Solutions to Lectures on Corporate Finance, Second Edition

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As an empirical investigation, check your local supermarket. Does 2 ketchup bottles of 0.5 litres cost the same as one ketchup bottle of 1 liter? What does this tell you about value additivity in financial markets?

3.2 *Milk* [2]

Why is skimmed milk always cheaper than regular milk even if it is healthier?

Solutions

3.1 *Ketchup* [2]

3.2 *Milk* [2]

Chapter 4

On the Efficient Markets Hypothesis

Problems

4.1 *Interest Rates* [2]

Consider the following statement.

> *Long term interest rates are at record highs. Most companies therefore find it cheaper to finance with common stock or relatively inexpensive short-term bank loans.*

What does the Efficient Market Hypothesis have to say about the correctness of this?

4.2 *Semistrong* [3]

Can you expect to earn excess returns if you make trades based on your broker's information about record earnings for a stock, rumors about a merger of a firm, or yesterday's announcement of a successful test of a new product, if the market is semi-strong form efficient?

4.3 *UPS* [3]

On 1/10/85, the following announcement was made: "Early today the Justice Department reached a decision in the UPC case. UPC has been found guilty of discriminatory practices in hiring. For the next five years, UPC must pay $2 million each year to a fund representing victims of UPC policies." Should investors not buy UPC stock after the announcement because the litigation will cause an abnormally low rate of return over the next five years?

4.4 *Management* [3]

Your broker claims that well–managed firms are not necessarily more profitable investment opportunities than firms with an average management. She cites an empirical study where 17 well–managed firms and a control group of 17 average firms were followed for 8 years after the former were reported in the press to be "excelling" as far as management is concerned. Is this evidence that the stock market does not recognize good management?

4.5 *TTC* [3]

TTC has released this quarter's earning report. It states that it changed how it accounts for inventory. The change does not change taxes, but the resulting earnings are 20% higher than what it would have been under the old accounting system. There is no other surprises in the earnings report.

1. Would the stock price now jump on the release of this earnings report?

4.6 *Investing?* [3]

Does the following statement make sense in view of the Efficient Markets Hypothesis (EMH)?

> *The Japanese economy has deep structural problems, which the Japanese seem reluctant to overcome. We do not see any major change in this situation over the next two to three years. Hence, we advise against investing in the Tokyo stock market, because we expect returns to be below average for the next two to three years.*

Solutions

4.1 *Interest Rates* [2]

Remember the first lesson about market efficiency: Markets have no memory. Just because long-term interest rates are high relative to past levels does not mean that they won't go higher still. Unless you have special information indicating that long-term rates are *too* high, issuing long-term bonds should be a zero-NPV transaction. So should issuing short-term debt or common stock.

4.2 *Semistrong* [3]

All of these are public information, you do not expect them to explain *future* changes in stock price. Hence, you can not expect to make excess returns using this information. You can only use private information to generate excess returns.

4.3 *UPS* [3]

Once the announcement is made and the price has reacted (downward) to the lower (discounted) future dividend stream, there is no further effect. The average return over the next five years will still be determined solely by risk and not by the fact that dividends will be $2 million lower. For instance, if risk continues to be high, average returns will also be.

4.4 *Management* [3]

The evidence that the 17 well-managed firms did not outperform the market merely confirms an implication of the efficient markets hypothesis: *Average returns are only determined by risk*. The fact that the firms were run by good managers was known 8 years earlier and already properly reflected in prices at that time.

4.5 *TTC* [3]

1. The changes in the accounting treatment do not change cash flows, even if reported earnings change. A possible channel for cash flow changes could have been the timing of taxes, but that is explicitly ruled out.

   The stock price should therefore not be affected by the accounting change.

4.6 *Investing?* [3]

No. Average returns are determined solely by risk. If there is a lot of risk, average returns are high, etc. The economic situation does not affect average returns.

Chapter 5

Present Value

Problems

5.1 *Present Value* [3]

You are given the following prices P_t today for receiving risk free payments t periods from now.

t	=	1	2	3
P_t	=	0.95	0.9	0.85

1. Calculate the implied interest rates and graph the term structure of interest rates.

2. Calculate the present value of the following cash flows:

t	=	1	2	3
X_t	=	100	100	100

5.2 *Borrowing* [2]

BankTwo is offering personal loans at 10%, compounded quarterly. BankThree is offering personal loans at 10.5%, compounded annually. Which is the better offer?

5.3 *Arbitrage* [4]

You are given the following prices P_t today for receiving risk free payments t periods from now.

t	=	1	2	3
P_t	=	0.95	0.9	0.95

There are traded securities that offer $1 at any future date, available at these prices.

How would you make a lot of money?

5.4 *Bank Loans* [2]

Your company is in need of financing of environmental investments. Three banks have offered loans. The first bank offers 4.5% interest, with biannual compounding. The second bank offers 4.3% interest, with monthly compounding. The third bank offers 4.25% with annual compounding.

Determine which is is the best offer.

5.5 *Stock* [4]

A stock has just paid a dividend of 10. Dividends are expected to grow with 10% a year for the next 2 years. After that the company is expecting a constant growth of 2% a year. The required return on the stock is 10%. Determine todays stock price.

5.6 *Bonds* [6]

You observe the following three bonds:

Bond	Price	Cashflow in period		
		1	2	3
A	95	100	0	0
B	90	10	110	0
C	85	10	10	110

1. What is the current value of receiving one dollar at time 3?

Consider now the bond D, with the following characteristics

Bond	Cashflow in period		
	1	2	3
D	20	20	520

2. What is the current price of bond D?

Consider next bond E, which last for four periods. Bond E has the following characteristics:

Bond	Cashflow in period			
	1	2	3	4
E	10	10	10	110

3. If the market does not allow any free lunches (arbitrage), what is the *maximal* price that bond E can have?

5.7 *Growing Perpetuity* [8]

The present value of a perpetuity that pays X_1 the first year and then grows at a rate g each year is:

$$PV = \sum_{t=1}^{\infty} \frac{X_1(1+g)^{t-1}}{(1+r)^t}$$

Show that this simplifies to

$$PV = \frac{X_1}{r-g}$$

5.8 *Annuity* [6]

Show that the present value of an annuity paying X per period for T years when the interest rate is r can be simplified as

$$PV = \sum_{t=1}^{T} \frac{X}{(1+r)^t} = X\left[\frac{1}{r} - \frac{1}{r}\frac{1}{(1+r)^T}\right]$$

5.9 *Stock* [2]

The current price for a stock is 50. The company is paying a dividend of 5 next period. Dividend is expected to grow by 5% annually. The relevant interest rate is 14%. In an efficient market, can these numbers be sustained?

