Lecture

Estimating CAPM using GMM

Testing the CAPM in a GMM setting

Consider the CAPM relationship, specified by the "moment condition"

$$E[er_{it}] = \beta_i E[er_{mt}]$$

where er_{it} is the excess return on an asset or a portfolio, and er_{mt} the excess return on the market portfolio.
One can alternatively specify the CAPM more generally as

$$E[r_{it}] = E[r_{zt}] + \beta_i (E[r_{mt}] - E[r_{zt}])$$

where we let r_{zt} be the return on a "zero covariance" portfolio.

The typical way of testing this relationship, the Black et al. [1972] method, estimates the corresponding regression

$$er_{it} = \alpha_i + \beta_i er_{mt} + \varepsilon_{it}$$

and tests whether $\alpha_i = 0$ on an equation by equation basis.
Econometically wasteful: the restriction on the constant will hold for all assets, one want to test this jointly, in a multivariate setting.
Doing this was proposed by Gibbons [1982], who showed how one could construct a multivariate statistic for testing this. His method was later expanded upon by Gibbons et al. [1989].
These statistic were developed under distributional assumptions that allowed us to use Maximum Likelihood, namely multivariate normality.

What if these are not fulfilled, can we still construct a similar test statistic?

This is done in MacKinlay and Richardson [1991] (MR). They construct a test statistic that essentially tests the same restriction, that $\alpha_I = 0$ or that $\alpha_i = E[r_{zt}](1 - \beta_i)$, but in a GMM framework, not a ML.

The setup:
The usual regression

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}$$

Assume that

$$E[\varepsilon_{it} | r_{mt}] = 0$$

Two moment restrictions for each asset i:

$$E[\varepsilon_{it}] = E[(r_{it} - \alpha_i - \beta_i r_{mt})] = 0$$

$$E[\varepsilon_{it} r_{mt}] = E[(r_{it} - \alpha_i - \beta_i r_{mt}) r_{mt}] = 0$$

The model is exactly identified. We can "stack" these moment conditions and estimate the parameters $\{\alpha_i, \beta_i\}$ of the model, by the usual formulation using sample moments.

The tests discussed in the paper are different ways of testing the parametric restriction $\alpha_i = 0$.
We will show how the GMM framework can be used to test the CAPM in a couple of examples involving US and Norwegian Data.

Exercise

Consider the 5 (ew) size based portfolios at the US. Estimate the CAPM using GMM with the Mackinlay method.
Use data 1926-2012.

1. Does the CAPM seem like a sufficient model for pricing this crossection of stock returns?

Exercise Solution

R code to do analysis

Read data

```
source("read_pricing_factors.R")
source("read_size_portfolios.R")
eRi <- FFSize5EW - RF
data <- merge(eRi,RMRF,all=FALSE)
eRi <- as.matrix(data[,1:5])
eRm <- as.matrix(data[,6])
```

```
     Index           Lo20                Qnt2                Q
 Min.   :1926   Min.   :-32.010   Min.   :-31.9600   Min.
 1st Qu.:1948   1st Qu.: -3.110   1st Qu.: -2.8650   1st Qu
 Median :1970   Median :  0.960   Median :  1.1700   Mediar
 Mean   :1970   Mean   :  1.373   Mean   :  0.9704   Mean
 3rd Qu.:1991   3rd Qu.:  4.695   3rd Qu.:  4.5350   3rd Qu
 Max.   :2013   Max.   :110.670   Max.   : 81.1900   Max.
      Qnt4              Hi20              RMRF
 Min.   :-29.760   Min.   :-30.100   Min.   :-28.980
 1st Qu.: -2.470   1st Qu.: -2.195   1st Qu.: -2.105
 Median :  1.160   Median :  0.930   Median :  1.010
 Mean   :  0.787   Mean   :  0.655   Mean   :  0.628
 3rd Qu.:  4.125   3rd Qu.:  3.640   3rd Qu.:  3.655
 Max.   : 50.010   Max.   : 41.790   Max.   : 37.770
```

Exercise Solution

First look at the OLS estimates,
low20: (smallest size)

```
Call:
lm(formula = Lo20 ~ eRm)

Residuals:
    Min       1Q   Median       3Q      Max
-18.245  -2.920  -0.618   1.890  76.656

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.47537    0.19852   2.395   0.0168 *
eRm          1.42903    0.03633  39.334   <2e-16 ***

Residual standard error: 6.344 on 1033 degrees of freedom
Multiple R-squared: 0.5996,Adjusted R-squared: 0.5992
F-statistic:  1547 on 1 and 1033 DF,  p-value: < 2.2e-16
```

