The Black et al. [1972] approach

We discuss the testing approach which stems from the analysis in Black et al. [1972].

Start with tests of the CAPM.

First consider a time series regression.

$$r_{jt} = \alpha_j + \beta_{jm} r_{mt} + \varepsilon_{jt}$$

where r_{jt} is the return of stock j at time t, and r_{mt} the corresponding return.

In this formulation, the CAPM imposes

$$\alpha_j = r_{zc}(1 - \beta_{jm})$$

Why?

$$r_{jt} = r_{zc} + (r_m - r_{zc})\beta_{jm} = r_{zc} - \beta_{jm} r_{zc} + \beta_{jm} r_{mt} = r_{zc}(1 - \beta_{jm}) + \beta_{jm} r_{mt}$$

To test the CAPM, test whether

$$E\left[\frac{\alpha_j}{1 - \beta_{jm}}\right] = r_{zc}, \text{ or } \frac{1}{N}\sum_{i=1}^{N} \frac{\alpha_j}{1 - \beta_{jm}} = r_{zc}$$

An alternative specification is to do this in excess return form, where we find the *excess return* er_{it} as

$$er_{it} = r_{it} - r_{ft}$$

Consider the time series regression

$$er_{jt} = \alpha_j + \beta_{jm} er_{mt} + \varepsilon_{jt}$$

In this formulation, the CAPM imposes that

$$\alpha_j = 0$$

The method of Black et al. [1972] is to run such time series regression, often on stock portfolios instead of individual stocks, and test the restrictions on the constant term.

Estimating CAPM on the US Cross Section

Use data library provided by Ken French

Exercise

Suppose the CAPM holds. What restriction does this impose on the following relationship:

$$E[r_i] - r_f = \alpha_i + \beta_i(E[r_m] - r_f)$$

Test this restriction using Ken French's 5 size sorted portfolios. Use data for 1980-2012. Do the tests for each portfolio separately.

Exercise Solution

Reading the data

```
# make sure that the first date do not change, this hardcod
library(zoo)
FFSizeEW <- read.table("../data/Portfolios_Formed_on_ME_mon
                        header=TRUE,skip=11)
FFSize3EW <- zooreg(FFSizeEW[,2:4],start=c(1926,7),frequenc
FFSize5EW <- zooreg(FFSizeEW[,5:9],start=c(1926,7),frequenc
FFSize10EW <- zooreg(FFSizeEW[,10:19],start=c(1926,7),frequ
```

Exercise Solution - ctd

Doing the analysis, taking the subperiod 1980-2012.

```
> FFSize5EW <- window(FFSize5EW,start=c(1980,1),end=c(2012,
> data <- merge.zoo(FFSize5EW,RMRF,RF,all=FALSE)
> summary(data)
     Index               Lo20                  Qnt2              Qnt
 Min.    :1980    Min.    :-28.120    Min.    :-29.11    Min.    :
 1st Qu.:1988    1st Qu.: -2.340    1st Qu.: -2.48    1st Qu.:
 Median :1996    Median :  1.260    Median :  1.89    Median  :
 Mean    :1996    Mean    :  1.215    Mean    :  1.11    Mean    :
 3rd Qu.:2004    3rd Qu.:  4.580    3rd Qu.:  4.61    3rd Qu.:
 Max.    :2013    Max.    : 31.780    Max.    : 24.60    Max.    :
      Qnt4                Hi20                 RMRF
 Min.    :-25.00    Min.    :-21.320    Min.    :-23.240    Min.
 1st Qu.: -2.04    1st Qu.: -1.860    1st Qu.: -2.100    1st Q
 Median :  1.60    Median :  1.460    Median :  1.090    Media
 Mean    :  1.14    Mean    :  1.057    Mean    :  0.586    Mean
 3rd Qu.:  4.43    3rd Qu.:  3.880    3rd Qu.:  3.520    3rd Q
 Max.    : 20.67    Max.    : 17.640    Max.    : 12.460    Max.
```