5.10 *Growing Annuity* [6]

Consider an T-period annuity that pays X next period. After that, the payments grows at a rate of g per year for the next T years.

The present value of the annuity is

$$PV = \sum_{t=1}^{T} \frac{X(1+g)^{(t-1)}}{(1+r)^t}$$

Can you find a simplified expression for this present value?

5.11 *Jane* [3]

Jane, a freshman in college, needs 55000 in 4 years to start studying for an MBA. Her investments earn 5% interest per year.

1. How much must she invest today to have that amount at graduation?

2. If she invested once a year for four years beginning today until the end of the 4 years how much must she invest?

5.12 *Bonds* [3]

The current interest rate is 7%. Given the opportunity to invest in one of the three bonds listed below, which would you buy? Sell short?

Bond	Face value	Annual coupon rate	Maturity	Price
A	1000	4%	1 year	990
B	1000	7.5%	17 years	990
C	1000	8.5%	25 years	990

Solutions

5.1 *Present Value* [3]

1. The implied interest rates r_t are found as:

$$P_1 = 0.95 = \frac{1}{1 + r_1}$$

$$r_1 = \frac{1}{0.95} - 1 = 5.26\%$$

$$P_2 = 0.9 = \left(\frac{1}{1 + r_2}\right)^2$$

$$r_2 = \sqrt{\frac{1}{0.9}} - 1 = 5.41\%$$

$$P_3 = 0.85 = \left(\frac{1}{1 + r_3}\right)^3$$

$$r_3 = \sqrt[3]{\frac{1}{0.85}} - 1 = 5.57\%$$

2. The present value is found as

$$PV = 100 \cdot 0.95 + 100 \cdot 0.9 + 100 \cdot 0.8 = 265$$

or alternatively, using interest rates, as

$$PV = 100 \left(\frac{1}{1.0526}\right) + 100 \left(\frac{1}{1.0541}\right)^2 + 100 \left(\frac{1}{1.0557}\right)^3 \approx 265$$

5.2 *Borrowing* [2]

The offers need to be made comparable. This is typically done by finding the equivalent annual interest rates and comparing those. In annual terms, BankTwo is charging

$$\left(1 + \frac{0.1}{4}\right)^4 - 1 \approx 10.38\%$$

When borrowing, one wants the lowest possible interest rate, which means that the 10.5% interest rate offered by BankThree is dominated by the 10.38% interest rate (in annual terms) offered by BankTwo.

5.3 *Arbitrage* [4]

This data implies an arbitrage opportunity. Note that the price of the risk free security offering $1 in period 3 is higher than the price of the risk free security offering $1 in period 2. What does this mean? It means you have to pay less today for receiving money sooner! To make a lot of money, short the risk free security for period 3, and use $0.9 of the $0.95 proceeds to buy the period 2 risk free security. The $1 you get in period 2 can be kept as money and used to cover your obligation in period 3. For each of these transactions you get $0.05 now. To get very rich, do a lot of these transactions.

5.4 *Bank Loans* [2]

1. Annualize the interest rates to make them comparable:

   First bank:

   $$\left(1 + \frac{0.045}{2}\right)^2 - 1 = 4.55\%$$

   Second bank:

   $$\left(1 + \frac{0.043}{12}\right)^{12} - 1 = 4.39\%$$

   Third bank

   $$4.25\%$$

   The offer from the third bank is preferable.

5.5 *Stock* [4]

$$D_1 = 10(1.1) = 11$$

$$D_2 = 10(1.1)^2 = 12.1$$

Calculate the time 2 value of the stock as a growing perpetuity.

$$D_3 = D_2(1.02) = 12.1 \cdot 1.02 = 12.34$$

$$P_2 = \frac{D_3}{r - g} = \frac{12.34}{0.1 - 0.02} = 154.27$$

Then find the current stock price as the present value:

$$P_0 = \frac{D_1}{1.1} + \frac{D_2 + P_2}{1.1^2} = \frac{11}{1.1} + \frac{12.1 + 154.27}{1.1^2} = 10 + 137.5 = 147.5$$

5.6 *Bonds* [6]

This is easiest solved using the current prices P_t of securities offering \$1 in period t.

For simplicity, let us formulate this as a linear algebra problem.

Let the matrix $\mathbf{C}$ contain the bond payoffs:

$$\mathbf{C} = \begin{bmatrix} 100 & 0 & 0 \\ 10 & 110 & 0 \\ 10 & 10 & 110 \end{bmatrix}$$

and the vector $\mathbf{B}$ contain the bond prices:

$$\mathbf{B} = \begin{bmatrix} 95 \\ 90 \\ 85 \end{bmatrix}$$

For a given vector $\mathbf{P}$ of prices of zero coupon bonds

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

we leave it to the reader to confirm that the bond prices $\mathbf{B}$ can be found as:

$$\mathbf{B} = \mathbf{C}\mathbf{P}.$$

It is then a standard linear algebra operation to calculate $\mathbf{P}$ as

$$\mathbf{P} = \mathbf{C}^{-1}\mathbf{B}$$

The actual calculations are below shown as extracts from a matlab-like environment.

Defining $\mathbf{C}$ and $\mathbf{B}$:

```
C =
   100     0     0
    10   110     0
    10    10   110

B =
   95
   90
   85
```

calculate $\mathbf{P}$ as:

```
P = inv(C)*B
   0.95000
   0.73182
   0.61983
```

The current value of receiving one dollar at time 3 is thus 0.61983.

If we now consider bond E. It lasts for 4 periods, one period more than we have data for. Since holding money is always an option, the current value of receiving one dollar at time 4 can never be *more* than the current value of receiving one dollar at time 3, which is 0.61983. This can easily be argued by arbitrage arguments. An alternative way of making the argument is to say that interest rates can not go below zero.

We can therefore use the discount factors P^* given by

```
Pstar =
   0.95000
   0.73182
   0.61983
   0.61983
```

to find the maximal price the bond can have (the upper bound).

Defining the cash flows of the new bond as

```
C =
   10   10   10   110
```

we find the bond's highest possible value as

```
C*dstar
  = 91.198
```

The highest possible price of bond E is 91.198.

5.7 *Growing Perpetuity* [8]

1. This is shown the following way:

$$PV = X_1 \left[\frac{1}{1+r} + \frac{(1+g)}{(1+r)^2} + \frac{(1+g)^2}{(1+r)^3} + \cdots \right]$$

At time 1, the present value starting at that date is $(1+g)PV$.

