# Fama Macbeth type of analysis 

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## 1 Fama-MacBeth type of analysis

The paper by Fama and MacBeth (1973) is important in empirical finance, much because of their methodological innovation.

Let us look at this first in the context of the original paper, designed to test the CAPM.

## 2 CAPM estimation

Let us introduce some notation
$r_{j t}$ is the return on stock $j$ at time $t$.
$r_{m t}$ is the return on a stock market index $m$ at time $t$.
$r_{f t}$ is the risk free interest rate over the same period.
Define the excess return as the return in excess of the risk free return.

$$
\begin{aligned}
e r_{j t} & =r_{j t}-r_{f t} \\
e r_{m t} & =r_{m t}-r_{f t}
\end{aligned}
$$

The CAPM specifies

$$
E\left[r_{j t}\right]=r_{f t}+\left(r_{m t}-r_{f t}\right) \beta_{j m},
$$

where $\beta_{j m}$ can be treated as a constant.
This can be rewritten as

$$
E\left[r_{j t}\right]-r_{f t}=\left(r_{m t}-r_{f t}\right) \beta_{j m}
$$

or, in excess return form

$$
E\left[e r_{j t}\right]=E\left[e r_{m t}\right] \beta_{j m}
$$

Consider now estimating the crossectional relation

$$
\left(r_{j t}-r_{f t}\right)=a_{t}+b_{t} \beta_{j \hat{m}}+u_{j t} j=1,2, \ldots, N
$$

or in excess return form

$$
e r_{j t}=a_{t}+b_{t} \beta_{j \hat{m}}+u_{j t} j=1,2, \ldots, N
$$

Comparing this to the CAPM prediction

$$
e r_{j t}=e r_{m t} \beta_{j m}
$$

we see that the prediction of the CAPM is:

$$
\begin{gathered}
E\left[a_{t}\right]=0 \\
E\left[b_{t}\right]=\left(E\left[r_{m}\right]-r_{f}\right)>0
\end{gathered}
$$

To test this, average estimated $a_{t}, b_{t}$ :

Test whether

$$
\begin{aligned}
& E\left[a_{t}\right]=0, \quad \frac{1}{T} \sum_{t=1}^{T} a_{t} \rightarrow 0 \\
& E\left[b_{t}\right]>0, \\
& \frac{1}{T} \sum_{t=1}^{T} b_{t}>0
\end{aligned}
$$

To do these tests we need an estimate of $\beta_{j \hat{m}}$. The "usual" approach is to use time series data to estimate $\beta_{j \hat{m}}$ from the "market model"

$$
r_{j t}=\alpha_{j}+\beta_{j m} r_{m t}+\varepsilon_{j t}
$$

on data before the crossection.

### 2.1 The Fama and MacBeth (1973) paper.

Let us first look at the original paper (Fama and MacBeth, 1973), and use their notation
The usual theoretical restriction:

$$
E\left[R_{i}\right]=E\left[R_{0}\right]+\beta_{i}\left(E\left[R_{m}\right]-E\left[R_{0}\right]\right)
$$

Implications of the model tested in the paper:
C 1 : Risk-return relation is linear.
$\mathrm{C} 2: \beta_{i}$ is sufficient to describe expected return.
C3: Positive market premium.
Consider the multivariate regression:

$$
R_{i t}=\gamma_{0 t}+\gamma_{1 t} \beta_{i}+\gamma_{2 t} \beta_{i}^{2}+\gamma_{3 t} s_{i}+\eta_{i t}
$$

where $s_{i}$ is some measure of non-beta risk.
What is actually tested?
C1: Linearity: Test $\gamma_{2 t}=0$.
C2: $\beta_{i}$ sufficient: Test $\gamma_{3 t}=0$.
C3: Positive market premium. Test $\gamma_{1 t}>0$.
Note that these are test of the null against specific alternatives.
Implementation:

