Dummy Variables

November 24, 2021

1 Dummies in regressions

(Not regressions for dummies)

> Let us remember the unfortunate econometrician who, in one of the major functions of his system, had to use a proxy for risk and a dummy for sex.
>
> Fritz Machlup, Journal of Political Economy, 1974.

In regression specifications, a tool that is very flexible and useful is the concept of a dummy, or binary variable.

This is a variable that takes one out of two values, zero or one.

$$D = \begin{cases} 1 & \text{if something} \\ 0 & \text{if not that something} \end{cases}$$

while this is a very simple concept, the number of possible uses of it in econometrics is virtually unlimited.

We will essentially go over som typical examples of the use of dummy variables in regression settings, and some of the typical problems which these methods help solving.

Some uses for dummy variables

1. Allowing for differences in the intercept term
2. Allowing for differences in slopes
3. Test for stability of regression coefficients
4. Ameliating outliers
5. Panel data (fixed effects)

1.1 Dummy variables for changes in the intercept term

Suppose we have two groups of observatons

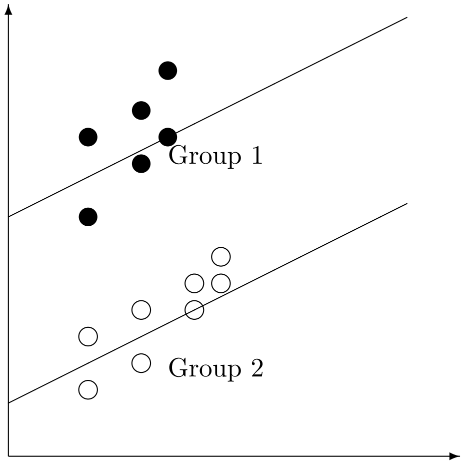
$$y = \begin{cases} \alpha_1 + \beta x + e & \text{first group} \\ \alpha_2 + \beta x + e & \text{second group} \end{cases}$$

Instead of estimating this separately for the two samples, we define a dummy variable

$$D = \begin{cases} 1 & \text{if sample is from first group} \\ 0 & \text{if sample is from second group} \end{cases}$$

and combine the regression into one equation.

$$y = \alpha_1 + (\alpha_2 - \alpha_1)D + \beta x + e$$



Any number of dummy variables may be added in such a formulation, if the samples can be grouped into more groups.

A well known use of dummies is to correct for seasonalities. If the behaviour of a variable varies across quarters, say, we can introduce quarterly dummies.

$$D_1 = \begin{cases} 1 & \text{if observation is from first quarter} \\ 0 & \text{otherwise} \end{cases}$$

$$D_2 = \begin{cases} 1 & \text{if observation is from second quarter} \\ 0 & \text{otherwise} \end{cases}$$

$$D_3 = \begin{cases} 1 & \text{if observation is from third quarter} \\ 0 & \text{otherwise} \end{cases}$$

$$D_4 = \begin{cases} 1 & \text{if observation is from fourth quarter} \\ 0 & \text{otherwise} \end{cases}$$

One important practical consideration.

If the groupings are exhaustive (span the whole sample) as the quarterly dummies do (A date can only be in one out of four quarters) the formulation has to be estimated either

- with a constant and one less dummy than groups (The last group is called the control group)
- without a constant term and dummy variables for all possible groups.

This is because running a regression with a constant and an exhaustive set of dummies will introduce perfect multicolleraity, and $\mathbf{X}'\mathbf{X}$ will not be invertible.

Let us now show some examples of this type of usage.

Exercise 1.

In finance one has identified various "calendar anomalies", that stock returns depend on calendar time in surprising ways. One of these is the "January effect," that stock returns seem to be higher in January.

Using returns for the S&P 500 in the period after 1950, test the hypothesis that the returns in January is different from other months.

In implementing this use indicator variables in a regression framework, where January is the only explanatory variable. Implement the tests in R.

Solution to Exercise 1.

We ask whether returns in january are fundamentally different from the rest. Regression to run

$$r_m = E[r_m] + \beta D_{january} + e$$

If january is different, $\beta \neq 0$.