Hence

$$PV = \frac{X_1 + (1+g)PV}{1+r}$$

Solving for PV

$$PV \left(1 - \frac{1+g}{1+r} \right) = X_1$$

and simplifying

$$PV = \frac{X_1}{1 - \frac{(1+g)}{(1+r)}} = \frac{X_1}{(1+r) - (1+g)} = \frac{X_1}{r-g}$$

5.8 *Annuity* [6]

We find the annuity formula the following way:

We know that a perpetuity that pays X per period indefinitely has a present value of

$$PV = \frac{X}{r}$$

We know that an annuity pays X per period for T periods. One way to achieve this is to *buy* a perpetuity that pays X and starts next period, and *sell* a perpetuity that starts running at time $(T+1)$:

Time	1	$\cdots$	T	$T+1$	$\cdots$		Value
Perpetuity from $t=1$ onwards	X	$\cdots$	X	X	$\cdots$	$=$	$X \cdot \left[\frac{1}{r} \right]$
$-$Perpetuity from $t=T+1$ onwards	0	$\cdots$	0	X	$\cdots$	$=$	$X \cdot \left[\frac{1}{(1+r)^T} \frac{1}{r} \right]$
$=$ Annuity	X	$\cdots$	X	0	$\cdots$	$=$	$X \cdot \left[\frac{1}{r} - \frac{1}{(1+r)^T} \frac{1}{r} \right]$

5.9 *Stock* [2]

The stock price should increase to

$$P = \frac{5}{0.14 - 0.05} = 55.56$$

5.10 *Growing Annuity* [6]

Remember what we did in the case of an annuity which is not growing.

Time	1	$\cdots$	T	$T+1$	$\cdots$		Value
Perpetuity from 1 onwards	X	$\cdots$	X	X	$\cdots$	$=$	$X \cdot [\frac{1}{r}]$
$-$Perpetuity from T+1 onwards	0	$\cdots$	0	X	$\cdots$	$=$	$X \cdot [\frac{1}{(1+r)^T} \frac{1}{r}]$
Annuity	X	$\cdots$	X	0	$\cdots$	$=$	$X \cdot \left[\frac{1}{r} - \frac{1}{(1+r)^T} \frac{1}{r} \right]$

Now we want to adjust this for the fact that the annuity also grows by g each period

Time	1	2	$\cdots$	T	$T+1$	$\cdots$		Value
Growing perpetuity from 1 onwards	X	$X(1+g)$	$\cdots$	$X(1+g)^{T-1}$	$X(1+g)^T$	$\cdots$	$=$	$X \cdot [\frac{1}{r-g}]$
$-$Growing perpetuity from $T+1$ onwards	0	0	$\cdots$	0	$X(1+g)^T$	$\cdots$	$=$	$X \cdot \left[\frac{(1+g)^T}{(1+r)^T} \frac{1}{r-g} \right]$
Annuity	X	$X(1+g)$	$\cdots$	$X(1+g)^{T-1}$	0	$\cdots$	$=$	$X \cdot \left[\frac{1}{r-g} - \left(\frac{1}{1+r}\right)^T \frac{(1+g)^T}{r-g} \right]$

5.11 *Jane* [3]

1. Solve for X in the following expression:

   $$X(1 + 0.05)^4 = 55000$$

   $$X = \frac{55000}{(1 + 0.05)^4} = 45248.64$$

   She needs to invest 45248.64 today at 5% to have 55000 4 years from now.

2. Let us illustrate two ways of doing this.

   a) Direct solution.

   Let C be the amount invested.

   $$C \left(1.05^4 + 1.05^3 + 1.05^2 + 1.05 \right) = 55000$$

   $$C = \frac{55000}{4.5256}$$

   $$C = 12,153$$

   b) Using annuities.

This can also be calculated using annuities from an annuity table. The annuity factor A for an interest rate of $r = 0.05$ and $n = 4$ years is 3.54595.

$$PV = C \times A(1 + r)$$

The $(1 + r)$ is because the investment is at the beginning of the period.

$$(1 + r)^4 PV = FV = 55000 = C \times A(1 + r)(1 + r)^4$$

$$C = \frac{55000}{A(1 + r)^4} = \frac{55000}{3.54595 \cdot (1 + 0.05)^5} = 12,153.$$

5.12 *Bonds* [3]

Let us first check that the bonds are correctly priced:

With an interest rate of 7% the correct bond prices should be:

$$P_A = \frac{1040}{1.07} = 971.96$$

$$P_B = 75 A_{r=7\%, n=17} + \frac{1000}{1.07^{17}} = 75 \cdot 9.76322 + 316.57 = 1048.82$$

$$P_C = 85 A_{r=7\%, n=25} + \frac{1000}{1.07^{25}} = 85 \cdot 11.6536 + 184.25 = 1174.80$$

Compared to the correct prices bond A is overpriced and bonds B and C underpriced. To exploit this short A and go long B and C.

Chapter 6

Capital Budgeting

Problems

6.1 *Projects* [2]

Two projects A and B have the following cashflows:

	X_0	X_1	X_2
A	$-4,000$	$2,500$	$3,000$
B	$-2,000$	$1,200$	$1,500$

1. Find the Payback periods for the two projects. Which project has the shortest payback period?

2. Calculate the Internal Rate of Return on the two projects. Which project has the higher IRR?

3. The discount rate is 10%. How would the NPV rule rank these two projects?

6.2 *Projects* [3]

A project costs 100 today. The project has positive cash flows of 100 in years one and two. At the end of the life of the project there are large environmental costs resulting in a negative cash flow in year 3 of -95. Determine the internal rate(s) of return for the project.

6.3 *Bonds* [4]

You are given the following information about three bonds.

Bond	Year of Maturity	Coupon	Yield to Maturity	Bond Price
A	2	10%	7.5862%	$1,043.29$
B	2	20%	7.6746%	$1,220.78$
C	3	8%	9.7995%	995.09

Coupons are paid at the end of the year (including the year of maturity). All three bonds have a face value of 1,000 at maturity.

1. Find the time zero prices, P_1, P_2, and P_3, of one dollar to be delivered in years 1, 2, and 3, respectively.

2. Find the 1, 2, and 3 year spot rates of interest r_1, r_2 and r_3.

6.4 *Machine* [4]

A company is considering its options for a machine to use in production. At a cost of 47 they can make some small repairs on their current machine which will make it last for 2 more years. At a higher cost of 90 they can make some more extensive repairs on their current machine which will make it last for 4 more years. A new machine costs 300 and will last for 8 years. The company is facing an interest rate of 10%. Determine the best action.