- Control for nonstationarity of betas: Use portfolios instead of individual securities.
- $\beta_{i}$ is not known, replace with an estimate $\hat{\beta}_{i}$. How to get this estimate? Implement by using data for previous periods to do beta estimation, i.e. $\beta_{i t}$ is estimated using return data for periods $t-61 \cdots t-1$.
- What to use as a measure of non-beta risk? In the paper, they estimate the (own) variance of the security.
How do we find this estimate of the variance? Consider running a regression on the market, or the market model:

$$
r_{i t}=\alpha_{i}+\beta_{i} r_{m t}+\varepsilon_{i t}
$$

In this context, the (own) variance is $E\left[\varepsilon_{i t}^{2}\right]$. FM uses the obvious estimate of the variance, by looking at the sample analogs of this for the same periods that the betas are estimated over.

$$
\hat{s}_{i t}=\frac{1}{60} \sum_{j=1}^{60}\left(r_{i, t-j}-\hat{\alpha}_{i}-\hat{\beta}_{i} r_{m, t-j}\right)^{2}
$$

This is then the setup.
The results are summarized as:

- $\sigma^{2}$ does not add significant forecasting ability, ie $\gamma_{3 t}=0$ can not be rejected.
- $\beta^{2}$ does not have significant predictive power, ie $\gamma_{2 t}=0$ can not be rejected.
- Can not reject a positive relation between $\beta$ and return, ie $\gamma_{1 t}>0$ can not be rejected.
- However, do reject that $\gamma_{0 t}=0$, find that $\gamma_{0 t}>0$, ie $E\left[R_{i t}\right]-E\left[R_{z t}\right]>0$.


## 3 The mechanics of doing this type of analysis

We will be replicating the Fama MacBeth type of analysis in R.
The mechanics of doing someting like this is a bit involved, one need to loop over estimations.
Exercise 1.
Gather the returns of 10 size based portfolios from Ken French website. Using the data from 1926-2013, do a Fama-MacBeth analysis, i.e.

Estimate

$$
e r_{i t}=a_{t}+b_{t} \beta_{i t}+e_{i t}
$$

and test whether $a_{t}=0$ and $b_{t}>0$.
In doing this use the previous five years to estimate betas using the market model.
Solution to Exercise 1.
The following computer code will do the trick

```
source ("../data/read_pricing_factors.R")
source ("../data/read_size_portfolios.R")
eR <- (FFSize10EW - RF)/100.0
eRm <- RMRF/100.0
n <- length(eRm)
B <- NULL
for (n2 in 61:n) {
    n1 <- n2-60
    regr <- lm(eR[n1:(n2-1),]~eRm[n1:(n2-1)])
    betai <- regr$coefficients[2,]
    eRi <- eR[n2,]
    attributes(betai) <- NULL
    attributes(eRi) <- NULL
    regr <- lm(eRi ~ betai)
    b <- regr$coefficients
    B <- rbind (B,b)
}
head(B)
colMeans(B)
t.test(B[,1])
t.test(B[,2])
```

Let us go over this in come detail.
The data has the form

| $>$ head (eR) |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Lo10 | Dec2 | Dec3 | Dec4 | Dec5 | Dec6 | Dec7 | Dec8 |
| $1926(7)$ | -0.0141 | -0.0183 | 0.0135 | 0.0124 | 0.0083 | 0.0178 | 0.0127 | 0.0127 |
| $1926(8)$ | 0.0478 | 0.0239 | 0.0370 | 0.0328 | 0.0281 | 0.0434 | 0.0126 | 0.0110 |
| $1926(9)$ | -0.0048 | -0.0111 | -0.0232 | -0.0097 | -0.0065 | 0.0021 | -0.0184 | 0.0073 |
| $1926(10)$ | -0.0443 | -0.0327 | -0.0306 | -0.0522 | -0.0343 | -0.0315 | -0.0381 | -0.0319 |
| $1926(11)$ | -0.0150 | 0.0010 | 0.0010 | 0.0289 | 0.0304 | 0.0365 | 0.0338 | 0.0264 |
| $1926(12)$ | -0.0327 | 0.0581 | 0.0421 | 0.0211 | 0.0066 | 0.0095 | 0.0207 | 0.0293 |
|  | Dec9 | Hi10 |  |  |  |  |  |  |
| $1926(7)$ | 0.0306 | 0.0307 |  |  |  |  |  |  |


