Performance evaluation of managed portfolios

The business of evaluating the performance of a portfolio manager has developed a rich set of methodologies for testing whether a manager is skilled or not.
The goal is to identify whether the manager has a skill that goes beyond simple, well known strategies that can easily be implemented by unskilled investors. For example, portfolio tilts towards small stocks should not necessarily be viewed as skill.
The methods can be grouped into two major approaches

1. Returns-based performance evaluation
2. Portfolio holdings-based performance evaluation

Performance evaluation of managed portfolios

Pros and cons.

Returns-based:

1. Rely on less information
2. Returns are often available at higher frequencies than other information

Portfolio holdings-based

1. Will more clearly identify skill
2. Require more information than returns-based measures.

Benchmark

A benchmark is a measuring tape, a portfolio that is an alternative investment opportunity.
Good benchmarks should be

- Unambiguous
- Tradeable
- Measurable
- Appropriate
- Reflective of current investment opinions
- Specified in advance.

Performance measures

Chen and Knez (1996): Desirable properties of performance measures.

- Fit. Capture strategies relevant for uninformed investors. Have zero performance for simple strategies feasible for such investors.
- Be Scalable. Linear combinations of manager measures should equal the measure for the linear combination of manager portfolios
- Be continuous. Close skills/strategies should have close performance measures.
- Exhibit monotonicity. Assign higher measures for more skilled managers.

An added desirable property is manipulation-proofness. See Goetzmann, Ingersoll, Spiegel, and Welch (2007)

Overview of rest of talk

Show examples of methods used for doing portfolio performance evaluation.

Only two examples in the talk.

- Baseline Regression Model
- Stochastic Discount Factor based performance measurement

Returns-based analysis

Standard benchmark for academics – four-factor model of Carhart (1997).

$$eR_{pt} = \alpha + \beta \mathrm{RMRF}_t + s\mathrm{SMB}_t + h\mathrm{HML}_t + u\mathrm{UMD}_t + \varepsilon_{pt}$$

where

eR_{pt} is the month-t excess return on a the managed portfolio (net return minus T-bill return)

RMRF_t is the month-t excess return on a value-weighted aggregate market proxy portfolio; and

SMB_t , HML_t and UMD_t are month-t return on value-weighted zero-investment factor-mimicking portfolios for size, book-to-market (BTM) equity, and one-year momentum in stock returns, respectively.

One reason for the popularity of this model as a benchmark is the provision by Ken French of these factors on his homepage.

These factors applies to the cross-section of US stock returns. For other market places similar pricing factors applies, factors that captures predictable variation in asset returns.

Exercise

On the course homepage you will find returns for *Folketrygdfondet*, a Pension Fund controlled by the Ministry of Finance, primarily investing in the Norwegian equity markets. The file "folketrygdfondet_1998_2014.csv" contains data for 1998 to 2014. In this file, the first data column (labeled SPN), contains data for the norwegian equity part of the portfolio. With this data, do a performance analysis using one factor and three factor models

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + \varepsilon_t$$

$$eR_{pt} = \alpha_p + \beta_p eR_{mt} + b_s SMB_t + b_h HML_t + \varepsilon_t$$

Consider both an equally weighted and a value weighted market index.

Exercise Solution

You read in the data and align it.
Show reading the FTF data:

```
library(zoo)
datadir <- "/home/bernt/data/2015/folketrygdfondet/"
filename <- paste(datadir,"folketrygdfondet_1998_2014.csv",
data <- read.zoo(filename,format="%m/%d/%Y",skip=1,header=T
rets <- as.numeric(coredata(data$SPN))
SpnRets <- zoo(rets/100.0,order.by=as.yearmon(index(data))
head(SpnRets)
```

Exercise Solution

The resulting time series are summarized as

Statistic	N	Mean	St. Dev.	Min	Max
eRp	195	0.005	0.063	−0.245	0.141
eRmew	195	0.010	0.051	−0.188	0.119
eRmvw	195	0.014	0.061	−0.221	0.162
SMB	195	0.006	0.042	−0.171	0.133
HML	195	−0.001	0.046	−0.166	0.093