Exercise Solution - ctd

Looking first at the analysis for the low quintile:

```
> eri <- data$Lo20-data$RF
> erm <- data$RMRF
> reg <- lm(as.matrix(eri)~as.matrix(erm))

Residuals:
     Min       1Q   Median       3Q      Max
-10.9582  -2.3328  -0.3355   1.9856  27.8145

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)     0.19914    0.20927   0.952    0.342
as.matrix(erm)  1.03408    0.04514  22.907   <2e-16 ***

Residual standard error: 4.115 on 391 degrees of freedom
Multiple R-squared: 0.573,Adjusted R-squared: 0.5719
F-statistic: 524.7 on 1 and 391 DF,  p-value: < 2.2e-16
```

Exercise Solution - ctd

Summarizing the analysis for the five size portfolios:

Portfolio		Estimate	Std. Error	t value	Pr(>|t|)
1 - Lo20	(Intercept)	0.1991	0.2093	0.95	0.3419
	as.matrix(erm)	1.0341	0.0451	22.91	0.0000
2	(Intercept)	0.0009	0.1433	0.01	0.9948
	as.matrix(erm)	1.1926	0.0309	38.57	0.0000
3	(Intercept)	0.0539	0.1149	0.47	0.6391
	as.matrix(erm)	1.1700	0.0248	47.19	0.0000
4	(Intercept)	0.0703	0.0838	0.84	0.4020
	as.matrix(erm)	1.1256	0.0181	62.23	0.0000
5 - Hi20	(Intercept)	0.0426	0.0585	0.73	0.4671
	as.matrix(erm)	1.0317	0.0126	81.70	0.0000

Using this time period, none of the individual alphas are significant.

Estimating CAPM on the OSE

Exercise

If we take the strict version of the CAPM with the risk free rate r_f, the regression to consider is

$$er_{it} = \alpha_i + \beta_i er_{mt} + e_{it}$$

where

$$er_{it} = r_{it} - r_{ft}$$

and

$$er_{mt} = r_{mt} - r_{ft}$$

A testable implication of the CAPM is that $\alpha_i = 0$. Run this regression on the industry portfolios at the OSE. Test $\alpha_i = 0$ on a portfolio by portfolio basis. Use an equally weighted market index, and returns data 1980-2012.

Excercise Solution Reading in the data and running the regressions

```
library(zoo)
library(xtable)
Rets <- read.zoo("../../data/industry_portfolios_monthly_ew
                  header=TRUE,sep=";",format="%Y%m%d")
# last 2  industries are not complete
Rets <- Rets[,1:8];
Rf <- read.zoo("../../data/NIBOR_monthly.txt",
                header=TRUE,sep=";",format="%Y%m%d")
Rm <- read.zoo("../../data/market_portfolios_monthly.txt",
                header=TRUE,sep=";",format="%Y%m%d")
eRmew <- Rm$EW - lag(Rf,-1)
eR  <- Rets - lag(Rf,-1)
data  <- merge(eR,eRmew,all=FALSE)
er  <- as.matrix(data[,1:8])
erm <-as.matrix(data[,9])
reg=lm(er~erm)
```

Excercise Solution ctd. Regression for industry 10:

```
Response X10.Energy.ew. :

Residuals:
     Min       1Q   Median       3Q      Max
-0.17133 -0.02955  0.00259  0.02831  0.44248

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.001248   0.002823   0.442    0.659
erm         1.381646   0.047809  28.899   <2e-16 ***

Residual standard error: 0.05362 on 372 degrees of freedom
Multiple R-squared: 0.6918,Adjusted R-squared: 0.691
F-statistic: 835.2 on 1 and 372 DF,  p-value: < 2.2e-16
```

Excercise Solution ctd. Summarizing results for all 8 industries:

Industry		Estimate	Std. Error	t value	Pr(>|t|)
10	(Intercept)	0.0012	0.0028	0.44	0.6589
	erm	1.3816	0.0478	28.90	0.0000
15	(Intercept)	-0.0014	0.0052	-0.28	0.7801
	erm	1.2097	0.0875	13.83	0.0000
20	(Intercept)	0.0004	0.0014	0.30	0.7646
	erm	0.9875	0.0234	42.24	0.0000
25	(Intercept)	0.0001	0.0026	0.04	0.9710
	erm	0.9251	0.0446	20.75	0.0000
30	(Intercept)	0.0044	0.0024	1.84	0.0659
	erm	0.8315	0.0407	20.42	0.0000
35	(Intercept)	0.0036	0.0057	0.63	0.5304
	erm	0.9166	0.0966	9.49	0.0000
40	(Intercept)	-0.0023	0.0016	-1.50	0.1333
	erm	0.7472	0.0263	28.39	0.0000
45	(Intercept)	0.0032	0.0044	0.72	0.4726
	erm	1.2637	0.0746	16.93	0.0000

Here we never reject that the intercept is zero.