6.5 *PI and NPV* [2]

Show that a project with a positive NPV will always have a profitability index greater than 1.

6.6 *Project* [4]

A project has a cost of 240. It will have a life of 3 years. The cost will be depreciated straight–line to a zero salvage value, and is worth 40 at that time. Cash sales will be 200 per year and cash costs will run 100 per year. The firm will also need to invest 60 in working capital at year 0. The appropriate discount rate is 8%, and the corporate tax rate is 40%. What is the project's NPV?

6.7 *C&C* [4]

The C&C company recently installed a new bottling machine. The machine's initial cost is 2000, and can be depreciated on a straight–line basis to a zero salvage in 5 years. The machine's per year fixed cost is 1500, and its variable cost is 0.50 per unit. The selling price per unit is 1.50. C&C's tax rate is 34%, and it uses a 16% discount rate.

1. Calculate the machine's *accounting* break–even point on the new machine (i.e., the production rate such that the accounting profits are zero).

2. Calculate the machine's *present value* break–even point(i.e., the production rate such that the NPV is zero).

6.8 *PillAdvent* [6]

After extensive medical and marketing research, PillAdvent Inc, believes it can penetrate the pain reliever market. It can follow one of two strategies. The first is to manufacture a medication aimed at relieving headache pain. The second strategy is to make a pill designed to relieve headache and arthritis pain. Both products would be introduced at a price of 4 per package in real terms. The broader remedy would probably sell 10 million packages a year. This is twice the sales rate for the headache–only medication. Cash costs of production in the first year are expected to be 1.50 per package in real terms for the headache–only brand. Production costs are expected to be 1.70 in real terms for the more general pill. All prices and costs are expected to rise at the general inflation rate of 5%.

Either strategy would require further investment in plant. The headache–only pill could be produced using equipment that would cost 10.2 million, last three years, and have no resale value. The machinery required to produce the broader remedy would cost 12 million and last three years. At this time the firm would be able to sell it for 1 million (in real terms). The production machinery would need to be replaced every three years, at constant real costs. For both projects the firm will use straight–line depreciation. The firm faces a corporate tax rate of 34%. The firm believes that the appropriate discount rate is 13%. Capital gains are taxed at the ordinary corporate tax rate of 34%.

What pain reliever should the firm produce?

Solutions

6.1 *Projects* [2]

1. Payback. For project A it takes 1.5 years before the initial investment is recouped, for project B it takes 1.53 years.

Project	Payback
A	1.5 years
B	1.53 years

2. Internal rate of return

Project A:

$$0 = \frac{2500}{1 + IRR} + \frac{3000}{(1 + IRR)^2} - 4000$$

$$IRR = 23.32\%$$

Project B:

$$0 = \frac{1200}{1 + IRR} + \frac{1500}{(1. + IRR)^2} - 2000$$

$$IRR = 21.65\%$$

3. Net present value

$$NPV_A = \frac{2500}{1.1} + \frac{3}{(1.1)^2} - 4000 \approx 752$$

$$NPV_B = \frac{1200}{1.1} + \frac{1.5}{(1.1)^2} - 2000 \approx 330$$

6.2 *Projects* [3]

The following picture shows the NPV as a function of the interest rate, and illustrates the fact that there are two
solutions y to the problem of solving

$$0 = -100 + \frac{100}{1 + y} + \frac{100}{(1 + y)^2} + \frac{-95}{(1 + y)^3}$$
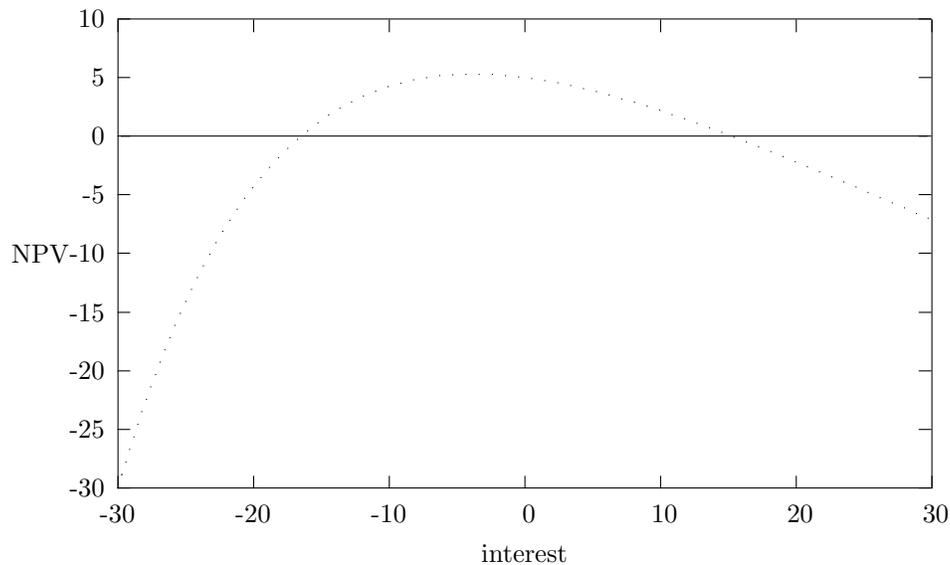
6.3 *Bonds* [4]

Consider the the cash flows of the bonds

		Period		
		1	2	3
Bond	1	100	1100	
	2	200	1200	
	3	80	80	1080

and let B_i be the price of bond i.

1. Prices of one dollar to be received at times 1, 2 and 3:

   We use the following system to solve for these:

   $$
   \begin{aligned}
   B_1 &= 100P_1 + 1100P_2 \\
   B_2 &= 200P_1 + 1200P_2 \\
   B_3 &= 80P_1 + 80P_2 + 1080P_3
   \end{aligned}
   $$

   We first use the prices of the first two bonds:

   $$
   \begin{aligned}
   B_1 &= 100P_1 + 1100P_2 \\
   B_2 &= 200P_1 + 1200P_2
   \end{aligned}
   $$