Exercise Solution

Doing the regressions. One factor model

```
eRp <- SpnRets - Rf
data <- merge(eRp,eRmew,eRmvw,all=FALSE)
eRp <- data$eRp
eRmEW <- data$eRmew
eRmVW <- data$eRmvw

regrEW <- lm(eRp ~ eRmEW)
regrVW <- lm(eRp ~ eRmVW)
```

Exercise Solution

Doing the regressions, Three factor model

```
data <- merge(eRp,eRmew,eRmvw,SMB,HML,all=FALSE)
eRp <- data$eRp
eRmEW <- data$eRmew
eRmVW <- data$eRmvw
SMB <- data$SMB
HML <- data$HML

regrEW3 <- lm(eRp ~ eRmEW+SMB+HML)
regrVW3 <- lm(eRp ~ eRmVW+SMB+HML)
```

Exercise Solution

The results are summarized as

	Model 1	Model 2	Model 3	Model 4
(Intercept)	-0.005^{*}	-0.008^{***}	-0.001	-0.007^{***}
	(0.002)	(0.001)	(0.002)	(0.001)
eRmEW	1.076^{***}		0.981^{***}	
	(0.041)		(0.030)	
eRmVW		0.988^{***}		0.959^{***}
		(0.019)		(0.022)
SMB			-0.534^{***}	-0.092^{**}
			(0.036)	(0.031)
HML			0.001	0.018
			(0.032)	(0.025)
Adj. R^2	0.776	0.936	0.896	0.938
Num. obs.	195	195	195	195

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Stochastic Discount Factors

An alternative formulation of the performance estimation problem comes from adapting the methods used for estimating asset pricing model.
Any asset pricing model can be written as a condition on the stochastic discount factor m_t that prices the risk in the economy at time t.

$$E[\mathbf{m}_t \mathbf{R}_t - 1] = 0$$

This relationship must also hold for any managed portfolio p

$$E[\mathbf{m}_t R_{pt} - 1] = 0$$

or, in conditional form,

$$E[Z_{t-1} \mathbf{m}_t R_{pt} - Z_{t-1} 1] = 0$$

Stochastic Discount Factors

Suppose we estimate the discount factor $\hat{m}$ using a crossection of assets. This *empirical stochastic discount factor* can then be used to evaluate any *other* assets, such as a portfolio.
Performance measurement is then a matter of calculating:

$$\alpha_p = \hat{m}_t R_{pt} - 1$$

When R_{pt} is a gross return (Unconditional), or

$$\alpha_p = \hat{m}_t R_{pt}$$

When R_{pt} is an excess return (Unconditional).
With conditioning information we would use:

$$\alpha_p = E[Z_{t-1} \hat{m}_t R_{pt} - Z_{t-1}],$$

Exercise

On the course homepage you will find returns for *Folketrygdfondet*, a Pension Fund controlled by the Ministry of Finance, primarily investing in the Norwegian equity markets. The file "folketrygdfondet_1998_2014.csv" contains data for 1998 to 2014. In this file, the first data column (labeled SPN), contains data for the norwegian equity part of the portfolio. With this data you want to do a portfolio performance analysis.

You want to use a SDF approach to evaluate the portfolio. To this end you first estimate a SDF using the crossection of 10 size based portfolios in the Norwegian Equity Market, i.e. you evaluate

$$E_{t-1}\left[\mathbf{m}_t \mathbf{eR}_{it}\right] = 0$$

using data for the Norwegian Equity Market 1980–2014, where $\mathbf{eR}_{it}$ is excess return on the set of 10 size sorted portfolios.

Exercise

You parameterize $\mathbf{m}_t$ as follows

$$\mathbf{m}_t = 1 + b_1 eR_{mt} + b_2 SMB_t + b_3 HML_t,$$

where eR_{mt} is excess return for an (equally weighted) market index, and *SMB* and *HML* are Norwegian versions of the Fama-French factors.
You use data for the Norwegian crossection to estimate the parameters $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$. This estimation is done with GMM.
Given the estimated parameters, you calculate the *empirical sdf* $\hat{\mathbf{m}}$:

$$\hat{\mathbf{m}}_t = 1 + \hat{b}_1 eR_{mt} + \hat{b}_2 SMB_t + \hat{b}_3 HML_t$$

