# Intro

"Performance question": How well did a given equity portfolio perform?

We observe the actual portfolio return.

How "good" was this return?

Need: A theoretical framework.

Mean Variance framework: The classical measures: Sharpe, Treynor and Jensen.

Many alternatives.

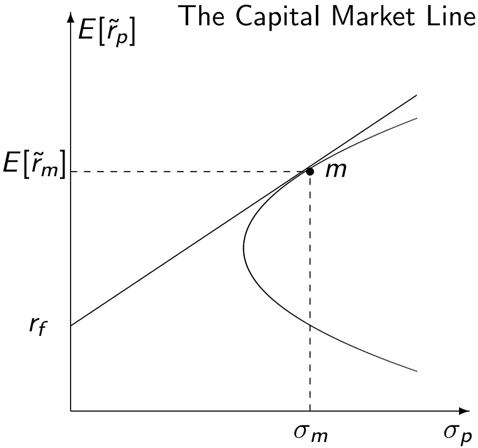
- Modifications of the classical measures.
  Example: Jensen alpha - based on the CAPM.
  Alternative: Alpha measure using alternative models for required returns.
- Alternatives bringing more information into the evaluation of the portfolio.
  What if not only using returns?
  Another piece of information one can potentially bring into the analysis is the actual portfolio decisions, when stocks are bought and sold.

# Classical (absolute) measures of performance.

- Sharpe Ratio
- Treynor Ratio
- Jensen's Alpha

# The Sharpe Ratio

How far is an asset $p$ from the Capital Market Line?



# The Sharpe Ratio ctd

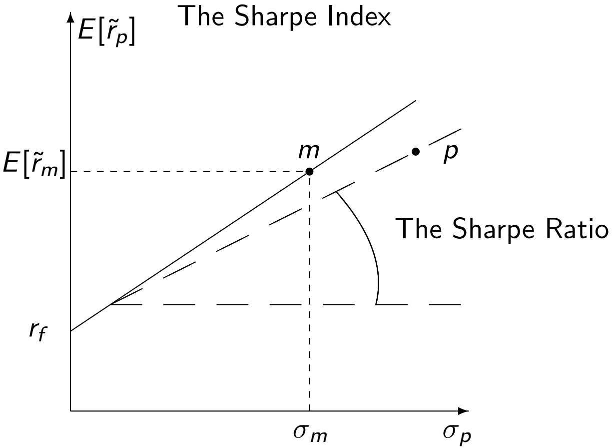
$$r_p - r_f = S\sigma_p$$

Here $S$ is the slope of the line from the risk free rate through $p$. From the equation for this line solve for $S$:

$$S = \frac{r_p - r_f}{\sigma_p}$$

# The Sharpe Ratio ctd

Sharpe index used for comparisons.
For example to the market



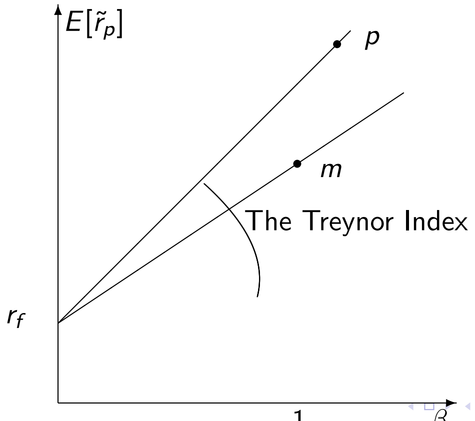

Sharpe is primarily used for undiversified portfolios.

# The Treynor index

$$T = \frac{r_p - r_f}{\beta_p}$$

This is the slope of a line in mean-beta space starting at $r_f$.
Treynor index used for comparisons, e.g. to market

The Treynor Index



# The Jensen alpha

Does the return on a portfolio/asset exceed its *required* return?

$$\alpha_p = r_p - \text{required return} = r_p - \hat{r}_p$$

To find an estimate of requred return an asset pricing model is required.