   Multiply the first equation by 2 and subtract the second:

   $$2B_1 - B_2 = 1000P_2$$

   Solve for P_2:

   $$P_2 = \frac{2 \cdot 1043.29 - 1220.78}{1000} = 0.8658$$

   Use this to find P_1:

   $$B_1 = 100P_1 + 1100P_2$$

   $$1043.29 = 100P_1 + 1100 \cdot 0.8658$$

   $$P_1 = 0.9091$$

   Then find P_3 using the third bond price:

   $$B_3 = 80P_1 + 80P_2 + 1080P_3$$

   $$995.09 = 80 \cdot 0.9091 + 80 \cdot 0.8658 + 1080P_3$$

   $$P_3 = 0.7529$$

2. From these we find the spot rates of interest

   $$P_1 = 0.9091 = \frac{1}{1 + r_1}$$

   $$r_1 = \frac{1}{0.9091} - 1 = 9.89\%$$

   $$P_2 = 0.8658 = \left(\frac{1}{1 + r_2}\right)^2$$

   $$r_2 = \sqrt{\frac{1}{0.8658}} - 1 = 7.47\%$$

   $$P_3 = 0.7529 = \left(\frac{1}{1 + r_3}\right)^3$$

   $$r_3 = \sqrt[3]{\frac{1}{0.7529}} - 1 = 9.92\%$$

This exercise could alternatively have been solved using matrix algebra, as the following excerpts from a matrix environment illustrates

```
B =
   1043.29
   1220.78
    995.09
C =
   100   1100      0
   200   1200      0
    80     80   1080
>> p=inv(C)*B
p =
   0.90910
   0.86580
   0.78991
>> r=p.^(-1./t)-1
r =
   0.099989   0.074710   0.081787
```

6.4 *Machine* [4]

The two alternatives need to be made comparable some way. One way to go about it make the alternatives comparably by repeating the projects to make each last 8 years. Then you have two ways of getting the same services from the two alternatives.

Small repairs

$t =$	0	2	4	6
$C_t =$	47	47	47	47

$$NPV = 47 + \frac{47}{(1+0.1)^2} + \frac{47}{(1+0.1)^4} + \frac{47}{(1+0.1)^6} = 144.475$$

Larger repairs

$t =$	0	1	2	3	4
$C_t =$	90	0	0	0	90

$$NPV = 90 + \frac{90}{(1+0.1)^4} = 151.471$$

The cheapest is the small repairs.

An alternative would have been to calculate the equivalent annual annuity of the two alternatives.

6.5 *PI and NPV* [2]

$$PI = \frac{PV}{C_0}$$

$$NPV = PV - C_0$$

If $NPV > 0$ then $PV > C_0$ and hence $\frac{PV}{C_0} > 1$.

6.6 *Project* [4]

Cash flows

$$c_0 = -240 - 60 = -300$$

$$c_t = (200 - 100)(1 - 0.4) + \frac{240}{3} \cdot 0.4 = 92 \quad (t = 1, 2)$$

$$c_3 = c_2 + 40(1 - 0.4) + 60 = 92 + 84$$

Net present value

$$NPV = -300 + \left(\frac{1}{1.08} + \frac{1}{1.08^2} + \frac{1}{1.08^3} \right) 92 + \frac{84}{1.08^3} = 4$$

6.7 *C&C* [4]

Accounting break-even:

$$-\frac{2000}{5} - 1500 + x(15 - 0.5) = 0$$

$$x = 1900$$

NPV break even (note use of accounting profits to compute taxes):

$$c_0 = -2000$$

$$c_t = x(1.5 - 0.5) - 1500 - 0.34 \left(x(1.5 - 0.5) - 1500 - \frac{2000}{5} \right)$$

$$= x \cdot 0.66 - 1500 \cdot 0.66 + 400 \cdot 0.34$$

$$0 = -2000 + \left(\frac{1}{1.16} + \frac{1}{1.16^2} + \frac{1}{1.16^3} + \frac{1}{1.16^4} + \frac{1}{1.16^5} \right) (x \cdot 0.66 - 1500 \cdot 0.66 + 400 \cdot 0.34)$$

$$x = 2219$$

6.8 *PillAdvent* [6]

1. Headache pain reliever

    $X_0 = -10.2$ million.
    cash inflow at $t = 1, 2, 3 : \$(4 - 1.50) \cdot 5 \cdot 10^6 = 12.5$ million.
    taxes: 12.5 million at 34% = 4.25 million
    tax "rebate" because of depreciation (straight-line): $\frac{10.2}{3}$ million at 34% = 1.156 million. (the tax "rebate" is a *nominal* cash flow, so will have to be discounted at a nominal rate.)

    Summary (in millions)

    - Real cash flows: $X_0 = -10.2, X_1 = X_2 = X_3 = 8.25$
    - Nominal cash flows: $X_0 = 0, X_1 = X_2 = X_3 = 1.156$
    - Real interest rate=13%.
    - Nominal interest rate $\approx (13 + 5)\% = 18\%$.

$$NPV = -10.2 + 8.25 \left(\frac{1}{1.13} + \frac{1}{1.13^2} + \frac{1}{1.13^3} \right) + 1.156 \left(\frac{1}{1.18} + \frac{1}{1.18^2} + \frac{1}{1.18^3} \right)$$

$$= -10.2 + 8.25 A_{n=3, r=13\%} + 1.156 A_{n=3, r=18\%}$$

$$= -10.2 + 8.25 \cdot 2.36115 + 1.156 \cdot 2.17427 = 11.7934 \text{ million}$$

2. Broad pain reliever $X_0 = -12$ million.
   cash inflow at $t = 1, 2, 3 : \$(4 - 1.70) \cdot 10 \cdot 10^6 = 23$ million.
   taxes: 23 million at 34% = 7.82 million
   tax "rebate" because of depreciation (straight-line) (again, nominal): $\frac{12}{3}$ million at 34% = 1.36 million.
   Salvage value: 1 million, taxed at 34%.

   Summary (in millions)

   - Real cash flows: $X_0 = -12, X_1 = X_2 = 15.18; X_3 = 15.18 + 1 - 0.34 = 15.84$
   - Nominal cash flows: $X_0 = 0, X_1 = X_2 = X_3 = 1.36$

$$NPV = -12 + 15.18 \left(\frac{1}{1.13} + \frac{1}{1.13^2} \right) + 15.84 \left(\frac{1}{1.13^3} \right) + 1.36 \left(\frac{1}{1.18} + \frac{1}{1.18^2} + \frac{1}{1.18^3} \right)$$

$$= -12 + 15.18 \cdot 1.6681 + 15.84 \cdot 0.6931 + 1.36 \cdot 2.17427 = 27.2575 \text{ million}$$

Conclusion: go for (b)

Chapter 7

Valuation Under Uncertainty: The CAPM

Problems

7.1 *Portfolio* [3]
You can choose to invest in two shares, A and B:

	$E[r]$	σ
A	12.5%	15%
B	16%	20%

Plot, in a mean standard deviation diagram, expected return and standard deviation for portfolios with weights in A of 0, 0.25, 0.5, 0.75 and 1.0. Do this for correlations between the returns of A and B of -1, 0 and $+1$. What does this tell you about how diversification possibilities varies with covariances?