Reading in data and generating

```
library(zoo)
library(xts)
INSP500d <- read.zoo("../data/sp500_daily.csv",
                     format="%Y-%m-%d",sep=",",header=TRUE)
sp500d <- as.xts(INSP500d[,6])
sp500m <-sp500d[endpoints(sp500d,on="months")]
Rsp500m <- diff(log(sp500m))
```

Now, using this return series:

```
> Rm <- Rsp500m;
> dates  <-  as.POSIXlt(index(Rm))
> jan <- as.numeric(dates$mon==0)
> reg <- lm(Rm~jan)
> summary(reg)

Call:
lm(formula = Rm ~ jan)

Residuals:
      Min        1Q    Median        3Q       Max
-0.250944 -0.023895  0.003518  0.028917  0.145527

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.005516   0.001603   3.442  0.00061 ***
jan         0.004578   0.005551   0.825  0.40982

Residual standard error: 0.04219 on 754 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared: 0.0009012,Adjusted R-squared: -0.0004239
F-statistic: 0.6801 on 1 and 754 DF,  p-value: 0.4098
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0055	0.0016	3.44	0.0006
jan	0.0046	0.0056	0.82	0.4098

There is an economically larger return in january, but not statistically significant.

Exercise 2.

In finance one has identified various “calendar anomalies”, that stock returns depend on calendar time in surprising ways. One of these is the “Day of the week effect,” that stock returns seem to be different across days of the week.

Using returns for the S&P 500, test the hypothesis that the expected return is different across days of the week.

In implementing this use indicator variables in a regression framework.

Implement the analysis in R.

Solution to Exercise 2.

Preliminary, reading the data

```
> library(xtable)
> library(car)
> source("read.R")
> Rm <- Rsp500d;
> dates  <-  as.POSIXlt(index(Rm))
> Rm <- as.matrix(Rm)
> mon <- as.numeric(dates$wday==1)
> tue <- as.numeric(dates$wday==2)
> wed <- as.numeric(dates$wday==3)
> thu <- as.numeric(dates$wday==4)
> fri <- as.numeric(dates$wday==5)
```

First estimate dummy for each day, with no constant term.

```
> reg1 <- lm(Rm~0+mon+tue+wed+thu+fri)
> summary(reg1)

Residuals:
      Min        1Q    Median        3Q       Max
-0.228293 -0.004455  0.000164  0.004691  0.110276

Coefficients:
      Estimate Std. Error t value Pr(>|t|)
mon -0.0007038  0.0001772  -3.971 7.19e-05 ***
tue  0.0003388  0.0001723   1.967  0.04919 *
wed  0.0007367  0.0001721   4.279 1.89e-05 ***
thu  0.0003510  0.0001733   2.026  0.04279 *
fri  0.0006426  0.0001739   3.695  0.00022 ***

Residual standard error: 0.009784 on 15852 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared: 0.003502,Adjusted R-squared: 0.003188
F-statistic: 11.14 on 5 and 15852 DF,  p-value: 9.737e-11
```

	Estimate	Std. Error	t value	Pr(>|t|)
mon	-0.0007	0.0002	-3.97	0.0001
tue	0.0003	0.0002	1.97	0.0492
wed	0.0007	0.0002	4.28	0.0000
thu	0.0004	0.0002	2.03	0.0428
fri	0.0006	0.0002	3.70	0.0002

in this setting need to construct hypothesis tests for equality

```
> C <- c(c(1, -1, 0, 0, 0), c(0, 1, -1, 0, 0 ), c(0, 0 ,1, -1, 0), c(0, 0, 0, 1, -1))
> C <- matrix(C,nrow=4,ncol=5,byrow=TRUE)
> r <- c(0, 0, 0, 0)
> linearHypothesis(reg1,hypothesis.matrix=C,rhs=r)
Linear hypothesis test

Hypothesis:
mon - tue = 0
tue - wed = 0
wed - thu = 0
thu - fri = 0

Model 1: restricted model
Model 2: Rm ~ 0 + mon + tue + wed + thu + fri

  Res.Df    RSS Df Sum of Sq     F    Pr(>F)
1  15856 1.5214
2  15852 1.5174  4 0.0040662 10.62 1.362e-08 ***
```

Reject the null of equality

Estimate regression with constant term, leaving out one observation (constant = monday)