Exercise Solution

	1	2	3	4	5
(Intercept)	0.475**	0.096	0.096	0.056	−0.004
	(0.199)	(0.077)	(0.077)	(0.051)	(0.029)
eRm	1.429***	1.260***	1.260***	1.164***	1.049***
	(0.036)	(0.014)	(0.014)	(0.009)	(0.005)
Adj. R^2	0.599	0.886	0.886	0.939	0.974
Num. obs.	1035	1035	1035	1035	1035

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Exercise Solution

```
gmm(g = eRi ~ eRm, x = eRm)

Coefficients:
                    Estimate     Std. Error    t value      Pr(>
Lo20_(Intercept)    4.7537e-01   1.8122e-01    2.6232e+00    8.
Qnt2_(Intercept)    1.3007e-01   1.0484e-01    1.2406e+00    2.
Qnt3_(Intercept)    9.5565e-02   7.1679e-02    1.3332e+00    1.
Qnt4_(Intercept)    5.6185e-02   5.1781e-02    1.0850e+00    2.
Hi20_(Intercept)   -3.7811e-03   2.7477e-02   -1.3761e-01    8.
Lo20_eRm            1.4290e+00   1.0552e-01    1.3543e+01    8.
Qnt2_eRm            1.3382e+00   5.9089e-02    2.2646e+01   1.5
Qnt3_eRm            1.2601e+00   3.6174e-02    3.4833e+01   7.7
Qnt4_eRm            1.1637e+00   1.9760e-02    5.8889e+01    0.
Hi20_eRm            1.0490e+00   1.1175e-02    9.3867e+01    0.

J-Test: degrees of freedom is 0
                 J-test               P-value
Test E(g)=0:     1.7729137049673e-23  *******
```

Exercise Solution

	Model 1
Lo20 (Intercept)	$0.475\ (0.181)^{***}$
Qnt2 (Intercept)	$0.130\ (0.105)$
Qnt3 (Intercept)	$0.096\ (0.072)$
Qnt4 (Intercept)	$0.056\ (0.052)$
Hi20 (Intercept)	$-0.004\ (0.027)$
Lo20 eRm	$1.429\ (0.106)^{***}$
Qnt2 eRm	$1.338\ (0.059)^{***}$
Qnt3 eRm	$1.260\ (0.036)^{***}$
Qnt4 eRm	$1.164\ (0.020)^{***}$
Hi20 eRm	$1.049\ (0.011)^{***}$
Criterion function	0.000
Num. obs.	1035

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Exercise Solution

Now, let us get to the test of whether the intercept is zero:
Need to formulate the linear restrictions in matrix form.

```
> R  <- cbind(diag(5),matrix(0,5,5))
> c  <- rep(0,5)
> print(R)
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
[1,]    1    0    0    0    0    0    0    0    0     0
[2,]    0    1    0    0    0    0    0    0    0     0
[3,]    0    0    1    0    0    0    0    0    0     0
[4,]    0    0    0    1    0    0    0    0    0     0
[5,]    0    0    0    0    1    0    0    0    0     0
> print(c)
[1] 0 0 0 0 0
```

Exercise Solution

and then perform the test:

```
> linearHypothesis(res,R,c,test="F")
Linear hypothesis test

Hypothesis:
Lo20_((Intercept) = 0
Qnt2_((Intercept) = 0
Qnt3_((Intercept) = 0
Qnt4_((Intercept) = 0
Hi20_((Intercept) = 0

Model 1: restricted model
Model 2: eRi ~ eRm

  Df  Chisq Pr(>Chisq)
1
2  5 13.588    0.01845 *
```

Fisher Black, Michael Jensen, and Myron Scholes. The capital asset pricing model, some empirical tests. In Michael C Jensen, editor, *Studies in the theory of capital markets*. Preager, 1972.

Michael R Gibbons. Multivariate tests of financial models, a new approach. *Journal of Financial Economics*, 10:3–27, March 1982.

Michael R Gibbons, Stephen A Ross, and Jay Shanken. A test of the efficiency of a given portfolio. *Econometrica*, 57:1121–1152, 1989.

A Craig MacKinlay and Matthew P Richardson. Using generalized method of moments to test mean-variance efficiency. *Journal of Finance*, 46:511–27, 1991.