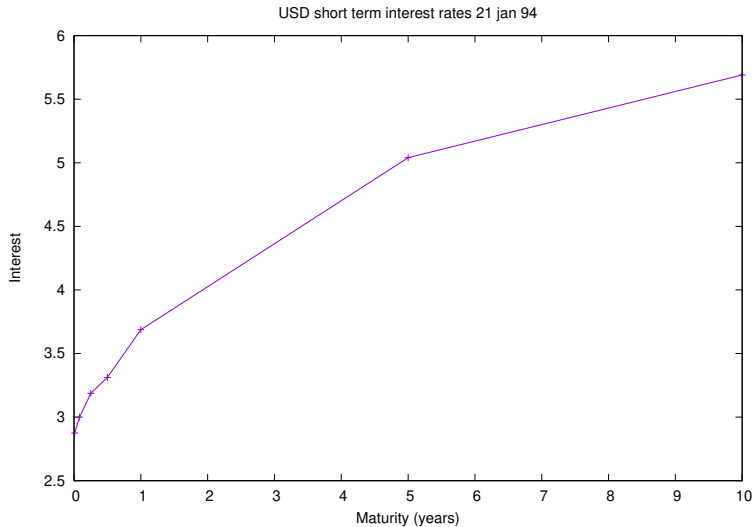
This interest rate varies with the period we discount over.

Find the term structure by looking at prices for *bonds* with various maturities.

For US data, we estimate the term structure from Treasury Bills and Bonds, at various maturities.

Interest rates implied in US treasury prices on 12 jan 1994

Maturity	yield
	21 jan 94
1 week	2.875
1 month	3
3 months	3.1875
6 months	3.3125
1 year	3.6875
5 years	5.04
10 years	5.69

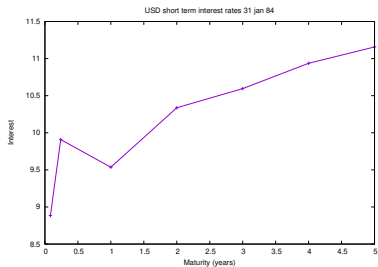
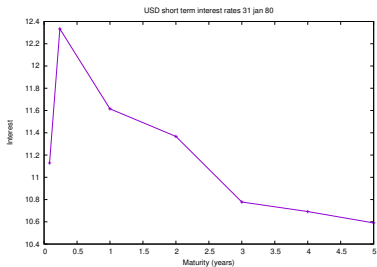


Term structure 21 jan 1994

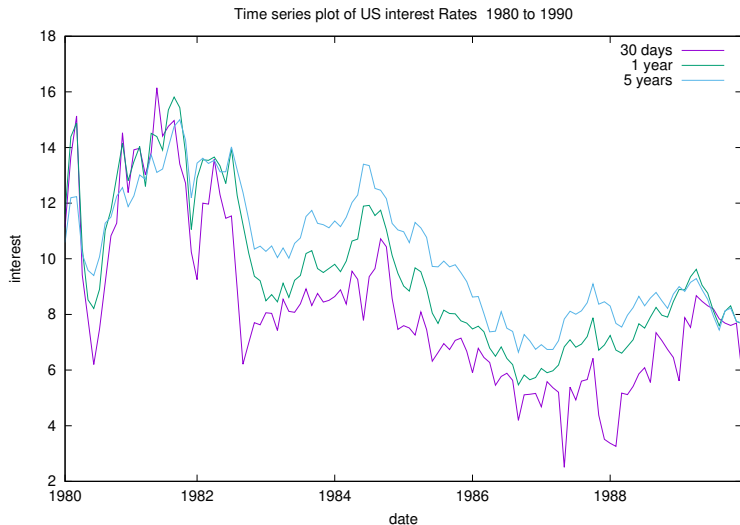
Interest rates implied in US treasury prices in 1980 and 1984

Maturity	yield	
	31 jan 80	31 jan 84
1 month	11.129	8.884
3 months	12.334	9.908
1 year	11.615	9.536
2 years	11.367	10.337
3 years	10.778	10.595
4 years	10.692	10.937
5 years	10.590	11.157

Example term structure in 1980 and 1984.



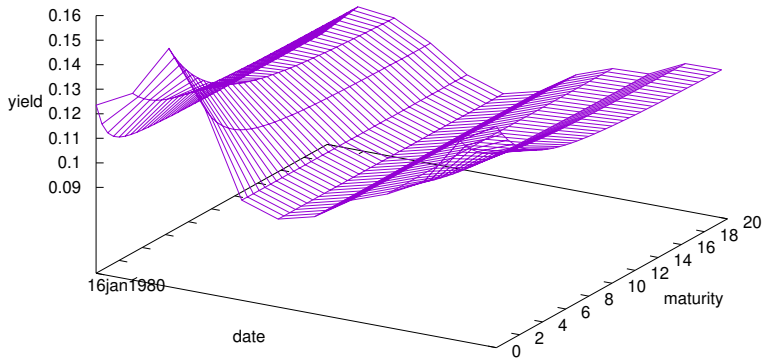
The time series evolution of selected US interest rates



The plots shows the time series evolution of 30 days, 1 year and 5 years interest rates.