```
> reg2 <- lm(Rm~tue+wed+thu+fri)
> summary(reg2)

Residuals:
      Min        1Q    Median        3Q       Max
-0.228293 -0.004455  0.000164  0.004691  0.110276

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0007038  0.0001772  -3.971 7.19e-05 ***
tue          0.0010427  0.0002472   4.219 2.47e-05 ***
wed          0.0014405  0.0002471   5.830 5.65e-09 ***
thu          0.0010549  0.0002479   4.256 2.10e-05 ***
fri          0.0013464  0.0002483   5.423 5.96e-08 ***

Residual standard error: 0.009784 on 15852 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared: 0.002673,Adjusted R-squared: 0.002421
F-statistic: 10.62 on 4 and 15852 DF,  p-value: 1.362e-08
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.0007	0.0002	-3.97	0.0001
tue	0.0010	0.0002	4.22	0.0000
wed	0.0014	0.0002	5.83	0.0000
thu	0.0011	0.0002	4.26	0.0000
fri	0.0013	0.0002	5.42	0.0000

to test whether friday different,

```
> reg3 <- lm(Rm~fri)
> summary(reg3)

Call:
lm(formula = Rm ~ fri)

Residuals:
      Min        1Q    Median        3Q       Max
-0.229190 -0.004432  0.000190  0.004675  0.109379
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.928e-04  8.694e-05   2.218   0.0266 *
fri         4.497e-04  1.946e-04   2.311   0.0208 *

Residual standard error: 0.009794 on 15855 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared: 0.0003368,Adjusted R-squared: 0.0002738
F-statistic: 5.342 on 1 and 15855 DF,  p-value: 0.02082
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0002	0.0001	2.22	0.0266
fri	0.0004	0.0002	2.31	0.0208

Find support to think friday is different.

1.2 Dummy variables for changes in slope coefficients

Suppose we also think that the slope varies between groups.

$$\begin{cases} y = \alpha_1 + \beta_1 x + e & \text{group 1} \\ y = \alpha_2 + \beta_2 x + e & \text{group 2} \end{cases}$$

This can be compactly written by introducing the dummy

$$D = \begin{cases} 0 & \text{group 1} \\ 1 & \text{group 2} \end{cases}$$

$$y = \alpha_1 + (\alpha_2 - \alpha_1)D + \beta_1 x + (\beta_2 - \beta_1)xD + e$$