| $1926(8)$ | 0.0070 | 0.0341 |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $1926(9)$ | -0.0124 | 0.0044 |  |  |  |
| $1926(10)$ | -0.0408 | -0.0275 |  |  |  |
| $1926(11)$ | 0.0337 | 0.0239 |  |  |  |
| $1926(12)$ | 0.0304 | 0.0273 |  |  |  |
| $>$ | head (eRm) |  |  |  |  |
| $1926(7)$ | $1926(8)$ | $1926(9)$ | $1926(10)$ | $1926(11)$ | $1926(12)$ |
| 0.0295 | 0.0263 | 0.0038 | -0.0324 | 0.0254 | 0.0262 |

Let us look at the first round of the loop,

```
n2 <- 61
> n1 <- 1
> regr <- lm(eR[n1:(n2-1),]~eRm[n1:(n2-1)])
```

This runs 10 different regressions on 10 size sorted portfolios

|  | Dependent variable: |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| $\overline{e R m[n 1:(n 2-1)] ~}$ | $\begin{gathered} 1.287^{* * *} \\ (0.141) \end{gathered}$ | $\begin{gathered} 1.312^{* * *} \\ (0.095) \end{gathered}$ | $\begin{gathered} 1.192^{* * *} \\ (0.080) \end{gathered}$ | $\begin{gathered} 1.049^{* * *} \\ (0.057) \end{gathered}$ | $\begin{gathered} 1.100^{* * *} \\ (0.049) \end{gathered}$ | $\begin{gathered} 1.129^{* * *} \\ (0.046) \end{gathered}$ | $\begin{gathered} 1.114^{* * *} \\ (0.036) \end{gathered}$ | $\begin{gathered} 1.085^{* * *} \\ (0.036) \end{gathered}$ | $\begin{gathered} 1.100^{* * *} \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.964^{* * *} \\ (0.017) \end{gathered}$ |
| Constant | $\begin{gathered} 0.001 \\ (0.010) \end{gathered}$ | $\begin{aligned} & -0.009 \\ & (0.006) \end{aligned}$ | $\begin{aligned} & -0.007 \\ & (0.005) \end{aligned}$ | $\begin{gathered} -0.008^{* *} \\ (0.004) \end{gathered}$ | $\begin{gathered} -0.006^{*} \\ (0.003) \end{gathered}$ | $\begin{aligned} & -0.003 \\ & (0.003) \end{aligned}$ | $\begin{gathered} -0.005^{* *} \\ (0.002) \end{gathered}$ | $\frac{-0.004^{*}}{(0.002)}$ | $\begin{aligned} & -0.002 \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.001 \\ (0.001) \end{gathered}$ |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
|  | 0.590 | 0.766 | 0.795 | 0.854 | 0.896 | 0.911 | 0.944 | 0.940 | 0.950 | 0.982 |
| Adjusted R ${ }^{2}$ | 0.583 | 0.762 | 0.791 | 0.851 | 0.894 | 0.909 | 0.943 | 0.939 | 0.949 | 0.981 |
| Residual Std. Error ( $\mathrm{df}=58$ ) | 0.074 | 0.050 | 0.042 | 0.030 | 0.026 | 0.024 | 0.019 | 0.019 | 0.017 | 0.009 |
| F Statistic ( $\mathrm{df}=1 ; 58$ ) | 83.516*** | 189.944*** | $224.713^{* * *}$ | $338.565 * * *$ | 497.568*** | 591.560*** | 982.590*** | 903.319*** | 1,109.718*** | 3,122.788*** |

We now pull the vector of beta coefficients
> betai <- regr\$coefficients[2,]

| Lo10 | Dec2 | Dec3 | Dec4 | Dec5 | Dec6 | Dec7 | Dec8 | Dec9 | Hi10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.287 | 1.312 | 1.192 | 1.049 | 1.100 | 1.129 | 1.114 | 1.085 | 1.100 | 0.964 |

