

# Measuring Performance with Factor Models

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# The Jensen alpha

Does the return on a portfolio/asset exceed its *required* return?

$$\alpha_p = r_p - \text{required return} = r_p - \hat{r}_p$$

To find an estimate of required return an asset pricing model is required.

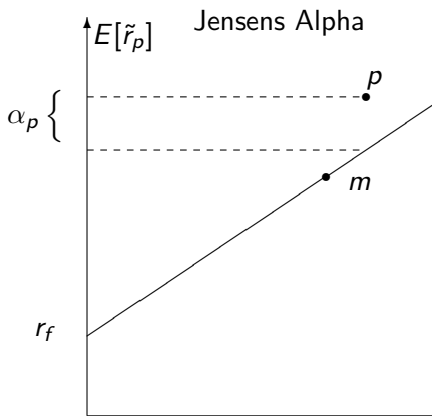
## The Jensen alpha ctd

The Classical such asset pricing model is the CAPM, which is what Jensen used

$$\hat{r}_p = (r_f + \beta_p(r_m - r_f))$$

Alpha is then

$$\alpha_p = r_p - (r_f + \beta_p(r_m - r_f))$$



## An alternative to the CAPM in Alpha calculations

The original Jensen measure is written in terms of the CAPM, but one can alternatively use another asset pricing model.

For example, we can write the alpha in terms of the Fama-French 3 factor model.

$$E[r_{i,t}] = r_{f,t} + (E[r_{m,t}] - r_{f,t})\beta_i + b_i^{hml} HML_t + b_i^{smb} SMB_t$$

The alpha for a portfolio  $p$  is then calculated as

$$\alpha_{p,t} = r_{p,t} - \left( r_{f,t} + \beta_i (r_{m,t} - r_{f,t}) + b_i^{hml} HML_t + b_i^{smb} SMB_t \right)$$

For a maintained hypothesis about asset pricing model, we can always calculate an alpha relative to the given model.

## Returns-based analysis

Standard benchmark for academics – four-factor model of Carhart [1997].

$$eR_{pt} = \alpha + \beta \text{RMRF}_t + s \text{SMB}_t + h \text{HML}_t + u \text{UMD}_t + \varepsilon_{pt}$$

where

$eR_{pt}$  is the month- $t$  excess return on a the managed portfolio (net return minus T-bill return)

$\text{RMRF}_t$  is the month- $t$  excess return on a value-weighted aggregate market proxy portfolio; and

$\text{SMB}_t$ ,  $\text{HML}_t$  and  $\text{UMD}_t$  are month- $t$  return on value-weighted zero-investment factor-mimicking portfolios for size, book-to-market (BTM) equity, and one-year momentum in stock returns, respectively.

One reason for the popularity of this model as a benchmark is the provision by Ken French of these factors on his homepage.

These factors applies to the cross-section of US stock returns. For other market places similar pricing factors applies, factors that captures predictable variation in asset returns.

## Example

We download the returns for *Folketrygdfondet*, a Pension Fund controlled by the Ministry of Finance, primarily investing in the Norwegian equity markets.

With this data, do a performance analysis using one factor and three factor models

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + \varepsilon_t$$

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + b_s SMB_t + b_h HML_t + \varepsilon_t$$

Consider both an equally weighted and a value weighted market index.

The results are summarized as

	Model 1	Model 2	Model 3	Model 4
(Intercept)	-0.005*	-0.008***	-0.001	-0.007***
	(0.002)	(0.001)	(0.002)	(0.001)
eRmEW	1.076***		0.981***	
	(0.041)		(0.030)	
eRmVW		0.988***		0.959***
		(0.019)		(0.022)
SMB			-0.534***	-0.092**
			(0.036)	(0.031)
HML			0.001	0.018
			(0.032)	(0.025)
Adj. R <sup>2</sup>	0.776	0.936	0.896	0.938
Num. obs.	195	195	195	195

\*\*\*  $p < 0.001$ , \*\*  $p < 0.01$ , \*  $p < 0.05$

## So what are we assuming when doing this?

If we want to interpret alpha as a measure of superior performance  
We put a lot of belief in the ability of asset managers.

- ▶ We assume the portfolio manager understands the process generating returns (i.e. what asset pricing model applies.)
- ▶ The portfolio manager must choose correctly/underpriced assets relative to the asset pricing model. (i.e. be able to estimate all the parameters of the model for a given asset, and use that to find the correct required return for the asset – alternatively believe the market prices things correctly, with a few exceptions that the manager has identified.)
- ▶ The portfolio manager must choose a portfolio that correctly aligns the factor risks with the desired factor exposures.
- ▶ These calculations are viewed as mechanical ways of adjusting the portfolio, the asset manager should not be rewarded unless one has gone long assets underpriced according to the model (alternatively shorted/underweighted overpriced assets)



## Is this reasonable?

If these calculations are limited to beta, it seems feasible, but once we go to three and four factors...

So, one should be careful in interpreting alpha as superior performance.

Particularly when one moves away from the U.S. crosssection where these models were developed...

However, the estimate of alpha is not the only interesting piece of information in factor regressions.

The *coefficent* estimates is also informative.

Knowing that e.g. the coefficient estimate on **SMB** is significant: The portfolio captures some of the risk premium associated with size.

Mark M Carhart. On persistence in mutual fund performance. *Journal of Finance*, 52(1):57–82, March 1997.