the Black Jensen Scholes analysis

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1 Introduction

We discuss the testing approach which stems from the analysis in Black, Jensen, and Scholes (1972).

Start with tests of the CAPM.

First consider a time series regression.

$$r_{jt} = \alpha_j + \beta_{jm} r_{mt} + \varepsilon_{jt}$$

where r_{jt} is the return of stock j at time t, and r_{mt} the corresponding return.

In this formulation, the CAPM imposes

$$\alpha_j = r_{zc}(1 - \beta_{jm})$$

Why?

$$r_{jt} = r_{zc} + (r_m - r_{zc})\beta_{jm} = r_{zc} - \beta_{jm} r_{zc} + \beta_{jm} r_{mt} = r_{zc}(1 - \beta_{jm}) + \beta_{jm} r_{mt}$$

To test the CAPM, test whether

$$E\left[\frac{\alpha_j}{1 - \beta_{jm}}\right] = r_{zc}, \text{ or } \frac{1}{N}\sum_{i=1}^{N} \frac{\alpha_j}{1 - \beta_{jm}} = r_{zc}$$

An alternative specification is to do this in excess return form, where we find the *excess return* er_{it} as

$$er_{it} = r_{it} - r_{ft}$$

Consider the time series regression

$$er_{jt} = \alpha_j + \beta_{jm} er_{mt} + \varepsilon_{jt}$$

In this formulation, the CAPM imposes that

$$\alpha_j = 0$$

The method of Black et al. (1972) is to run such time series regression, often on stock portfolios instead of individual stocks, and test the restrictions on the constant term.

2 Estimating CAPM on the US Crossection

Let us now look at CAPM type estimation with some examples from the US, using Ken French Data. We use 5 industries, data for the whole period covered by the French data (which starts in 1926).

	Dependent variable:				
	eRi[, 1]	eRi[, 3]		eRi[, 4]	eRi[, 5]
	Cnsmr	Manuf	HiTec	Hlth	Other
	(1)	(2)	(3)	(4)	(5)
eRm	1.175***	1.362***	1.362***	1.063***	1.223***
	(0.019)	(0.023)	(0.023)	(0.024)	(0.023)
Constant	0.001	0.002*	0.002*	0.004***	0.002
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	1,126	1,126	1,126	1,126	1,126
Adjusted R^2	0.771	0.752	0.752	0.644	0.720

Note: *p<0.1; **p<0.05; ***p<0.01

Here is the R code for doing this.

```
library(gmm)
library(car)
library(stargazer)
outdir <- "../../results/2020_09_bjs/"
source("/home/bernt/data/2020/french_us_data/read_5_industries.R")
source("/home/bernt/data/2020/french_us_data/read_3_pricing_factors.R")
                                        # then take 5 portfolios, start in 1990
industries <- names(FF5IndusEW)
eRi <-FF5IndusEW-RF
#eRi <- window(eRindus,start=c(1970,1))
data <- merge(eRi,RMRF,all=FALSE)
                                        # estimate as separate linear regressions
eRi <- as.matrix(data[,1:5])
eRm <- as.matrix(data[,6])
summary(lm(eRi~eRm))
results <- list(lm(eRi[,1]~eRm),lm(eRi[,3]~eRm),lm(eRi[,3]~eRm),lm(eRi[,4]~eRm),lm(eRi[,5]~eRm))
ofilename <- paste0(outdir,"bjs_regressions_5_industries.tex")
stargazer(results,
          out=ofilename,
          column.labels=industries,
          float=FALSE,
          omit.stat=c("rsq","f","ser"))
```

References

Fisher Black, Michael Jensen, and Myron Scholes. The capital asset pricing model, some empirical tests. In Michael C Jensen, editor, *Studies in the theory of capital markets*. Preager, 1972.