The cases where we multiply the dummy with x are often called interaction terms.

1.3 Dummy variables for parameter stability testing

When we consider time series, a question often asked is whether the underlying model is stable. Are the parameters changing at certain dates?

One way that test is implemented is the Chow test, which tests for changes in *all* the coefficients of a regression simultaneously.

However, not necessarily so that *all* coefficients change, can test for stability of one (or more) coeffients by a suitable dummy.

Example, NHY market model

$$r_{it} = a + b_1 r_{mt} + e_{it}$$

in the first period

$$r_{it} = a + b_2 r_{mt} + e_{it}$$

in the second period Define

$$D = \begin{cases} 1 & \text{period 1} \\ 0 & \text{period 2} \end{cases}$$

And estimate

$$r_{it} = a + b_1 r_{mt} + (b_2 - b_1) D r_{mt} + e_{it}$$

Testing for parameter stability involves the coefficient of the dummy.

Exercise 3.

You are investigating the market model

$$r_{it} = a + b r_{mt} + e_{it}$$

in the Norwegian Market, and apply it to the company Norsk Hydro (NHY). Collect monthly returns for NHY for the period 1980-2006, and monthly returns for a value weighted market index for the same period.

After having estimated the model you worry that the NHY beta (The parameter b) has changed over time. You therefore split the sample into two periods, 1980–1989 and 1990–2006.

Test whether there are reasons to believe the b parameter has changed different in the two periods.

Solution to Exercise 3.

Consider the following regression.

$$r_{it} = a + b_1 r_{mt} + b_2 D r_{mt} + e_{it}$$

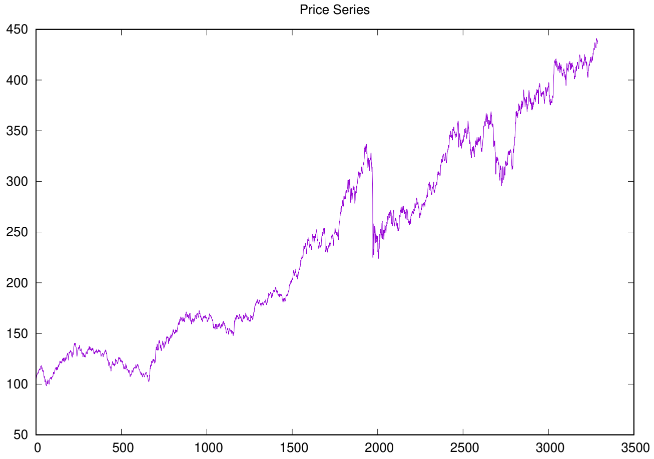
This achieve the desired test, by testing wheter $b_2 = 0$ we test the null of no change in beta.

Variable	coeff	serr	t-val	p-val(t)
Constant	-0.00814	0.00318	-2.56	0.011
Rm	1.19274	0.06825	17.48	0.000
D	-0.14959	0.09054	-1.65	0.099
R^2	0.631	F	274.40	
$Adj\bar{R}^2$	0.629	pval F	0.0000	
DW	1.88			