This is the explanatory variable in a regression in next-period returns
> eRi <- eR[n2,]
> attributes(betai) <- NULL
> attributes(eRi) <- NULL
> regr <- lm(eRi ~ betai)
(the attributes part is to allow the coefficients to be used as an explanatory variable.
The results of this regression is

|  | Dependent variable: |
| :--- | :---: |
|  | eRi |
| betai | 0.041 |
|  | $(0.033)$ |
| Constant | $-0.116^{* *}$ |
|  | $(0.038)$ |
| Observations | 10 |
| R $^{2}$ | 0.162 |
| Adjusted $\mathrm{R}^{2}$ | 0.057 |
| Residual Std. Error | $0.011(\mathrm{df}=8)$ |
| F Statistic | $1.544(\mathrm{df}=1 ; 8)$ |
| Note: | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |

The results of the loop in the Fama-Macbeth analysis is then doing this over and over again, moving a "window" of time over which we estimate the beta coefficients using the market model, and using this beta coefficient to predict the return.

```
> head(B)
    (Intercept) betai
b -0.11625046 0.041478451
b 0.19939307-0.192615845
b -0.29409924 -0.009725232
b 0.07420183 0.021346338
b -0.13722297 0.040692233
b 0.22861050 -0.372036413
> colMeans(B)
    (Intercept) betai
-0.006526818 0.013277653
```

Testing whether the mean is different from zero

```
> t.test(B[,1])
One Sample t-test
data: B[, 1]
t = -1.4332, df = 989, p-value = 0.1521
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
    -0.015463375 0.002409738
sample estimates:
    mean of x
-0.006526818
```

Regarding the test for the market risk premium we need to specify the alternative differently, since we are explicitly testing whether it is positive.

```
> t.test(B[,2],alternative=c("greater"))
One Sample t-test
data: B[, 2]
t = 2.7907, df = 989, p-value = 0.002681
alternative hypothesis: true mean is greater than 0
95 percent confidence interval:
    0.005444296 Inf
sample estimates:
    mean of x
0.01327765
```

Summarizing the results

|  | constant | beta |
| :--- | :---: | :---: |
| average | -0.007 | 0.013 |
| p.value | 0.152 | 0.002 |

## 4 Econometric issues

So far have not gone into the econometrics of this type of analysis, simply done ordinary tests.
However, there are econometric issues in this type of analysis.
Best known: Errors in Variables, since betas are estimated
Solution used by Fama and MacBeth (1973): Group stocks into portfolios, reducing estimation error in betas.

A recent overview of econometrics of Panel data in finance, including Fama Macbeth: ?

## 5 Replicating Chen Roll and Ross

As a more involved example of using the Fama and Macbeth type of methodology, let us look at the replication of a well known empirical study. In 1986 Chen, Roll and Ross published a paper where they did a Fama MacBeth type of analysis of US stock market crossections, asking whether a number of explanatory variables were risk factors.

We will do a similar analysis updating their data set till today.
Specifically, they use the following explanatory variables

- US Inflation
- US Treasury bill rate (short term)
- US industrial production
- US Long term treasury rates
- Low-Grade bonds (Baa)
- Stock market return
- US Consumption (per capita)
- Oil Prices

They investigate to what degree these alternative "pricing factors" can explain the crossection of asset returns.

The factors they use are (slightly simplified)

- $\beta$ - Stock market beta
- $d I P$ - change in (log) Industrial Production
- Infl - Inflation (change in log cpi)
- $d I n f l$ - first difference of Inflation (not log)
- Term - Term Premium (TermSpread)
- Qual - Risk premium (Quality Spread)
- dCons - change in (log) Consumption
- dOil - change in (log) Oil prices

Construct these variables.
We will use as stock market return data for 49 different industry portfolios (ew) provided by Ken French, and use returns starting in 1970.