The time series evolution of US term structures (3d)

Term Structures 19800116 to 19801216



Plotting Norwegian term structures

- ▶ Similar data for Norway
- ▶ Use matlab / octave - matrix handler
- ▶ Use R - statistics package

Data: Selected maturities, from Norges Bank.

```
date;NIBOR T/N-nom;NIBOR 1U-nom;NIBOR 2U-nom;NIBOR 1M-nom;  
19911031;10.36;10.37;10.46;10.46;10.39;10.39;10.34;10.32;1  
19911130;8.58;9.74;9.83;10.18;10.28;10.34;10.37;10.36;10.3  
.....
```

Read into octave

```
> M = dlmread("term_stru_norway.txt", ";", 1, 0);
```

Setting up the data for plotting

```
> dates=M(:,1);
```

```
> yields=M(:,2:13);
```

```
> maturities=[1/250,1/52,2/52,1/12,2/12,3/12,6/12,9/12,1,3
```

Read into R

```
> termstru <- read.table("term_stru_norway.txt",header=TRUE)
```

Definitions

```
> nrows<-dim(termstru) [1]
```

```
> ncols<-dim(termstru) [2]
```

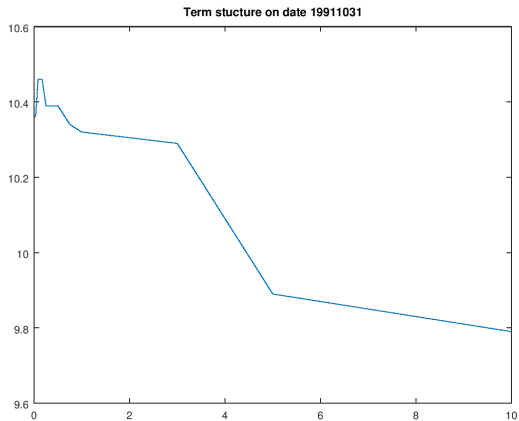
```
> times<-1:nrows
```

```
> mat<-c(1/360,1/52,2/52,1/12,2/12,3/12,6/12,9/12,1,3,5,10)
```

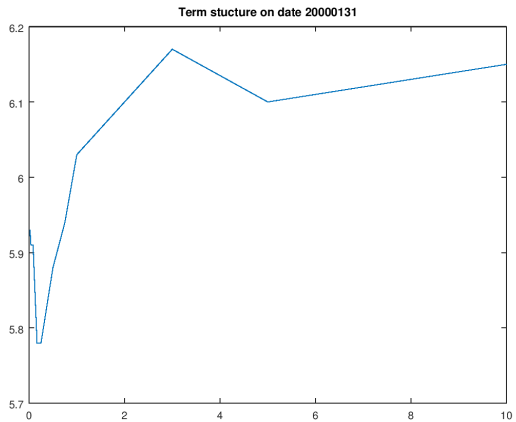
Single term structure using octave

```
> plot(maturities,yields(1,:))
```