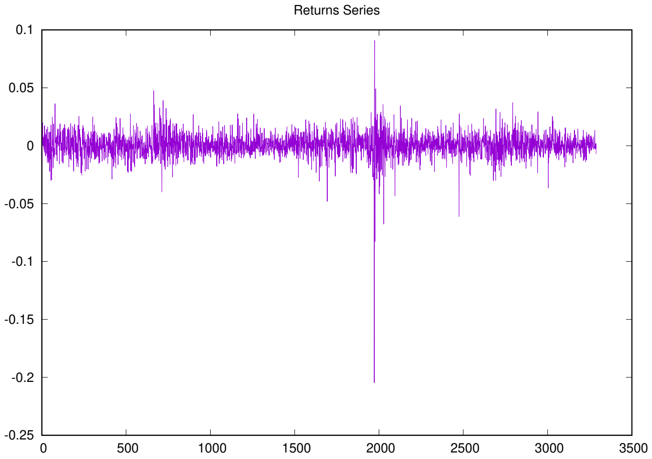
The p-value on the D is not significant at the 5% level. Do therefore not reject a null of no change.

1.4 Using dummies to ameliate outliers

You are not always justified in simply throwing out any observations you think are outliers. Consider the next picture, which is a time series of something...



There are what seems to be large jumps in the time series at a couple of points. If you were to difference this picture, as shown in the next figure,

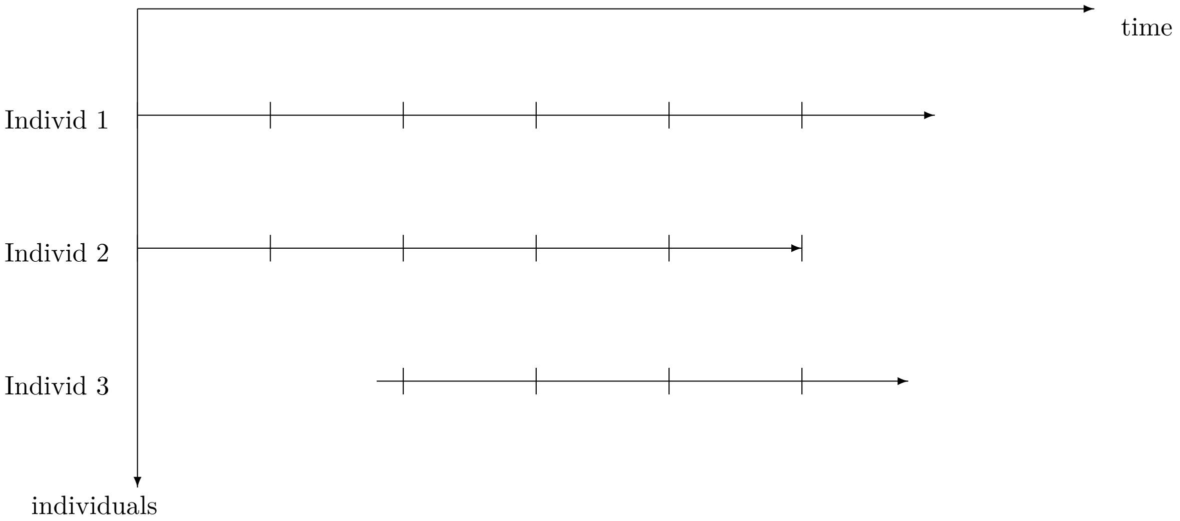


the large jump observations look like outliers.

However, that observation is actually a true observation. This time series is the evolution of the Standard & Poors 500 stock price index for 1980 to 1992. The large drops in the index is the 20sep87 "crash", and the "mini-crash" in 1989. What seems like "bad data" is not wrong data, it is just data that is hard to explain (We still can not explain the 1987 "crash.") If you want to calculate something that includes this as a datapoint, you can "remove" this by a dummy variable equal to one if 20sep87 is included.

1.5 Dummy variables in panel data

Panel data are obsevations of the same individual on different dates.



The panel is *balanced* if all individuals have a complete set of observations, otherwise the panel is unbalanced.

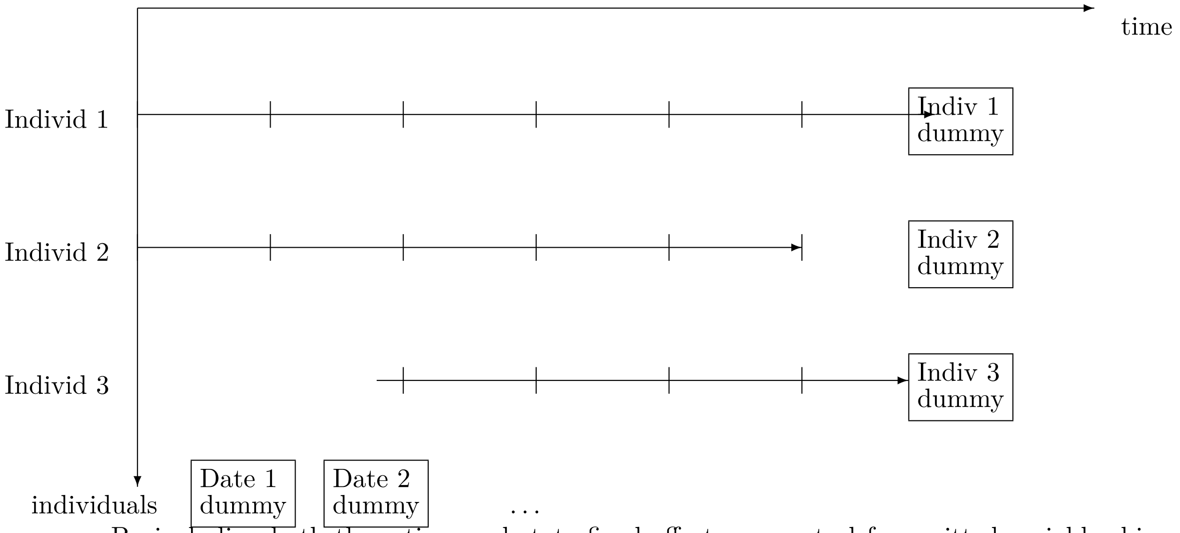
In such settings dummy variables can be used to control for unobserved heterogeneity

This is typically called fixed effects

We talk about

- State fixed effects
- Time fixed effects

or both



By including both these time and state fixed effects we control for omitted variables bias arising both from unobserved variables constant over time and from unobserved variables that are constant across states.

2 Concluding

Dummy variables have large potential as modelling devices, the sky is the limit.

Literature Stock and Watson (2019)

References

James H Stock and Mark W Watson. *Introduction to Econometrics.* Pearson, 4th edition, 2019.