7.2 *CAPM* [2]
The current risk free interest rate is 5%. The expected return on the market portfolio is 14%. What is the expected return of a stock with a beta value of 0.5?

7.3 *Beta* [2]
Stock A has an expected rate of return of 15%. Todays expected return on the market portfolio is 10%. The current risk free interest rate is 7,5%. What is the beta of stock A?

7.4 *Project* [3]
A project with a beta of 1.5 has cash flows 100 in year 1, 200 i year 3, 500 in year 4 and 100 in year 6. The current expected market return is 10%. The risk free interest rate is 5%. What is the highest cost that makes this project worth investing in?

7.5 *Portfolio* [2]
Stock A has an expected return of 10% and a standard deviation of 5%. Stock B has an expected return of 15% and a standard deviation of 20%. The correlation between the two is shares is 0.25. You can invest risk free at a 5% interest rate. What is the standard deviation for a portfolio with weights 25% in A, 25% in B and 50% in the risk free asset?

7.6 *Q* [2]
Equity in the company Q has an expected return of 12%, a beta of 1.4 and a standard deviation of 20%. The current risk free interest is 10%. What is the current expected market return?

7.7 *Line* [4]
You can invest in two assets. One is a risk free asset yielding an interest rate of r_f. The other is an asset with expected return $E[r_1]$ and standard deviation σ_1. Show that combinations of these two assets map as a straight line in a mean-standard deviation plot.

7.8 *1&2* [2]
A portfolio is made up of 125% of stock 1 and -25% of stock 2. Stock 1 has a standard deviation of 0.3, and stock 2 has a standard deviation of 0.05. The correlation between the stocks is -0.50. Calculate both the variance and the standard deviation of the portfolio.

7.9 *A, B & C* [4]

You are given the following information about three stocks

Stock	Expected Return	Standard Deviation
A	0.06	0
B	0.1	0.1
C	0.2	0.375

The correlation between B and C is 0.2

1. Suppose you desire to invest in any *one* of the stocks listed above (singly). Can any be recommended?

2. Now suppose you diversify into two securities. Given all choices, can any portfolio be eliminated? Assume equal weights.

7.10 *EotW* [5]

Eyes of the World Corporation has traditionally employed a firm–wide discount rate for capital budgeting purposes. However, its two divisions, publishing and entertainment, have different degrees of risk given by $\beta_P = 1$ and $\beta_E = 1.5$, where β_P is the beta for publishing and β_E the beta for entertainment. The beta for the overall firm is 1.3. The risk free rate is 5% and the expected return on the market is 15%. The firm is considering the following capital expenditures:

Publishing			Entertainment		
Proposed Project	Initial Investment	IRR	Proposed Project	Initial Investment	IRR
P1	1M	0.13	E1	4M	0.16
P2	3M	0.121	E2	6M	0.17
P3	2M	0.09	E3	5M	0.14

1. Which projects would it accept if it uses the opportunity cost of capital for the entire company?

2. Which projects would it accept if it estimates the cost of capital separately for each division?

7.11 *Misui* [4]

Misui, Inc is a levered firm with a debt–to–equity ratio of 0.25. The beta of common stock is 1.15, while that of debt is 0.3. The market premium (expected return in excess of the risk free rate) is 10% and the risk free rate is 6%. The CAPM holds.

1. If a new project has the same risk as the common stock of the firm, what discount rate should the firm use?

2. If a new project has the same risk as the entire firm (debt and equity), what discount rate should the firm use? (Hint: Betas are "additive." In the present context, this means the following. The value of the firm (V) equals the sum of the value of equity (E) and debt (B). That is, the firm is really a portfolio of equity and debt. As a result, the beta of the firm equals the weighted average of the betas of equity and debt, where the weights are given by the ratios of E over V and B over V, respectively.)

Solutions

7.1 *Portfolio* [3]

Diversification possibilities improve when covariance is lower.

7.2 *CAPM* [2]

$$E[r] = r_f + (E[r_m] - r_f)\beta = 0.05 + (0.14 - 0.05)0.5 = 9.5\%$$

7.3 *Beta* [2]

$$r = r_f + (E[r_m] - r_f)\beta$$

$$\beta = \frac{r - r_f}{(E[r_m] - r_f)} = \frac{0.15 - 0.075}{0.1 - 0.075} = 3$$

7.4 *Project* [3]

First find the required rate of return for the project

$$r = 0.05 + (0.1 - 0.05)1.5 = 12.5\%$$

The maximal cost equals the present value of the future cash flow.

$$PV = \frac{100}{1 + 0.125} + \frac{200}{(1.125)^3} + \frac{500}{(1.125)^4} + \frac{100}{(1.125)^6} = 590.83$$

Thus, the maximal cost is 590.83.

7.5 *Portfolio* [2]

Let r_A be the return on stock A and r_B the return on stock B. Since the return on the risk free asset is a constant, its variance equals zero, and its covariance with other assets is also zero, and we can calculate the portfolio variance as

$$\text{var} = 0.25^2\text{var}(r_A) + 2 \cdot 0.25 \cdot 0.25 \cdot \text{cov}(r_A, r_B) + 0.25^2\text{var}(r_B)$$

$$\text{var}(r_A) = 0.05^2 = 0.0025$$

$$\text{var}(r_B) = 0.2^2 = 0.04$$

$$\text{cov}(r_A, r_B) = 0.25 \cdot 0.05 \cdot 0.2 = 0.0025$$

$$\text{var}(r_p) = 0.0029688$$

$$\sigma(r_p) = \sqrt{\text{var}(r_p)} = \sqrt{0.0029688} = 0.545$$

7.6 *Q* [2]

$$E[r] = r_f + (E[r_m] - r_f)\beta$$

$$E[r_m] = \frac{E[r] - r_f}{\beta} + r_f = \frac{0.12 - 0.1}{1.4} + 0.1 = 11.4\%$$

7.7 *Line* [4]

Let w be the weight of the risky asset in the portfolio.

$$E[r_p] = (1 - w)r_f + wE[r_1] = r_f + w(E[r_1] - r_f)$$

$$\sigma^2(r_p) = (1 - w)\sigma^2(r_f) + 2(1 - w)w\text{cov}(r_f, r_1) + w^2\sigma^2(r_1) = (1 - w) \cdot 0 + 2(1 - w)w \cdot 0 + w^2\sigma^2(r_1) = w^2\sigma^2(r_1)$$

$$\sigma(r_p) = w\sigma(r_1)$$

Both mean and standard deviation are linear functions of w, and will map as a line in the mean-standard deviation plot.