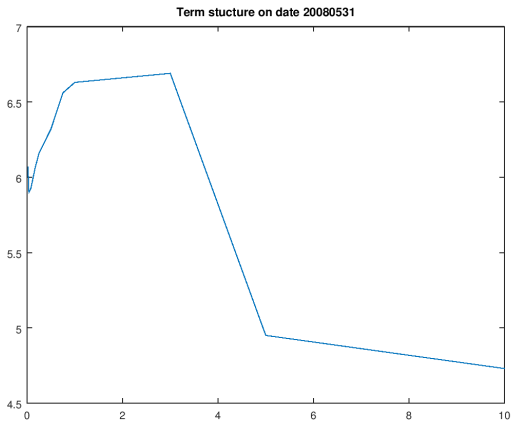
Term structure 31 oct 1991



Single term structure 31 jan 2000



Single term structure 31 may 2008

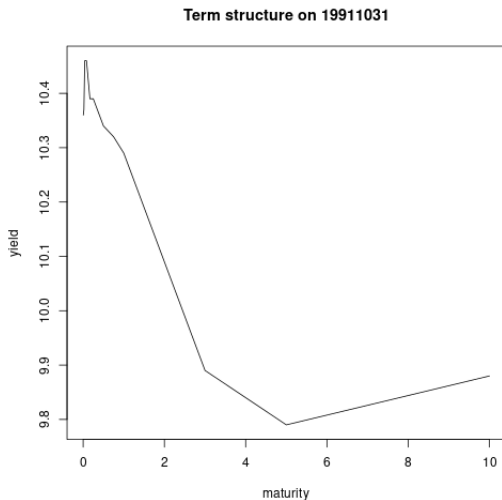


Single term structure using R

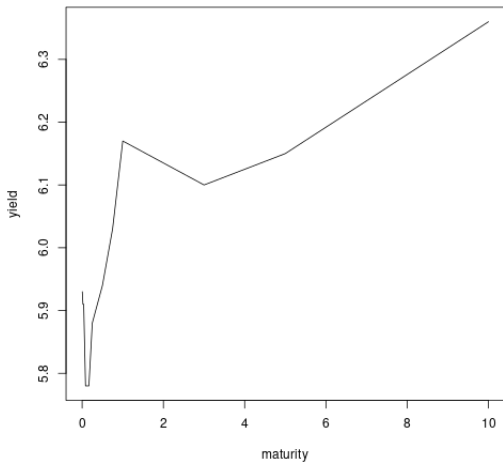
```
> row <- 1
```

```
> plot(mat,termstru[row,2:13],xlab="maturity",ylab="yield")
```

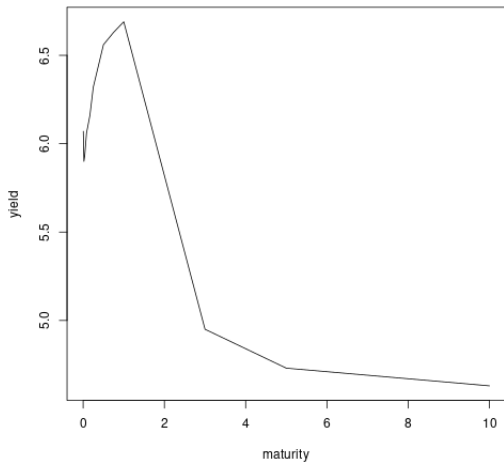
1991 observation



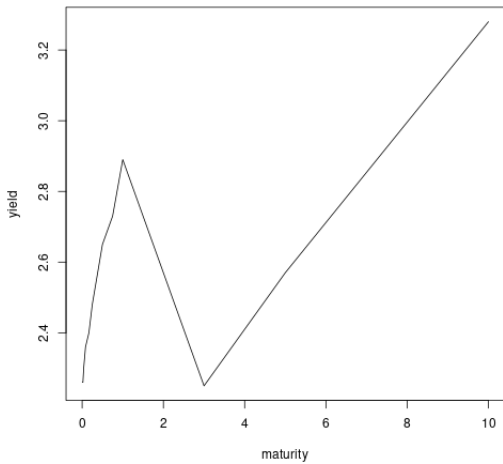
2000 observation



2008 observation



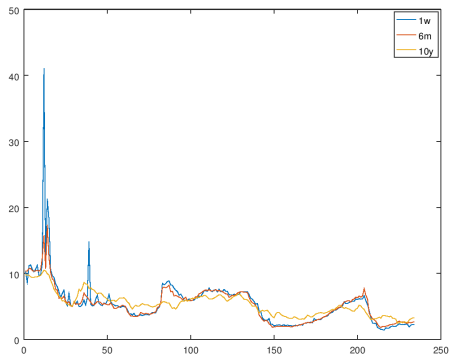
2010 observation



Time series evolution of selected interest rates

Octave commands

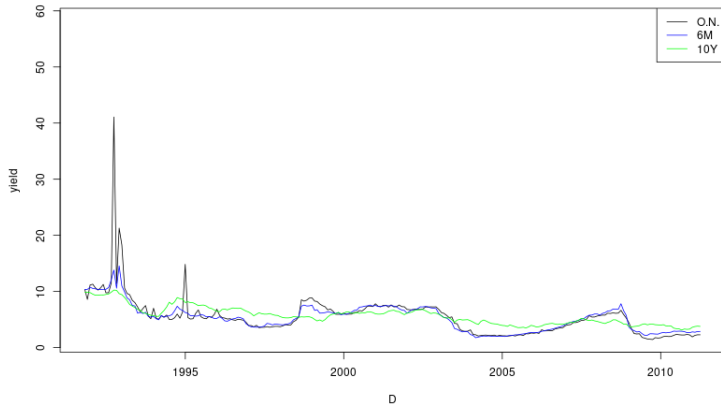
```
> t=1:rows(yields);  
> plot(t,yields(:,1),";o/n;",t,yields(:,7),";6m;",t,yields
```



