Crossectional Asset Pricing

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1 Introduction

This lecture gives a historical perspective on testing of asset pricing models explaining the crossection of stock returns.

The starting point is the CAPM.

Initial tests (late sixties, early seventies) were tests of the CAPM.

Late seventies, eighties, more and more additional variables seem important for the crossection. (Anomalies).

Theoretical developments: Multiperiod models where evolving investment opportunities matter (e.g. Merton (1973)), can have more than just the market portfolio affecting the crossection.

Fama and French (1996) introduce three-factor model summarizing the empirically most important factors: Firm Size and Book/Market.

One more important factor: Momentum (Carhart, 1997).

Empirical literature keep unearthing various additional ways of sorting stocks by characteristics, searching for additional/alternative pricing factors.

Some important ones

- Stock liquidity (specially after the financial crisis).
- Firm profitability
- Firm investments

Latest model to gain favour: Fama and French five factor model (Fama and French, 2015).

2 CAPM

2.1 Theory

Theoretical starting point: The Capital Asset Pricing Model (CAPM), due to Sharpe (1964), Lintner (1965) and Mossin (1966).

Want to value the whole market of equities.

In the theoretical development of the CAPM: This is a one period model.

Payouts from all stocks in the economy provides equity owners all that is available to consume in the economy.

Risk that matters to equity owners: how their future consumption opportunities is affected.

In the CAPM setting, consumption = aggregate stock market return.

Hence, the only relevant risk in the CAPM: covariability with aggregate stock market return,

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or

$$beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$$

2.2 Initial tests

Early testing of the CAPM.

Late sixties, early seventies.

CRSP, academic database with US stock returns, starting 1926.

Well known papers: Black, Jensen, and Scholes (1972) Fama and MacBeth (1973)

Initial results supportive of the CAPM. Returns linked to estimates of beta.

However, even in the early tests examples of additional explanatory variables. E.g. idiosyncratic risk seemed to matter.

3 Anomalies

After the success of the initial empirical tests, the work began on converting finance industry to using CAPM in their risk adjustment. Key here: The publication of the first edition of Brealey and Myers (1980).

Given the normative prescription that "all you need is beta", it was viewed as embarassing when it was shown empirically that "beta is not enough", by showing other variables beside beta affecting the cross-section.

First line of defence: Well, remember that beta is not observed, it is only estimated. We should be looking at the covariance of the firms' future business with the stock market. Using historical data to estimate beta assumes "nothing changes"

But even allowing for time-varying beta not enough to explain some glaring crossectional patterns in stock return.

Example: Size. Small firms have substantially higher returns than they should according to the CAPM. For some reason they were also consistently higher at the start of the year (The January Effect).

Aside: This is a pattern that is not isolated to the US. Figure 1 show similar numbers for Norway 1980–2015.

Averages of monthly (percentage) returns for 5 size portfolios for Norway. Averages shown for each month.

Month	1(small)	2	3	4	5(large)
Jan	8.6	5.5	5.7	4.1	2.8
Feb	4.7	2.9	3.6	2.3	1.2
Mar	3.1	2.5	2.1	2.1	1.7
Apr	3.5	3.2	3.4	2.9	3.0
May	1.9	1.5	1.5	1.9	1.0
Jun	0.3	-0.3	-0.6	-1.0	-1.4
Jul	3.4	2.2	2.3	2.5	3.3
Aug	1.1	0.1	0.0	-0.3	-0.8
Sep	0.7	-0.6	-1.1	-2.0	-1.7
Oct	0.4	1.2	1.2	0.9	0.7
Nov	1.7	-0.1	0.2	0.6	-0.1
Dec	1.0	0.8	2.9	3.5	2.7

Figure 1: Size and Calendar differences for Norway 1980–2015

Because this effect could not be reconciled with the CAPM, finance academics called this an "Anomaly"

Note however that this is only an "anomaly" relative to a strict interpretation of the CAPM, it may be reconciled with a broader underlying model.