Exercise

The CAPM is usually written as

$$E[r_{it}] = r_{ft} + \beta_i(E[r_{mt}] - r_{ft})$$

where r_{ft} is the risk free rate, r_{it} the return on an asset, and r_{mt} the market return. Rewriting in excess return form, where we find the excess asset return $er_{it} = r_{it} - r_{ft}$ and excess market return $er_{mt} = r_{mt} - r_{ft}$ we write the univariate regression.

$$er_{it} = \alpha_i + \beta_i er_{mt} + e_{it}$$

A testable implication of the CAPM is that $\alpha_i = 0$ in this regression. As first shown by Black et al. [1972] this can be tested by time series regressions.

Run these regressions on 10 (ew) size sorted portfolios at the OSE. Test $\alpha_i = 0$ on a portfolio by portfolio basis. Use an equally weighted market index, and returns data 1980-2012.

Exercise solution

Reading in the data and running the regressions

```
library(zoo)
Rets <- read.zoo("../../data/equity_size_portfolios_monthly_ew.t
                    header=TRUE,sep=";")
Rf <- read.zoo("../../data/NIBOR_monthly.txt",
                  header=TRUE,sep=";");
Rm <- read.zoo("../../data/market_portfolios_monthly.txt",
                  header=TRUE,sep=";");
eRmew <- Rm$EW - lag(Rf,-1)
eR  <- Rets - lag(Rf,-1)
data  <- merge(eR,eRmew,all=FALSE)
er  <- as.matrix(data[,1:10])
erm <-as.matrix(data[,11])
reg=lm(er~erm)
```

Exercise solution ctd

Results, Portfolio 1 (small firms)

```
Response X1..small.size. :

Residuals:
     Min       1Q   Median       3Q      Max
-0.15175 -0.03246 -0.00610  0.02541  0.33963

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.013229   0.002884   4.588 6.04e-06 ***
erm         0.811883   0.049781  16.309  < 2e-16 ***

Residual standard error: 0.05634 on 393 degrees of freedom
Multiple R-squared: 0.4036,Adjusted R-squared: 0.4021
F-statistic:   266 on 1 and 393 DF,  p-value: < 2.2e-16
```

Exercise solution

Portfolio 10, largest firms

```
Response X10..large.size. :

Residuals:
     Min       1Q   Median       3Q      Max
-0.17599 -0.02669  0.00022  0.02645  0.58592

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.005126   0.002854  -1.796   0.0733 .
erm          0.995196   0.049273  20.197   <2e-16 ***

Residual standard error: 0.05577 on 393 degrees of freedom
Multiple R-squared: 0.5093,Adjusted R-squared: 0.5081
F-statistic: 407.9 on 1 and 393 DF,  p-value: < 2.2e-16
```

Exercise solution

		Estimate	Std. Error	t value	Pr(>|t|)
1(small)	(Intercept)	0.0132	0.0029	4.59	0.0000
	erm	0.8119	0.0498	16.31	0.0000
2	(Intercept)	0.0061	0.0024	2.57	0.0105
	erm	0.8956	0.0413	21.68	0.0000
3	(Intercept)	-0.0004	0.0020	-0.21	0.8332
	erm	0.9798	0.0344	28.52	0.0000
4	(Intercept)	-0.0016	0.0021	-0.80	0.4267
	erm	1.0042	0.0358	28.08	0.0000
5	(Intercept)	0.0023	0.0021	1.11	0.2693
	erm	0.9938	0.0360	27.60	0.0000
6	(Intercept)	0.0004	0.0019	0.20	0.8417
	erm	0.9786	0.0323	30.34	0.0000
7	(Intercept)	-0.0040	0.0019	-2.09	0.0369
	erm	1.0802	0.0332	32.54	0.0000
8	(Intercept)	-0.0031	0.0019	-1.62	0.1050
	erm	1.0531	0.0325	32.38	0.0000
9	(Intercept)	-0.0071	0.0020	-3.55	0.0004
	erm	1.1838	0.0345	34.27	0.0000
10 (large)	(Intercept)	-0.0051	0.0029	-1.80	0.0733
	erm	0.9952	0.0493	20.20	0.0000