This empirical sdf is then used to estimate the alpha

$$\alpha_p = \hat{m}_t R_{pt}$$

Exercise Solution

First estimate the discount factor **m**.
Data for Norway is read in, not shown.
Excess returns for size portfolios in eR:

```
> eR <- SizeRets-Rf
> head(eR)
                     1           2           3           4
feb. 1980   0.09332633  0.12805033  0.09656333  0.01081033
mars 1980   0.04064733 -0.13399067 -0.11062267 -0.02122667
april 1980  0.04325900 -0.02528300  0.01138800 -0.02672600
mai 1980    0.13158033 -0.01072267  0.02496333  0.00331933
juni 1980  -0.07027333  0.05159967 -0.01640333  0.08002867
juli 1980   0.08894633  0.05146533  0.00258433 -0.01490567

....
```

Exercise Solution

Start by gathering all the necessary data into one matrix `X`:

```
data  <- merge(eR,eRm,SMB,HML,all=FALSE)
er  <- as.matrix(data[,1:10])
erm <- as.matrix(data[,11])
SMB <- as.matrix(data[,12])
HML <- as.matrix(data[,13])
X <- cbind(er,erm,SMB,HML)
```

Exercise Solution

To do the GMM estimation, set up moment conditions and rund GMM

```
g <- function (parms,X) {
  b1 <- parms[1]
  b2 <- parms[2]
  b3 <- parms[3]
  m <- 1 + b1 * X[,11] + b2 * X[,12] + b3 * X[,13]
  e <- m * X[,1:10]
  return (e);
}
library(gmm)
t0 <- c(0.1,0,0)
res <- gmm(g,X,t0)
```

Exercise Solution

The results of the GMM estimation

```
gmm(g = g, x = X, t0 = t0)
Method:  twoStep
Kernel:  Quadratic Spectral(with bw =  3.23446 )

Coefficients:
          Estimate     Std. Error   t value      Pr(>|t|)
Theta[1]  -3.96584720   1.18316431  -3.35189893   0.000802
Theta[2]  -4.62060402   1.35085274  -3.42050906   0.000625
Theta[3]  -8.93536075   3.51482567  -2.54219173   0.011015

J-Test: degrees of freedom is 7
                J-test     P-value
Test E(g)=0:    16.005459   0.025067

Initial values of the coefficients
  Theta[1]    Theta[2]    Theta[3]
 -2.650790  -5.875247 -13.255253
```

Exercise Solution

Summarizing the results

	Model 1
Theta[1]	$-3.966\ (1.183)^{***}$
Theta[2]	$-4.621\ (1.351)^{***}$
Theta[3]	$-8.935\ (3.515)^{*}$
Criterion function	4072.636
Num. obs.	393

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Exercise Solution

We can now construct an “ex post” **m**.

```
> print(res$coefficients)
 Theta[1]  Theta[2]  Theta[3]
-3.965847 -4.620604 -8.935361
> b <- as.numeric(res$coefficients)
> m <- 1 + b[1] * X[,11] + b[2] * X[,12] + b[3] * X[,13]
> m <- zoo(m,order.by=index(data))
> head(m)
  juli 1981   aug. 1981   sep. 1981   okt. 1981   nov. 198
 1.31491216 -0.02696329  0.96937468  0.82884810  0.4391530
```

Exercise Solution

This **m** is then used to estimate the alpha of the portfolio.
First align the data

```
>                                             # portfolio to be
> eRp <- SpnRets - Rf
>                                             # intersection of
> data <- merge(m,eRp,all=FALSE)
> head(data)
                     m         eRp
jan. 1998  1.4690374 -0.03175000
feb. 1998  0.9694579  0.03640000
mars 1998  1.3309379  0.07070833
april 1998 1.4671863  0.03778333
mai 1998   0.9361106 -0.08715833
juni 1998  1.0683310 -0.00269167
> mhat <- data$m
> eRp <- data$eRp
```

Exercise Solution

Then do calculation

```
>                                        # do alpha calcul
> alpha <- mhat*eRp
> head(alpha)
   jan. 1998    feb. 1998    mars 1998   april 1998     ma
-0.046641937  0.035288268  0.094108397  0.055435183 -0.081
> tail(alpha)
   okt. 2013    nov. 2013    des. 2013    jan. 2014    feb.
 0.033069149  0.020558175  0.003943663 -0.003177656  0.017
```