Another "anomaly": *Momentum* – past behaviour of stock prices could predict returns.

- Short term continuation: Jegadeesh and Titman (1993) – stocks with high returns over the past three to twelve months (winners) outperform stocks with low recent returns (losers) over the next three to twelve montsh.
- Long term reversals: DeBondt and Thaler (1985) – Winners over the past three to five years underperform losers.

And another: Book/Market – Sorting on Book/Market has predictive information for future returns.

4 Theory – multifactor models

On the theory side, CAPM is not the end.

The main problem of the CAPM: It is a single period model.

In the CAPM, the reason the market is so important is that it is what can be consumed at the end.

The actual world is multiperiod in nature, investments made in one period "live on."

So consumption is continually decided as what can be "taken out" from the investment economy.

And one may care about things affecting the intermediate investment economy because of the changes in future consumption it leads to.

This is one of the intuitions coming from the continous time model of Merton (1973).

So, since the corporate sector is where investments happen, may have several sources of risk there, affecting stock returns.

→ leads to a multifactor model for stock returns.

Some theory also argues that this multifactor model needs to be linear in the sources of uncertainty (The APT (Ross, 1978)).

So, empirical work went looking for multifactor models explaining the cross-section of stock returns, in the context of the APT. (Chen, Roll, and Ross, 1986)

5 The Fama and French three factor model

The anomalies that Fama and French found most worrying were the firm size and Book/Market sorts.

In Fama and French (1992) they return to the US crossection from 1926 till then. They evaluating the crossectional performance of the CAPM on newer data, and also using portfolios also split according to other criteria (size, B/M)

The results can be summarized as

- β does a poor job in explaining cross–section of asset returns.
- Size and B/M does a much better job.

Fama and French (1996) then formulate a model that they claim can account for most of the crossectional variation, where they add the two Fama and French factors `SMB` and `HML` that accounts for size and B/M differences.

The resulting model is essentially the CAPM plus two additional terms:

$$E[er_{it}] = \beta_i er_{mt} + b^{SMB} SMB_t + b^{HML} HML_t$$

These variables are best remembered from their mnemonics:

- SMB - Small Minus Big (i.e. small minus big companies)
- HML - High Minus Low (i.e. high B/M minus low B/M

		Book/Market		
		L (30%)	M (40%)	H(30%)
Size	Small (50%)	S/L	S/M	S/H
	Big (50%)	B/L	B/M	B/H

Figure 2: The construction of the two Fama and French (1996) factors

For the construction they split data for the US stock market as shown in figure 2.

The pricing factors are then constructed as:

$$\texttt{SMB} = \text{average}(S/L, S/M, S/H) - \text{average}(B/L, B/M, B/H)$$

$$\texttt{HML} = \text{average}(S/H, B/H) - \text{average}(S/L, B/L)$$

Note an important property of these factors:

They are zero investment investment strategies, a long position minus a short position.

In estimation settings we need the two factors SMB and HML. These are typically downloaded from Ken French homepage when dealing with US (and international) data, or alternatively constructed from the crossection.

(For Norway see my homepage).

6 Momentum

The Carhart factor `PR1YR`

Carhart (1997) introduced an additional factor that accounts for momentum. Figure 3 illustrates this factor construction. Each month the stock return is calculated over the previous eleven months. The returns are ranked, and split into three portfolios: The top 30%, the median 40% and the bottom 30%. The Carhart (1997) factor `PR1YR` is the difference between the average return of the top and the bottom portfolios. The ranking is recalculated every month.