We will first do the FM analysis for each of the variables. For example, for the CAPM beta we analyze

$$
e r_{i t}=a+b_{\beta} \widehat{\beta}_{i t}^{m}+e_{i t}
$$

where the betas are estimated using a MM type regression on data before $t$, for example five years.

$$
e r_{i \tau}=\alpha_{i}+\beta_{i t}^{m} e r_{m \tau}+\varepsilon_{i \tau}
$$

using observations $\tau=t-61, \cdots, t-1$.
For each of the non-beta variables, we will also do an analysis adding the variable to beta and investigate whether it adds explanatory power to the CAPM.

For example, for Industrial Production one will estimate

$$
e r_{i t}=a+b_{\beta} \widehat{\beta}_{i t}^{m}+b_{i p} \widehat{\beta}_{i t}^{i p}+e_{i t},
$$

where the betas are estimated using a MM type regression on data before $t$, for example five years.
Reading the original paper it is not clear which version of the MM regression they do, are we looking at a joint regression

$$
e r_{i \tau}=\alpha_{i}+\beta_{i t}^{m} e r_{m \tau}+\beta_{i t}^{i p} d I P_{\tau}+\varepsilon_{i \tau}
$$

using observations $\tau=t-61, \cdots, t-1$.
or a "factor by factor" type of analysis?

$$
\begin{aligned}
& e r_{i \tau}=\alpha_{i}+\beta_{i t}^{m} e r_{m \tau}+\varepsilon_{i \tau} \\
& e r_{i \tau}=\alpha_{i}+\beta_{i t}^{i p} d I P_{\tau}+\varepsilon_{i \tau}
\end{aligned}
$$

We will be using this latter version.

### 5.1 Gathering the data

In R, gathering this data is actually relatively simple, as they can be downloaded from the St. Louis Fed data library FRED. Specifically, we will download

- CPIAUCSL (Cpi)
- POP (Population)
- DNDGRA3M086SBEA (Real Consumption)
- INDPRO (Industrial Production)
- OILPRICE
- BAA
- DTB3 (3 month t bills)
- DGS10 (10 year treasuries)
and use these to construct the data series.
The following is the R code for doing the data variable construction

```
library(stargazer)
library(zoo)
library (quantmod)
source ("../data/read_pricing_factors.R")
source ("../data/read_industry_portfolios.R")
eR <- (FF49IndusEW - RF)/100.0
head(eR)
eR <- window(eR,start=c(1970,1))
Rm <- RMRF/100.0
names(eRm) <- "eRm"
getSymbols("CPIAUCSL",src="FRED") # cpi urban
head(CPIAUCSL)
cpi <- CPIAUCSL
head(cpi)
Infl <- diff(log(cpi))
head(Infl)
Infl <- zooreg(coredata(Infl),order.by=as.yearmon(index(Infl)))
names(Infl)<-"Infl"
head(Infl)
                            #plain difference, not log, inflation may be negative
dInfl <- diff(Infl)
names(dInfl)<-"dInfl"
getSymbols("POP",src="FRED")
Population <- zooreg(coredata(POP),order.by=as.yearmon(index(POP)))
names(Population) <- "Population"
head(Population)
getSymbols("DNDGRA3M086SBEA", src="FRED")
head (DNDGRA3M086SBEA)
```

```
RealConsump <- DNDGRA3M086SBEA
names(RealConsump) <- "RealConsump"
RealConsump <- zooreg(coredata(RealConsump),order.by=as.yearmon(index(RealConsump)))
head(RealConsump)
data <- merge(RealConsump,Population,all=FALSE)
RealConsPerCapita <- 1e8*data$RealConsump/data$Population
head(RealConsPerCapita)
dRealCons <- diff(log(RealConsPerCapita))
names(dRealCons) <- "dRealCons"
                            # industrial production index, US
getSymbols("INDPRO",src="FRED")
head(INDPRO)
IndProd <- INDPRO
IndProd <- zooreg(coredata(IndProd),order.by=as.yearmon(index(IndProd)))
head(IndProd)
dIndProd <- diff(log(IndProd))
names(dIndProd)<-"dIndProd"
# oil price
getSymbols("OILPRICE",src="FRED")
head(OILPRICE)
tail(OILPRICE)
OilPrice <- zooreg(coredata(OILPRICE),order.by=as.yearmon(index(OILPRICE)))
head(OilPrice)
dOilPrice <- diff(log(OilPrice))
names(dOilPrice)<-"dOilPrice"
    # interest rates
    # BAA, low quality bonds
getSymbols("BAA",src="FRED")
head (BAA)
Baa <- zooreg(coredata(BAA),order.by=as.yearmon(index(BAA)))
    # daily series, only starts in 1986, so do not use
etSymbols("DBAA",src="FRED")
head(DBAA)
    # 3 month t-bill, monthly
getSymbols("TB3MS",src="FRED")
head(TB3MS)
getSymbols("DTB3",src="FRED")
head(DTB3)
mTbill3m <- na.locf(DTB3)[endpoints(na.locf(DTB3),on="months")]
mTbill3m <- zooreg(coredata(mTbill3m),order.by=as.yearmon(index(mTbill3m)))
head(mTbill3m)
    # 10 year treasury rates
getSymbols("GS10",src="FRED")
head(GS10)
                            # 10 year treasury rate, daily
getSymbols("DGS10",src="FRED")
head(DGS10)
mTreas10y <- na.locf(DGS10) [endpoints(na.locf(DGS10), on="months")]
mTreas10y <- zooreg(coredata(mTreas10y),order.by=as.yearmon(index(mTreas10y)))
head(mTreas10y)
QualSpread <- Baa-mTreas10y
names(QualSpread) <- "QualSpread"
head(QualSpread)
TermSpread <- mTreas10y-mTbill3m
names(TermSpread) <- "TermSpread"
head(TermSpread)
```