Exercise Solution

This result in a time series of monthly alpha estimates.

```
> summary(alpha)
     Index          alpha
 Min.   :1998   Min.   :-0.343869
 1st Qu.:2002   1st Qu.:-0.021946
 Median :2006   Median : 0.007956
 Mean   :2006   Mean   : 0.004929
 3rd Qu.:2010   3rd Qu.: 0.037217
 Max.   :2014   Max.   : 0.303020
```

Exercise Solution

Superior performance is found if this on average is positive. To do a statistical test, treat each observation as independent, and test whether the mean is significantly positive.

```
> mean(alpha)
[1] 0.004929535
> t.test(alpha)

One Sample t-test


data:  alpha
t = 0.9721, df = 194, p-value = 0.3322
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.00507157  0.01493064
sample estimates:
  mean of x
0.004929535
```

Exercise Solution

Note that the previous test is a test against alpha equal to zero. If all we are concerned with is the ability to have *positive* alpha, we do a one sided test.

```
> t.test(alpha,alternative="greater")

One Sample t-test

data:  alpha
t = 0.9721, df = 194, p-value = 0.1661
alternative hypothesis: true mean is greater than 0
95 percent confidence interval:
 -0.003451319          Inf
sample estimates:
  mean of x
0.004929535
```

Holdings-based analysis

Do not just consider the portfolio returns, we use the complete records of the asset composition of the portfolios.
What can this achieve?

- It may alleviate the sensitivity of returns bases measures to choice of benchmark (the Roll critique).
- This approach may deal with nontrivial shifts in style allocations.
- One can look at performance before trading costs (which are incorporated in returns).
- One can decompose the sources of value added by a manager.
- Holdings-based analysis leads to more precise identification of manager ability, as observing performance on a security-by security basis increases the number of observations of ability.

Holdings-based analysis

holdings-bases measures $\rightarrow$ the covariance between lagged weights and current returns.

$$PHM_t = \text{cov}(w_{t-1}, R_t)$$

Intuition:
A skilled manager will have portfolio weights that move in the same direction as future returns.
Grinblatt and Titman (1993):

$$GT_t = \sum_j (w_{j,t-1} - w_{j,t-2}) R_{j,t}$$

Averaged across time

Stochastic Discount Factors and weight measures

Generate intuition
General relationship

$$E_t[m_{t+1} R_{t+1} | Z_t] = 1$$

where R_{+1} is the vector of primitive asset returns, m is the stochastic discount factor, and Z_t is conditioning information.
For a given portfolio p, Alpha is calculated as

$$\alpha_p = E_t[m_{t+1} R_{p,t+1} | Z_t] - 1$$

Stochastic Discount Factors and weight measures

asset manager: chooses a set of weights w_t
weights – function of the asset manager's information set Ω_t

$$w_t = w_t(\Omega_t)$$

The next period portfolio return $R_{p,t+1}$ is then

$$R_{p,t+1} = w_t(\Omega_t)R_{t+1}$$

Plugging this into the alpha calculation

$$\alpha_p = E_t[m_{t+1}w_t(\Omega_t)R_{t+1}|Z_t] - 1$$

from the definition of covariance

$$\text{cov}(m_{t+1} R_{t+1}, w_t(\Omega_t))$$

$$= E[m_{t+1} R_{t+1} w_t(\Omega_t)] - E[m_{t+1} R_{t+1}] E[w_t(\Omega_t)]$$

From the fundamental pricing relation

$$E[m_{t+1} R_{t+1}] = 1$$

the second term in the covariance is equal to 1
and we can express alpha as

$$\alpha_p = \text{cov}(m_{t+1} R_{t+1}, w_t(\Omega_t) | Z_t)$$

Interpretation: alpha — the covariance between the weights with the risk-adjusted returns

Mark M Carhart. On persistence in mutual fund performance. *Journal of Finance*, 52(1):57–82, March 1997.

Zhiwu Chen and Peter J. Knez. Portfolio performance measurement: Theory and applications. *Review of Financial Studies*, 9(2):511–555, Summer 1996.

W Goetzmann, J Ingersoll, M Spiegel, and I Welch. Portfolio performance manipulation and manipulation-proof performance measures. *Review of Financial Studies*, 20:1503–46, 2007.

Mark Grinblatt and Sheridan Titman. Performance measurement without benchmarks. *Journal of Business*, 66:47–68, 1993.