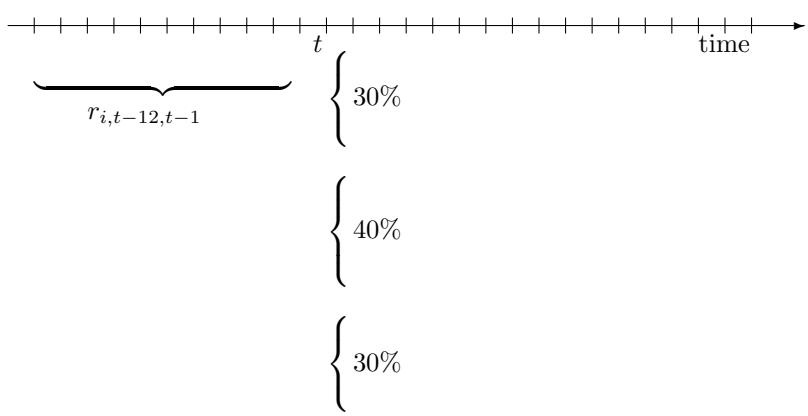

Figure 3: The construction of the Carhart (1997) factor PR1YR

So, this factor is also a zero investment portfolio.

One important thing about this portfolio: It needs to be rebalanced very often (each month).

7 Consumption Based Asset Pricing

The three and four factor models are linear multifactor models of asset returns.

One way these can be theoretically motivated is as consequences of the time-varying investment opportunities in the Merton (1973) Continous-time CAPM model. The theorists are not idle, though. They view this as an unsatisfactory (complex) model, and simplify it in the following manner: What decision makers in the economy cares about is future consumption. So all risks in the economy can really be summarized in their effect on future consumption opportunities.

Hence, all risk can be reduced to one number, a "consumption-beta", the covariance between an asset and consumption.

This leads to the "Consumption CAPM" (Breeden, 1979).

Empirical testing of the Consumption CAPM is however less than impressive...

The problem is that consumption tends to be pretty smooth. So it will not covary much with stock returns, which are volatile.

Testing the Consumption CAPM has lead to one of the more entertaining papers in finance, though. Savov (2011) uses the amount of garbage produced in New York as a better proxy for consumption. This is much more variable, and a better pricing variable than consumption...

But for crossectional asset pricing the consumption link does not lead anywhere.[1]

8 Searching for further factors

By 2000 asset pricing studies tended to use either the three or four factor models by default in any setting investigating the US crossection.

But it was not a satisfactory situation.

Essentially, the factors that are added to market are proxies for some unknown source of risk.

Theoretially, it is hard to see the link between these factors and any economic model.

Researchers keep trying alternative ways of sorting the crossection

- Either trying to replace the FF factors with ones more understandable in terms of economics.
- Or finding factors with additional/better explanatory power. (Factor Fishing)

Examples of things that are tried

- Macro Variables - can they replace the FF factors (Liew and Vassalou, 2000)
- Liquidity (Amihud and Mendelson, 1986) (size may be a proxy for illiquidity)
- Firm Investments (Looking for real options)
- ...

There are also some ways one can rationalize effects by other means.

A cute example is provided by Jonathan Berk (1995), showing how ones' net present value intuition is always useful.

Suppose we have two firms i, j with identical annual future cash flows X. They are then priced as

$$P_i = \frac{X}{r_i}$$

$$P_j = \frac{X}{r_j}$$

[1] See Breeden, Litzenberger, and Jia (2015a) and Breeden, Litzenberger, and Jia (2015b) for some alternative views.

The more risky firm (j,say) will have higher cost of capital r_j.

Since we are dividing by a larger number r_j, the *Price* P_j is *lower* than P_i.

This induces a link between firm size and riskiness. Small firms may just be more risky.

There is a mechanical link between firm size and expected returns.

9 The Fama and French five factor model (2015)

A recent step on the "lets add factors" road is Fama and French (2015). In this article Fama and French add two more variables, profitability and investment, to their three factor model.

These are again zero investment portfolios, but sorted on two different *accounting* variables.

- Operating profitability, OP, in the sort for June of year t is measured with accounting data for the fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity.
- Investment, Inv, is the rate of growth of total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$."