7.8 *1&2* [2]

$$X = 1.25, 1 - x = -0.25$$

$$\sigma_p^2 = x^2\sigma_1^2 + 2x(1-x)\rho\sigma_1\sigma_2 + (1-x)^2\sigma_2^2 = 0.15$$

$$\sigma_p = 0.39$$

7.9 *A, B & C* [4]

1. No, there is a well-defined risk(standard deviation) – return trade off.

2. Consider three equally weighted portfolios

   (a)

   $$r_p = \frac{1}{2}r_A + \frac{1}{2}r_B$$

   $$E[r_p] = 0.08, \quad \sigma(r_p) = 0.05$$

   (b)

   $$r_p = \frac{1}{2}r_A + \frac{1}{2}r_C$$

   $$E[r_p] = 0.13, \quad \sigma(r_p) = 0.19$$

   (c)

   $$r_p = \frac{1}{2}r_B + \frac{1}{2}r_C$$

   $$E[r_p] = 0.15, \quad \sigma(r_p) = 0.20$$

   No dominating choice, (c) *almost* dominates (b), but not quite so.

7.10 *EotW* [5]

cost of capital

(a) Firmwide

$$E[R] = 0.05 + \beta_F(E[R_m] - 0.05) = 0.05 + 1.3 \cdot 0.10 = 0.18$$

(b) P:

$$E[R] = 0.05 + \beta_P(E[R_m] - 0.05) = 0.05 + 1.0 \cdot 0.10 = 0.15$$

(c) E:

$$E[R] = 0.05 + \beta_E(E[R_m] - 0.05) = 0.05 + 1.5 \cdot 0.10 = 0.20$$

Using (a): Accept none
Using (b) for division P: accept none
Using (c) for division E: accept none

7.11 *Misui* [4]

1. $\beta = 1.15$, so discount rate $= 0.06 + 1.15 \cdot 0.10 = 0.175$.

2. β of firm $= \frac{4}{5}1.15 + \frac{1}{5}0.3 = 0.98$, so discount rate $= 0.06 + 0.98 \cdot 0.10 = 0.158$.

Chapter 8

Valuing Risky Cash Flows

Problems

8.1 *States* [4]

Two possible states can occur next period, A or B. You observe the following securities:

Security	Price	Payoff in state A	B
1	5	10	10
2	4	10	6

1. Determine the prices of digital securities that pays off in the two states.

2. Determine the current risk free interest rate.

3. What is the current price of a security that pays $ 20 in state A and $ 25 in state B?

8.2 *Probability* [6]

Let I^s be the current price of a digital option that pays 1 if state s occurs. p^s is the time 1 value of investing I^s at time 0, $p^s = I^s(1 + r)$, where r is the one period risk free interest rate. Show that the sum over all states s, $\sum_s p_s$, sum to one.

8.3 *Digital Options* [3]

There are three possible states next period. The risk free interest rate is 5%, and there are two digital options traded, with prices 0.43 and 0.33. What is the price of a digital option for the third state?

8.4 *Price* [2]

An asset has two possible values next period, $X^u = 50$ and $X^d = 500$. If you are told that the state price probability in the u state is 0.4 and the risk free interest rate is 10%, what is the value of the asset?

Solutions

8.1 *States* [4]

Note: The solution is shown by excerpts from the calculation as done in a matlab-like environment.

If we let A be the matrix of payoffs,

```
A =
   10   10
   10    6
```

and P be the current *prices* of the securities,

```
P =
   5
   4
```

1. Find the state price vector q as $A^{-1}P$:

```
q=inv(A)*P
  0.25000
  0.25000
```

The state price vector is thus $(0.25, 0.25)$.

2. The price P_s of a security that pays 1 in every state is the sum of the state prices:

```
> ps = [1 1]*q
ps = 0.50000
```

The current risk free interest rate r is then

$$P_s = \frac{1}{1+r}$$

$$r = \frac{1}{P_s} - 1 = \frac{1}{0.5} - 1 = 2 - 1 = 1 = 100\%$$

3. To find the price of a security that pays $C = (20, 25)$, just multiply q with the payoffs

```
> C
C =
   20   25
> q
q =
   0.25000
   0.25000
> C*q
ans = 11.250
```

8.2 *Probability* [6]

Know that

$$\sum_s I^s = \frac{1}{1+r}$$

Multiplying with $(1+r)$ on both sides:

$$(1+r)\sum_s I^s = (1+r)\frac{1}{1+r}$$

or

$$\sum_s I^s(1+r) = \frac{1+r}{1+r} = 1$$

Replacing $I^s(1+r)$ with p^s:

$$\sum_s p^s = 1$$

8.3 *Digital Options* [3]

Let P^* be the unknown price. Since $\frac{1}{1+r}$ equals the sum of the digital options for all states, solve for P^* as

$$\frac{1}{1+0.05} = 0.43 + 0.33 + P^*$$

$$P^* = \frac{1}{1+0.05} - 0.43 - 0.33 \approx 0.1924$$

8.4 *Price* [2]

$$\text{value} = \frac{1}{1+r}\left(p^u X^u + p^d X^d\right) = \frac{1}{1+0.1}\left(0.4 \cdot 50 + (1-0.4) \cdot 500\right) = \frac{320}{1.1} = 290.91$$

Chapter 9

Introduction to derivatives.

Problems

9.1 *XYZ Option* [2]

The current price of an American call option with exercise price 50, written on ZXY stock is 4. The current price of one ZXY stock is 56. How would you make a lot of money?

9.2 *Put Lower Bound* [5]

Show that the following is an lower bound on a put price

$$P_t \geq \frac{K}{(1+r)^{(T-t)}} - S_t$$

where P_t is the current put price, K is the exercise price, r is the risk free interest rate, $(T - t)$ is the time to maturity and S_t is the current price of the underlying security.

9.3 *Options* [4]

A put is worth \$10 and matures in one year. A call on the same stock is worth \$15 and matures in one year also. Both options are European. The put and call have the same exercise price of \$40. The stock price is \$50. The current price of a (risk free) discount bond (zero coupon bond) paying \$1 that matures in one year is \$0.90. How do you make risk free profits given these prices?

9.4 *Put Upper bound* [5]

Show that the following is an upper bound for the price of a put option

$$P_t \leq \frac{K}{(1+r)^{(T-t)}}$$

where P_t is the current put price, K is the exercise price, r is the risk free interest rate and $(T - t)$ is the time to maturity of the option.