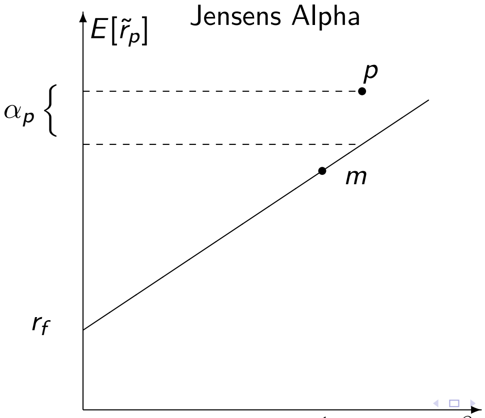
# The Jensen alpha ctd

The Classical such asset pricing model is the CAPM, which is what Jensen used

$$\hat{r}_p = (r_f + \beta_p(r_m - r_f))$$

Alpha is then

$$\alpha_p = r_p - (r_f + \beta_p(r_m - r_f))$$



# Exercise

You are given historical returns of two different equities, $r_A$ and $r_B$, as well as the market return $r_m$, and the risk free rate $r_f$.

| rA, | rB, | rm, | rf |
|---|---|---|---|
| 0.10, | 0.05, | 0.01, | 0.01 |
| 0.20, | 0.03, | -0.05, | 0.01 |
| -0.10, | -0.01, | -0.05, | 0.01 |
| 0.13, | 0.03, | 0.10, | 0.01 |
| 0.24, | 0.04, | 0.14, | 0.0140 |
| -0.08, | -0.05, | -0.02, | 0.02 |
| -0.15, | -0.02, | 0, | 0.02 |
| 0.15, | 0.12, | 0.10, | 0.01 |
| 0.45, | 0.15, | 0.05, | 0.01 |
| -0.10, | -0.10, | 0.04, | 0.02 |
| 0.01, | 0.01, | 0.03, | 0.01 |
| -0.05, | -0.01, | 0.01, | 0.01 |
| 0.20, | 0.11, | 0.05, | 0.02 |
| -0.05, | 0.12, | 0.05, | 0.01 |

# Exercise

Use matlab/octave to calculate

- Sharpe measures
- Treynor measures
- Jensen alphas (relative to the CAPM)

# Exercise Solution

```
> rets = dlmread("../data/example.txt",",",1,0);
rets =
    0.10000   0.05000   0.01000   0.01000
    0.20000   0.03000  -0.05000   0.01000
   -0.10000  -0.01000  -0.05000   0.01000
    0.13000   0.03000   0.10000   0.01000
    0.24000   0.04000   0.14000   0.01400
   -0.08000  -0.05000  -0.02000   0.02000
   -0.15000  -0.02000   0.00000   0.02000
    0.15000   0.12000   0.10000   0.01000
    0.45000   0.15000   0.05000   0.01000
   -0.10000  -0.10000   0.04000   0.02000
    0.01000   0.01000   0.03000   0.01000
   -0.05000  -0.01000   0.01000   0.01000
    0.20000   0.11000   0.05000   0.02000
   -0.05000   0.12000   0.05000   0.01000
```

# Exercise Solution

```
> rA = rets(:,1);
> rB = rets(:,2);
> rm = rets(:,3);
> rf = rets(:,4);
```

Calculating Sharpe Measures

```
> sA = mean(rA-rf)/std(rA)
sA =  0.32043
> sB = mean(rB-rf)/std(rB)
sB =  0.28515
> sm = mean(rm-rf)/std(rm)
sm =  0.35502
```

# Exercise Solution

Treynor measure, first need to estimate beta

```
> betaA = cov(rA,rm)/var(rm)
betaA =  1.3418
> betaB = cov(rB,rm)/var(rm)
betaB =  0.52031
> betam = 1
betam =  1
```

Then can calculate

```
> tA = mean(rA-rf)/betaA
tA =  0.040778
> tB = mean(rB-rf)/betaB
tB =  0.039262
> tm = mean(rm-rf)/betam
tm =  0.019714
```

# Exercise Solution

Alpha measure

```
> alphaA = mean(rA - (rf + betaA*(rm-rf)))
alphaA =  0.028262
> alphaB = mean(rB - (rf + betaB*(rm-rf)))
alphaB =  0.010171
```