Let us first illustrate the R code for doing the simplest possible Fama Macbeth analysis, a single estimation of the CAPM

```
library(stargazer)
library(zoo)
library(quantmod)
source ("../data/read_pricing_factors.R")
source ("../data/read_industry_portfolios.R")
R <- (FF49IndusEW - RF)/100.0
length(eR)
head(eR)
eRm <- RMRF/100.0
head(eRm)
eR <- window(eR,start=c(1970,1),end=c(2014,1))
data <- merge(eR,eRm,all=FALSE)
```

```
ER <- data[,1:49]
ERM <- data[,50]
head(ER)
head(ERM)
n <- length(ERM)
B <- NULL
Rsqrs <- NULL
n2<-61
for (n2 in 61:n)
{
    n1 <- n2-60
    regr <- lm(ER[n1:(n2-1),] ~ERM[n1:(n2-1)])
    betai <- regr$coefficients[2,]
    eRi <- ER[n2,]
    attributes(eRi) <- NULL
    attributes(betai) <- NULL
    regr <- lm(eRi ~ betai )
    b <- regr$coefficients
    B <- rbind(B,b)
    rsqr <- summary(regr)$adj.r.squared
    Rsqrs <- c(Rsqrs,rsqr)
}
head(B)
colMeans(B)
t.test(B[,1])
t.test(B[,2],alternative=c("two.sided"))
t.test(B[,2],alternative=c("greater"))
est <- colMeans(B)
p1 <- t.test(B[,1])$p.value
p2 <- t.test(B[,2],alternative=c("greater"))$p.value
test <- rbind(test,c(p1,p2))
colnames(test)<- c("constant","beta");
rownames(test)<- c("average","p.value");
print(test)
diagn <- c(nrow(B),mean(Rsqrs))
names(diagn) <- c("n","mean R2")
tabl1 <- stargazer(test,float=FALSE,summary=FALSE)
tabl2 <- stargazer(diagn,float=FALSE, summary=FALSE)
cat(tabl1,tabl2,file="../R_tables/indus_portf_49_capm_1970_.tex",sep="\n")
```

This results in the following output tables

|  | constant | beta |
| :--- | :---: | :---: |
| average | 0.008 | 0.002 |
| p.value | 0.001 | 0.229 |

n mean R2
$468 \quad 0.094$
As we see, the CAPM is not supported in this sample.
The next example shows the analysis of a model where we add the oil price to the market portfolio, and test whether the oil price is a priced risk factor.