Time series plot selected norwegian term structures

R commands

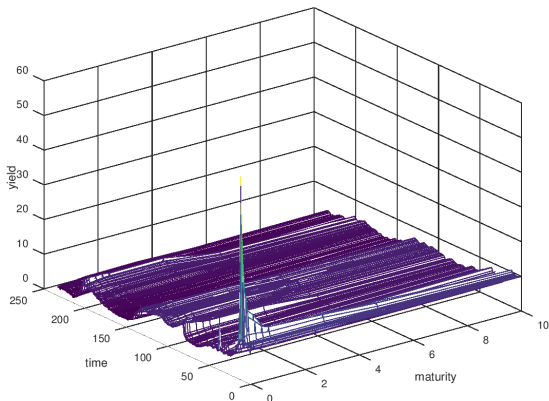
```
> plot(D,termstru$NIBOR.T.N.nom,ylab="yield",type="l",ylim=c(0,60))  
> lines(D,termstru$NIBOR.6M.nom)  
> lines(D,termstru$NIBOR.ST10.nom)
```



3D plot selected norwegian term structures

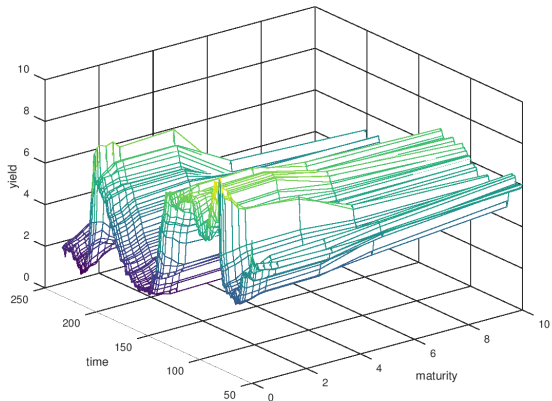
Octave commands

```
> mesh (maturities,t,yields)
> zlabel("yield");
> xlabel("maturity");
> ylabel("time");
```



3D plot, skipping the extreme observations

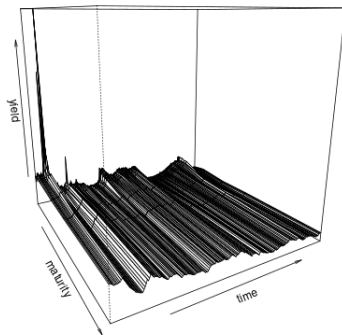
```
> mesh (maturities,t(50:T),yields(50:T,:))
```



3D plot Norwegian term structures

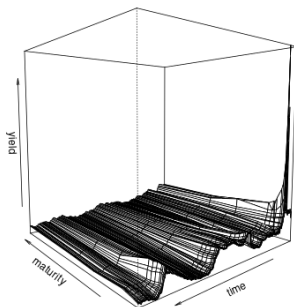
R commands

```
> persp(mat,times,t(termstru[,2:13]),ylab="yield",xlab="ma  
      zlab="yield",theta=60)
```



3D plot Norwegian term structures, alternative perspective

```
> persp(mat,times,t(termstru[,2:13]),ylab="time",xlab="mat",  
        zlab="yield",theta=-130,phi=5)
```



Plotting the Black Scholes option price

Consider the Black Scholes option price

$$c = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)}$$

where S : price underlying. K : exercise price. r : interest rate. σ volatility of underlying. T : maturity date. $(T-t)$: time to maturity.

Let $S = 100$, $X = 100$, $\sigma = 0.1$, time=1, and $r = 0.1$; Plot the Black Scholes value varying S from 80 to 120.

Define Black Scholes formula

```
function c = black_scholes_call(S,K,r,sigma,time)
    time_sqrt = sqrt(time);
    d1 = (log(S/K)+r*time)/(sigma*time_sqrt)+0.5*sigma*time_sqrt;
    d2 = d1-(sigma*time_sqrt);
    c = S * normcdf(d1) - K * exp(-r*time) * normcdf(d2);
endfunction
```

Code to do plot

```
X=100;
sigma=0.1;
time=1;
r=0.1;
Srange=[80:0.2:120];
C=[];
Tight=[];
Exercise=[];
for S=Srange
    C = [C black_scholes_call(S,X,r,sigma,time)];
    Tight = [Tight max(S-exp(-r*time)*X,0)];
    Exercise = [Exercise max(S-X,0)];
endfor;
plot(Srange,C,Srange,Tight,Srange,Exercise)
xlabel("S");
ylabel("C");
print("black_scholes_function_s.png","-dpng");
```

Results in the following picture