# Exercise

You are given historical returns of two different equities, $r_A$ and $r_B$, as well as the market return $r_m$, and the risk free rate $r_f$.

```
   rA,      rB,      rm,     rf
 0.10,    0.05,    0.01,   0.01
 0.20,    0.03,   -0.05,   0.01
-0.10,   -0.01,   -0.05,   0.01
 0.13,    0.03,    0.10,   0.01
 0.24,    0.04,    0.14,   0.0140
-0.08,   -0.05,   -0.02,   0.02
-0.15,   -0.02,       0,   0.02
 0.15,    0.12,    0.10,   0.01
 0.45,    0.15,    0.05,   0.01
-0.10,   -0.10,    0.04,   0.02
 0.01,    0.01,    0.03,   0.01
-0.05,   -0.01,    0.01,   0.01
 0.20,    0.11,    0.05,   0.02
-0.05,    0.12,    0.05,   0.01
```

Use R to calculate

- Sharpe measures
- Treynor measures
- Jensen alphas (relative to the CAPM)

# Solution

```
> data <- read.table("../data/example.txt",
                     header=TRUE,sep=",")
> head(data)
     rA    rB    rm    rf
1  0.10  0.05  0.01 0.010
2  0.20  0.03 -0.05 0.010
3 -0.10 -0.01 -0.05 0.010
...

> rA <- data$rA
> rB <- data$rB
> rm <- data$rm
> rf <- data$rf
```

## Sharpe Ratio

```
> sA <- mean(rA-rf)/sd(rA-rf)
> print(sA)
[1] 0.3175401
> sB <- mean(rB-rf)/sd(rB-rf)
> print(sB)
[1] 0.2767519
> sm <- mean(rm-rf)/sd(rm-rf)
> print(sm)
[1] 0.352184
```

Beta

```
> betaA <- cov(rA,rm)/var(rm)
> print(betaA)
[1] 1.341768
> betaB <- cov(rB,rm)/var(rm)
> print(betaB)
[1] 0.5203136
> betam <- 1
```

Treynor

```
> tA <- mean(rA-rf)/betaA
> print(tA)
[1] 0.04077777
> tB <- mean(rB-rf)/betaB
> print(tB)
[1] 0.03926204
> tm <- mean(rm-rf)/1
> print(tm)
[1] 0.01971429
```

Alpha

```
> alphaA <- mean(rA - (rf + betaA*(rm-rf)))
> print(alphaA)
[1] 0.0282623
> alphaB <- mean(rB - (rf + betaB*(rm-rf)))
> print(alphaB)
[1] 0.01017096
```

Summarizing the calculated numbers

| | $r_A$ | $r_B$ | $r_m$ |
|---|---|---|---|
| Sharpe | 0.318 | 0.277 | 0.352 |
| $\beta$ | 1.342 | 0.520 | |
| Treynor | 0.041 | 0.039 | 0.020 |
| $\alpha$ | 0.028 | 0.010 | |

# Exercise

Download monthly returns for 10 Norwegian Industry Portolios 1980–2013. Also download returns for a broad Norwegian market index for the same period, and an estimate of the one month risk free rate.

1. Calculate the Sharpe ratios for the industry portfolios.

# Solution

Go through the R steps
The data come as time series, so one should read them into a time series, such as for example `zoo`:
First, we read in the industry portfolios. (Note, show only head of 2 (of 10) columns).

```
> library(zoo)
> IndPortf <- read.zoo("../data/industry_portfolios_monthly
+                     format="%Y%m%d",header=TRUE,sep=",")
> head(IndPortf[,1:2])
           X10.Energy.ew. X15.Material.ew.
1980-01-31       0.097561       0.01221640
1980-02-29       0.011111       0.07595600
1980-03-31      -0.098901      -0.10693300
1980-04-30       0.091463       0.02555040
1980-05-31       0.131844       0.01895950
1980-06-30      -0.036269       0.00775375
```

```
> MarketPortf <- read.zoo("../data/market_portfolios_monthl
+                         sep=",",header=TRUE,format="%Y%m%
> head(MarketPortf)
                  EW        VW Allshare OBX
1980-01-31  0.021660  0.023249       NA  NA
1980-02-29  0.055595 -0.042630       NA  NA
1980-03-31 -0.053663 -0.186248       NA  NA
1980-04-30  0.013371  0.098598       NA  NA
1980-05-31  0.043773  0.112954       NA  NA
1980-06-30 -0.003351 -0.013420       NA  NA
> ew <- MarketPortf$EW
> head(ew)
1980-01-31 1980-02-29 1980-03-31 1980-04-30 1980-05-31 198
  0.021660   0.055595  -0.053663   0.013371   0.043773  -0.
```