Here we find the following results:
First, just oil, without the market portfolio.

|  | constant | oil |
| :--- | :---: | :---: |
| average | 0.010 | 0.003 |
| p.value | 0.00002 | 0.688 |


| n | mean R2 |
| :---: | :---: |
| 463 | 0.076 |

Then the two versions of the analysis adding oil to the market portfolio.

|  | constant | beta | oil |
| :--- | :---: | :---: | :---: |
| average | 0.006 | 0.004 | 0.002 |
| p.value | 0.006 | 0.129 | 0.739 |

n mean R2
$463 \quad 0.155$

|  | constant | beta | oil |
| :--- | :---: | :---: | :---: |
| average | 0.006 | 0.004 | 0.002 |
| p.value | 0.006 | 0.090 | 0.807 |

n mean R2
$463 \quad 0.155$
Oil does not seem to add much explanatory power, but note that including it in the analysis makes the market portfolio much more significant, the p-value falls to 0.054 (in the version with betas estimated from separate MM regressions).

There does not seem to be a big difference between the analysis using a "joint" MM regression and the "factor by factor" regresssions. In the following we will therefore use this formulation.

We now report the results for each of the alternative explanatory variables, first just the variable itself, and then adding the variable to the stock market.

### 5.2 Industrial Production

|  | constant | ind.prod |
| :--- | :---: | :---: |
| average | 0.009 | -0.002 |
| p.value | 0.0002 | 0.008 |


| n | mean R2 |  |
| :--- | :---: | :---: | :---: |
|  |  |  |
| 468 | 0.055 |  |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.136 |

### 5.3 Inflation

|  | constant | dInfl |
| :--- | :---: | :---: |
| average | 0.011 | -0.0003 |
| p.value | 0.00005 | 0.273 |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.070 |


|  |  |  |  |
| :--- | :---: | :---: | :---: |
|  | constant | beta | dInfl |
| average | 0.007 | 0.004 | 0.00001 |
| p.value | 0.002 | 0.125 | 0.969 |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.151 |

### 5.4 Qual Spread

|  | constant | QualSpread |  |
| :---: | :---: | :---: | :---: |
| average | 0.009 | 0.027 |  |
| p.value | 0.001 | 0.624 |  |
| n | mean R2 |  |  |
| 468 | 0.054 |  |  |
|  | constant | beta | QualSpread |
| average | - 0.008 | 0.001 | 0.016 |
| p.value | 0.0005 | 0.416 | 0.770 |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.133 |

### 5.5 Term Spread

|  | constant | TermSpread |
| :--- | :---: | :---: |
| average | 0.007 | 0.059 |
| p.value | 0.005 | 0.585 |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.059 |

$468 \quad 0.059$

|  | constant | beta | TermSpread |
| :--- | :---: | :---: | :---: |
| average | 0.006 | 0.001 | 0.028 |
| p.value | 0.004 | 0.389 | 0.801 |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.146 |

### 5.6 Consumption

|  | constant | dConsum |
| :--- | :---: | :---: |
| average | 0.009 | 0.0004 |
| p.value | 0.0005 | 0.524 |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.050 |


|  | constant | beta | Consum |
| :--- | :---: | :---: | :---: |
| average | 0.007 | 0.003 | -0.0003 |
| p.value | 0.003 | 0.187 | 0.643 |


| n | mean R2 |
| :---: | :---: |
| 468 | 0.133 |

### 5.7 Chen Roll Ross approximation

To gather the above analysis into a single analysis, we look at The formulation that Chen Roll and Ross focus on,

$$
R=a+b_{m p} M P+b_{d e i} D E I+b_{u i} U I+b_{u p r} U P R+b_{u t s} U T S
$$

where MP - montly change in industrial production
DEI - change in expected inflation
UI - unexpected inflation
UPR - risk premium (quality spread)
UST - term structure (term spread)
These are the risk premia associated with the various factors.
We will instead of their two inflation measures merely use one variable measuring inflation differences.
We therefore estimate using the following data
dIndProd - change in (log) industrial production
dInfl - change in inflation
QualSpread - Quality Spread
TermSpread - Term Spread
First, some summary stats of explanatory variables