Doing the usual double sort, Fama and French then introduce the additional factors

- RMW – Robust Minus Weak (profitability)
- CMA – Conservative Minus Aggressive (investments)

Resulting in their five factor model

$$E[r_{i,t}] = r_{f,t} + \beta_i E[er_{m,t}] + b_i^{SMB} SMB_t + b_i^{HML} HML_t + b^{RMW} RMW_t + b^{CMA} CMA_t$$

10 Conclusion

So, where are we today?

Four decades of research given us a "menu" of possible ways to describe the cross-section of asset returns (and the resulting expected return of an asset).

- CAPM
- Fama-French 3 factor
- Fama-French 3 factor + Momentum
- Fama-French 5 factor

Leaving out the academic side,

Where are these more complicated primarily used in practice:

Investment Allocation (large portfolios) – Investigating Portfolio Performance.

Note where it is *not* used: Corporate Finance / Valuation.

Whenever we start estimating cost of capital for valuation purposes – back to the CAPM.

One reason for this: Estimation Uncertainty.

Estimating parameters of five factors inherently a noisy process.

References

Yakov Amihud and Yakov Mendelson. Asset pricing and the bid/ask spread. *Journal of Financial Economics*, 17:223–249, 1986.

Jonathan Berk. A critique of size related anomalies. *Review of Financial Studies*, 8:275–86, 1995.

Fisher Black, Michael Jensen, and Myron Scholes. The capital asset pricing model, some empirical tests. In Michael C Jensen, editor, *Studies in the theory of capital markets*. Preager, 1972.

Richard A Brealey and Stewart C Myers. *Principles of Corporate Finance*. McGraw–Hill, first edition, 1980.

Douglas T Breeden. An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics*, 7:265–296, 1979.

Douglas T Breeden, Robert H Litzenberger, and Tingyan Jia. Consumption-based asset pricing, part 1: Classic theory and tests, measurement issues, and limited participation. In Lo and Merton (2015), pages 35–83.

Douglas T Breeden, Robert H Litzenberger, and Tingyan Jia. Consumption-based asset pricing, part 2: Habit formation, conditional risks, long-run risk, and rare disasters. In Lo and Merton (2015), pages 84–131.

Mark M Carhart. On persistence in mutual fund performance. *Journal of Finance*, 52(1):57–82, March 1997.

Nai fu Chen, Richard Roll, and Stephen Ross. Economic forces and the stock market. *Journal of Business*, 59:383–403, 1986.

Werner F. M. DeBondt and Richard H. Thaler. Does the stock market overreact? *Journal of Finance*, 40(3):793–808, 1985.

Eugene F Fama and Kenneth R French. The cross-section of expected stock returns. *Journal of Finance*, 47(2):427–466, June 1992.

Eugene F Fama and Kenneth R French. Multifactor explanations of asset pricing anomialies. *Journal of Finance*, 51(1):55–85, 1996.

Eugene F. Fama and Kenneth R. French. A five-factor asset pricing model. *Journal of Financial Economics*, 116(1):1 – 22, 2015.

Eugene F Fama and J MacBeth. Risk, return and equilibrium, empirical tests. *Journal of Political Economy*, 81:607–636, 1973.

Narasimhan Jegadeesh and Sheridan Titman. Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*, 48(1):65–91, March 1993.

J Liew and M Vassalou. Can book-to-market, size and momentum be risk factors that predict economic growth? *Journal of Financial Economics*, 57:221–245, 2000.

John Lintner. The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*, February 1965.

Andy Lo and Robert Merton, editors. *Annual Review of Financial Economics*, volume 7 of *Annual Reviews*. Annual Reviews, 2015.

Robert C Merton. An intertemporal capital asset pricing model. *Econometrica*, 41:867–888, September 1973.

Jan Mossin. Equilibrium in a capital asset market. *Econometrica*, October 1966.

S A Ross. A simple approach to the valuation of risky streams. *Journal of Business*, 51:453–75, 1978.

Alexi Savov. Asset pricing with garbage. *Journal of Finance*, 66(1):177–201, 2011.

William F Sharpe. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19: 425–42, September 1964.