Risk free rate. Note that the risk free rate is the interest rate on the given date, i.e. for 1 month *starting* on that date.

```
> Rf <- read.zoo("../data/Rf_monthly.txt",
+                 sep=",",header=TRUE,format="%Y%m%d")
> head(Rf)
1979-12-31 1980-01-31 1980-02-29 1980-03-31 1980-04-30 198
0.00818333 0.00826667 0.00821667 0.00827500 0.00834167 0.0
> Rf <- lag(Rf,-1)
> head(Rf)
1980-01-31 1980-02-29 1980-03-31 1980-04-30 1980-05-31 198
0.00818333 0.00826667 0.00821667 0.00827500 0.00834167 0.0
```

Let us now look at one estimation.
We first align the data.

```
> R <- merge(na.omit(IndPortf[,1]),ew,Rf,all=FALSE)
> head(R)
           na.omit(IndPortf[, 1])        ew         Rf
1980-01-31               0.097561  0.021660 0.00818333
1980-02-29               0.011111  0.055595 0.00826667
1980-03-31              -0.098901 -0.053663 0.00821667
1980-04-30               0.091463  0.013371 0.00827500
1980-05-31               0.131844  0.043773 0.00834167
1980-06-30              -0.036269 -0.003351 0.00828333

> ri <- R[,1]
> rm <- R[,2]
> rf <- R[,3]
```

Now ready to do the calculation.
Calculate excess return, and then the Sharpe Ratio

```
> eri <- ri-rf
> erm <- rm-rf
> Si <- mean(eri)/sd(eri)
> print(Si)
[1] 0.1660124
```

Doing this for 10 different portfolios, I use a for loop, stacking the results into a matrix S:

```
> S <- matrix(ncol=1,nrow=10)
> rownames(S) <- c("10 Energy",    "15 Material",
+                  "20 Industry",  "25 ConsDisc",
+                  "30 ConsStapl", "35 Health",
+                  "40 Finan",     "45 IT",
+                  "50 Telecom",   "55 Util")
> colnames(S) <- c("Sharpe")
```

The `for` loop:

```
> for (i in 1:10){
+     R <- merge(na.omit(IndPortf[,i]),ew,Rf,all=FALSE)
+     ri <- R[,1]
+     rm <- R[,2]
+     rf <- R[,3]
+     eri <- ri-rf
+     erm <- rm-rf
+     Si <- mean(eri)/sd(eri)
+     print(Si)
+     S[i,1] <- Si
+ }
```

The resulting matrix is

```
> print(S)
                 Sharpe
10 Energy     0.16601244
15 Material   0.08850231
20 Industry   0.17976648
25 ConsDisc   0.14025390
30 ConsStapl  0.20183111
35 Health     0.10685682
40 Finan      0.11968392
45 IT         0.16516336
50 Telecom    0.09734733
55 Util       0.05723917
```

## Summarizing Results

| | Sharpe Ratio |
|---|---|
| 10 Energy | 0.166 |
| 15 Material | 0.089 |
| 20 Industry | 0.180 |
| 25 ConsDisc | 0.140 |
| 30 ConsStapl | 0.202 |
| 35 Health | 0.107 |
| 40 Finan | 0.120 |
| 45 IT | 0.165 |
| 50 Telecom | 0.097 |
| 55 Util | 0.057 |

# An alternative to the CAPM in Alpha calculation

The original Jensen measure is written in terms of the CAPM, but one can alternatively use another asset pricing model.
For example, we can write the alpha in terms of the Fama-French 3 factor model.

$$E[r_{i,t}] = r_{f,t} + (E[r_m, t] - r_{f,t})\beta_i + b_i^{hml} HML_t + b_i^{smb} SMB_t$$

The alpha for a portfolio $p$ is then calculated as

$$\alpha_{p,t} = r_{p,t} - \left( r_{f,t} + \beta_i \left( r_{m,t} - r_{f,t} \right) + b_i^{hml} HML_t + b_i^{smb} SMB_t \right)$$

# Exercise

Download monthly returns for 10 Norwegian Industry Portolios 1980–2013. Also download returns for a broad Norwegian market index for the same period, and an estimate of the one month risk free rate.