| Table 1 |  |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Statistic | N | Mean | St. Dev. | Min | Max |  |
| eRm | 619 | 0.005 | 0.045 | -0.232 | 0.161 |  |
|  | dIndProd | 619 | 0.002 | 0.007 | -0.043 | 0.030 |
|  | dInfl | 619 | 0.00000 | 0.003 | -0.014 | 0.018 |
|  | QualSpread | 619 | 2.000 | 0.859 | 0.100 | 6.280 |
|  | TermSpread | 619 | 1.529 | 1.249 | -1.910 | 4.390 |
|  | dRealCons | 619 | 0.001 | 0.007 | -0.040 | 0.034 |
|  | dOilPrice | 619 | 0.006 | 0.075 | -0.396 | 0.853 |

Correlations

|  | eRm | dIndProd | dInfl | QualSpread | TermSpread | dRealCons | dOilPrice |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| eRm | 1 |  |  |  |  |  |  |  |
| dIndProd | -0.002 | 1 |  |  |  |  |  |  |
| dInfl | -0.062 | -0.014 | 1 |  |  |  |  |  |
| QualSpread | 0.073 | -0.318 | -0.041 | 1 | 1 | 1 |  |  |
| TermSpread | 0.069 | 0.022 | -0.031 | 0.464 | 0.03 | -0.039 | 1 |  |
| dRealCons | 0.163 | 0.153 | -0.18 | -0.057 | -0.069 | -0.103 |  |  |
| dOilPrice | 0.012 | 0.034 | 0.298 | -0.18 |  |  |  |  |

First we do the analysis without the market portfolio

|  | constant | dIndProd | dInfl | QualSpread | TermSpread |
| :--- | :---: | :---: | :---: | :---: | :---: |
| average | 0.008 | -0.002 | -0.0001 | -0.023 | 0.080 |
| p.value | 0.0004 | 0.007 | 0.829 | 0.776 | 0.511 |

n mean R2
$468 \quad 0.183$
Then we add the market portfolio to this analysis

|  | constant | betai | dIndProd | dInfl | QualSpread | TermSpread |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| average | 0.006 | 0.002 | -0.002 | 0.0001 | -0.003 | 0.026 |
| p.value | 0.010 | 0.255 | 0.017 | 0.761 | 0.967 | 0.819 |

n mean R2
$468 \quad 0.235$
Finally, add consumption as another potential explanatory variable

|  | constant | betai | dIndProd | dInfl | QualSpread | TermSpread | dConsum |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| average | 0.005 | 0.002 | -0.002 | 0.0003 | -0.023 | 0.063 | -0.0002 |
| p.value | 0.016 | 0.259 | 0.052 | 0.321 | 0.765 | 0.595 | 0.595 |


|  |  |
| :---: | :---: |
| n | mean R2 |
| 468 | 0.254 |

and oil as a final alternative explanatory variable

|  | constant | betai | dIndProd | dInfl | QualSpread | TermSpread | dOil |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| average | 0.006 | 0.004 | -0.002 | 0.0001 | -0.044 | 0.017 | 0.008 |
| p.value | 0.007 | 0.144 | 0.002 | 0.693 | 0.565 | 0.882 | 0.882 |


| n | mean R 2 |
| :---: | :---: |
| 463 | 0.258 |

Interestingly, Oil seems to "destroy" dIndProd as an explanatory variable. Let us look at just those in isolation

|  | constant | beta | IP | oil |
| :--- | :---: | :---: | :---: | :---: |
| average | 0.004 | 0.006 | -0.002 | 0.001 |
| p.value | 0.069 | 0.037 | 0.008 | 0.813 |


| n | mean R2 |
| :---: | :---: |
| 463 | 0.181 |

## 6 Literature

The original paper Fama and MacBeth (1973)
Textbook treatment (Campbell, Lo, and MacKinlay, 1997, pg 215)
Econometrics of using Fama-MacBeth standard errors ?

## References

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