Exercise

The CAPM is usually written as

$$E[r_{it}] = r_{ft} + \beta_i(E[r_{mt}] - r_{ft})$$

where r_{ft} is the risk free rate, r_{it} the return on an asset, and r_{mt} the market return. Rewriting in excess return form, where we find the excess asset return $er_{it} = r_{it} - r_{ft}$ and excess market return $er_{mt} = r_{mt} - r_{ft}$ we write the univariate regression.

$$er_{it} = \alpha_i + \beta_i er_{mt} + e_{it}$$

A testable implication of the CAPM is that $\alpha_i = 0$ in this regression. As first shown by Black et al. [1972] this can be tested by time series regressions. You have run these regressions on 10 (ew) size sorted portfolios at the OSE, and tested $\alpha_i = 0$ on a portfolio by portfolio basis. The data is an equally weighted market index, and returns data 1980-2012.

Exercise

Summarizing the results, you find for the first two portfolios (the smallest).

		Estimate	Std. Error	t value	Pr(>|t|)
1(small)	(Intercept)	0.0132	0.0029	4.59	0.0000
	erm	0.8119	0.0498	16.31	0.0000
2	(Intercept)	0.0061	0.0024	2.57	0.0105
	erm	0.8956	0.0413	21.68	0.0000
	...				

These results are from standard OLS regressions. You are concerned about the sensitivity of the tests to the OLS assumption, in particular the time series nature of the data leads to concern about autocorrelation of errors. Investigate to what extent the time series nature of the data changes your conclusions by looking at HAC corrected estimates of variances.

Exercise solution

The conclusions we are concerned with here concerns the estimates of α, whether we reject that $\alpha = 0$. The test statistic we construct to do the test is

$$z = \frac{\hat{\alpha}}{\text{std}(\hat{\alpha})}$$

What one does to correct for possible autocorrelation is to replace the OLS estimate of the variance-covariance matrix by a HAC estimate.

Exercise solution ctd

In R this is in the package `Sandwich`, where, after running the regression, we can calculate the HAC robust estimate by `varHAC`. Let us calculate the z statistic used to evaluate $\alpha = 0$ for these two portfolios.

```
> reg1 <- lm(er[,1]~erm)
> vcov(reg1)
              (Intercept)           erm
(Intercept)  8.314717e-06 -0.0000262456
erm         -2.624560e-05  0.0024781863
> vcovHAC(reg1)
             (Intercept)          erm
(Intercept) 7.357656e-06 5.502103e-05
erm         5.502103e-05 5.504716e-03
```

Exercise solution ctd

And then look at the influence of the revised estimates on the test statistic

```
> reg1$coefficients[1]/sqrt(vcov(reg1)[1,1])
(Intercept)
   4.587717
> reg1$coefficients[1]/sqrt(vcovHAC(reg1)[1,1])
(Intercept)
   4.876976
```

The z statistic actually increases, so we clearly are not going to change our rejection.

Exercise solution ctd

Doing the same thing for portfolio 2

```
> reg2 <- lm(er[,2]~erm)
> reg2$coefficients[1]/sqrt(vcov(reg2)[1,1])
(Intercept)
   2.570447
> reg2$coefficients[1]/sqrt(vcovHAC(reg2)[1,1])
(Intercept)
   2.749426
```

There too the z statistic increases.

Fisher Black, Michael Jensen, and Myron Scholes. The capital asset pricing model, some empirical tests. In Michael C Jensen, editor, *Studies in the theory of capital markets*. Preager, 1972.