1. Calculate Jensen's alpha using CAPM as asset pricing model.
2. Calculate Jensen's alpha using the Fama French three factor model as asset pricing model.

# Solution

Read in the data

```
> library(zoo)
> IndPortf <- read.zoo("../data/industry_portfolios_monthly
+                         format="%Y%m%d",header=TRUE,sep=",")
> head(IndPortf[,1:2])
           X10.Energy.ew. X15.Material.ew.
1980-01-31       0.097561       0.01221640
1980-02-29       0.011111       0.07595600
1980-03-31      -0.098901      -0.10693300
1980-04-30       0.091463       0.02555040
1980-05-31       0.131844       0.01895950
1980-06-30      -0.036269       0.00775375
```

```
> MarketPortf <- read.zoo("../data/market_portfolios_monthl
+                            sep=",",header=TRUE,format="%Y%m%
> head(MarketPortf)
                  EW        VW Allshare OBX
1980-01-31  0.021660  0.023249       NA  NA
1980-02-29  0.055595 -0.042630       NA  NA
1980-03-31 -0.053663 -0.186248       NA  NA
1980-04-30  0.013371  0.098598       NA  NA

> ew <- MarketPortf$EW
> head(ew)
1980-01-31 1980-02-29 1980-03-31 1980-04-30 1980-05-31 1980
  0.021660   0.055595  -0.053663   0.013371   0.043773  -0.
```

```
> Rf <- read.zoo("../data/Rf_monthly.txt",
+                 sep=",",header=TRUE,format="%Y%m%d")
> head(Rf)
1979-12-31 1980-01-31 1980-02-29 1980-03-31 1980-04-30 1980
0.00818333 0.00826667 0.00821667 0.00827500 0.00834167 0.00
> Rf <- lag(Rf,-1)
> head(Rf)
1980-01-31 1980-02-29 1980-03-31 1980-04-30 1980-05-31 1980
0.00818333 0.00826667 0.00821667 0.00827500 0.00834167 0.00
>
> eRm <- ew-Rf
> eRi <- IndPortf-Rf
>
> head(eRm)
 1980-01-31  1980-02-29  1980-03-31  1980-04-30  1980-05-3
 0.01347667  0.04732833 -0.06187967  0.00509600  0.0354313
```

Do the regression for the first industry:

```
> data1 <- merge(na.omit(eRi[,1]),eRm,all=FALSE)
> eri <- data1[,1]
> erm <- data1[,2]
> names(eri) <- "eri"
> names(erm) <- "erm"
> regr1 <- lm(eri~erm)
> summary(regr1)
```

The results

```
Call:
lm(formula = eri ~ erm)

Residuals:
     Min       1Q   Median       3Q      Max
-0.17072 -0.02880  0.00187  0.02618  0.44307

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0005646  0.0026026   0.217    0.828
erm         1.3822523  0.0453677  30.468   <2e-16 ***


Residual standard error: 0.05163 on 406 degrees of freedom
Multiple R-squared:  0.6957,Adjusted R-squared:  0.695
F-statistic: 928.3 on 1 and 406 DF,  p-value: < 2.2e-16
```

To get this into a table, use the stargazer package.

```
> library(stargazer)
> collabels <- c("10 Enrgy",    "15 Matr",
+                "20 Indus",  "25 ConsDisc",
+                "30 ConStapl", "35 Hlth",
+                "40 Finan",     "45 IT",
+                "50 Tele",    "55 Util")
> rowlabels <-c("beta","alpha")
> stargazer(regr1,regr2,regr3,regr4,regr5,regr6,regr7,regr8
+           column.labels=collabels,
+           covariate.labels=rowlabels,
+           omit.stat=c("f","rsq","ser"),
+           digits=3,
+           float=FALSE,
+           header=FALSE)
```

Which results in the following table

| | *Dependent variable:* | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | eri | | |
| | 10 Enrgy | 15 Matr | 20 Indus | 25 ConDisc | 30 ConStapl | 35 Hlth | 4 |
| | (1) | (2) | (3) | (4) | (5) | (6) | |
| beta | 1.382*** | 1.188*** | 0.994*** | 0.926*** | 0.843*** | 0.926*** | 0 |
| | (0.045) | (0.085) | (0.022) | (0.043) | (0.040) | (0.066) | |
| alpha | 0.001 | −0.002 | 0.0003 | 0.0001 | 0.004* | −0.0003 | |
| | (0.003) | (0.005) | (0.001) | (0.002) | (0.002) | (0.004) | |
| Observations | 408 | 408 | 408 | 408 | 408 | 408 | |
| Adjusted $R^2$ | 0.695 | 0.324 | 0.828 | 0.528 | 0.516 | 0.328 | |

*Note:*

The Fama French model
Reading the Fama French factors

```
> head(PricFacts)
           SMB HML      PR1YR        UMD         LIQ
1981-01-31  NA  NA         NA         NA  0.20882000
1981-02-28  NA  NA  0.2025280  0.0718885  0.26717800
1981-03-31  NA  NA  0.1741180  0.1947210  0.02982810
1981-04-30  NA  NA  0.0289189  0.1102370  0.14572500
1981-05-31  NA  NA -0.0262097 -0.0171490 -0.04324430
1981-06-30  NA  NA -0.0212579  0.0160817  0.00845631
> SMB <- na.omit(PricFacts$SMB)
> HML <- na.omit(PricFacts$HML)
```

Doing the regression on the first portfolio

```
> eri <- data1[,1]
> erm <- data1$eRm
> names(eri) <- "eri"
> names(erm) <- "erm"
> smb <- data1$SMB
> hml <- data1$HML
> names(smb) <- "smb"
> names(hml) <- "hml"
> regr1 <- lm(eri~erm+smb+hml)
```

Which result in the following results

```
> summary(regr1)

Call:
lm(formula = eri ~ erm + smb + hml)

Residuals:
      Min        1Q    Median        3Q       Max
-0.157188 -0.026138  0.001237  0.026013  0.127511

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.001958   0.002394   0.818   0.4139
erm          1.308991   0.040551  32.280   <2e-16 ***
smb         -0.122480   0.051245  -2.390   0.0173 *
hml         -0.054556   0.045237  -1.206   0.2286


Residual standard error: 0.04429 on 368 degrees of freedom
Multiple R-squared:  0.7461 Adjusted R-squared:  0.744
```

# Collecting the results

| | *Dependent variable:* | | | | | |
|---|---|---|---|---|---|---|
| | eri | | | | | |
| | 10 Enrgy | 15 Matr | 20 Indus | 25 ConDisc | 30 ConStapl | 35 Hlth |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| beta | 1.309*** | 1.163*** | 0.993*** | 0.934*** | 0.839*** | 0.949*** |
| | (0.041) | (0.087) | (0.023) | (0.045) | (0.042) | (0.067) |
| smb | −0.122** | −0.256** | 0.010 | 0.052 | −0.135** | −0.018 |
| | (0.051) | (0.110) | (0.028) | (0.057) | (0.053) | (0.084) |
| hml | −0.055 | 0.462*** | 0.071*** | 0.004 | −0.026 | −0.476*** |
| | (0.045) | (0.097) | (0.025) | (0.051) | (0.046) | (0.074) |
| alpha | 0.002 | −0.001 | −0.0002 | −0.001 | 0.005** | 0.003 |
| | (0.002) | (0.005) | (0.001) | (0.003) | (0.002) | (0.004) |
| Observations | 372 | 372 | 372 | 372 | 372 | 372 |
| Adjusted $R^2$ | 0.744 | 0.377 | 0.844 | 0.534 | 0.536 | 0.385 |

*Note:*