9.5 *American Put* [4]

An American put option with exercise price 50 has a time to maturity of one year. The price of the underlying security has fallen to 10 cents. The risk free interest rate is 5%.

Show that it is optimal to exercise this option early.

9.6 *Convexity* [8]

Consider three European options written on the same underlying security. The options mature on the same date. The options have different exercise prices X_1, X_2 and X_3. Assume $X_1 < X_2 < X_3$. All other features of the options are identical.

Let ω be a number between 0 and 1 satisfying

$$X_2 = \omega X_1 + (1 - \omega)X_3$$

Show that the following inequality must hold to avoid arbitrage (free lunches):

$$C(X_2) \leq \omega C(X_1) + (1 - \omega)C(X_3),$$

where $C(X)$ is the price for an option with exercise price X.

9.7 *Options* [4]

Suppose a share of stock is trading at 30, a put with a strike of 28 is trading at 1 and a call with strike 29 at 8. The maturity of both options is 1 period. The risk free rate is 20%. Is there an arbitrage opportunity (free lunch)?

9.8 *M$ Option* [4]

American call options written on Microsoft's common stock are trading for $8. They carry a strike price of $100, and expire 6 months from today. Microsoft does not pay dividends. At present, its stock price is $104. Hence, the calls are $4 in the money. The annualized six-month risk free rate is 10%. Find an arbitrage opportunity (free lunch) in these numbers and explain how you would exploit it.

9.9 *BoA* [4]

In-the-money American call options written on BoA's common stock carry a strike price of of $55 and expire in 6 months. The annualized six-month risk free rate is 10%. BoA's common stock will go "ex" dividend tomorrow. BoA will pay a $2 dividend. Ms. Johnson holds an option and wonders whether to exercise it. She is worried, because she knows that BoA's stock price will drop tomorrow with $2, making it less likely that her call option will expire in the money. Explain to Ms. Johnson why she should not exercise.

Solutions

9.1 *XYZ Option* [2]

Buy the option for 4, exercise immediately paying 50 to get the stock, sell the stock for 56. Net proceeds $-4 - 50 + 56 = 2$. Repeat indefinitely.

9.2 *Put Lower Bound* [5]

To show this, consider the following strategies.

		Payments	
		Time t	Time T
A	Short stock	S	$-S_T$
	Invest K	$-K\frac{1}{(1+r)^{(T-t)}}$	K
	Total	$S - Ke^{r(T-t)}$	$K - S_T$
B	Buy put	$-p$	$\max(0, K - S_T)$

Strategy B has higher future payoff

S_T:	$S_T < K$	$S_T \geq K$
A:	$K - S_T (> 0)$	$K - S_T (\leq 0)$
B:	$K - S_T$	0

Therefore the cost of entering into B should be higher, to avoid arbitrage.

$$-p \leq S - K\frac{1}{(1+r)^{(T-t)}}$$

$$p \geq K\frac{1}{(1+r)^{(T-t)}} - S$$
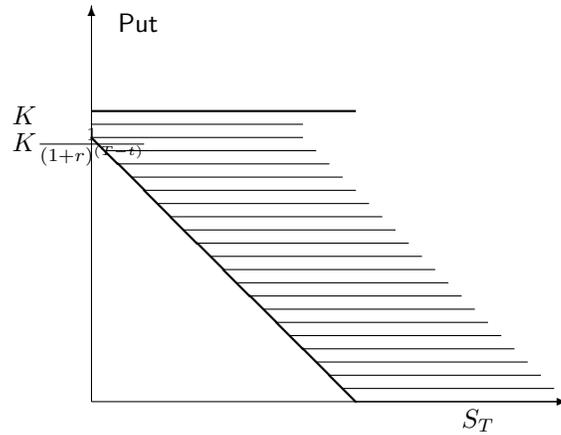
9.3 *Options* [4]

Put call parity is violated.

$$C - P = 15 - 10 = 5$$

$$S - PV(X) = 50 - PV(40) \geq 10$$

No *need* to calculate PV(40), since it must be less than or equal 40, but can do it using

$$r = \frac{1}{0.9} - 1 = 11.11\%$$

$$S - PV(X) = 50 - \frac{40}{1 + 0.1111} = 14$$

To make money, buy sheep and sell deer, i.e. go long the relatively cheap strategy of $C - P$ and short the relatively expensive strategy $S - PV(X)$.

9.4 *Put Upper bound* [5]

The most you can get at exercise is K (if stock price is zero).
The value today is less than the present value of this, or

$$p \leq K \frac{1}{(1 + r)^{(T-t)}}$$

Put

$$K\frac{1}{(1+r)^{(T-t)}}$$
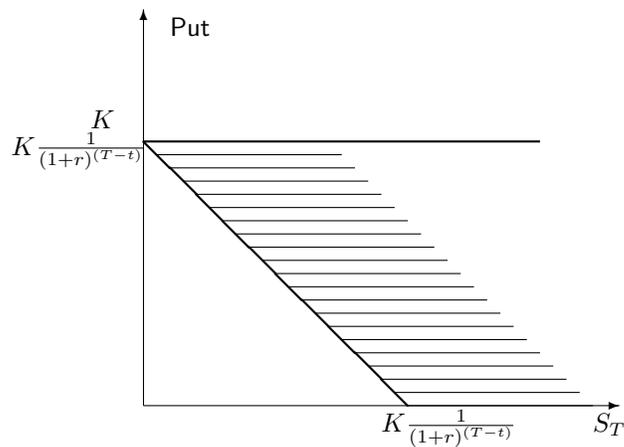
9.5 American Put [4]

The payoff from exercising now is 49.90. If you keep the option, the most you can get one year from now is 50, which you get if the price of the underlying falls to zero. If instead you exercise the option and invest the proceeds at the risk free rate, you get $49.90 \cdot 1.1 = 54.89$ one year from now. It clearly pays to exercise immediately.

9.6 Convexity [8]

This is true.

If not, immediate arbitrage:

Suppose

$$C(X_2) > \omega C(X_1) + (1 - \omega)C(X_3)$$

Consider the following transactions.

Time:	t		T		
State:		$S_T < X_1$	$X_1 \leq S_T < X_2$	$X_2 \leq S_T < X_3$	$X_3 \leq S_T$
Sell	$C(X_2)$	0	0	$-(S_T - X_2)$	$-(S_T - X_2)$
Buy	$-\omega C(X_1)$	0	$\omega(S_T - X_1)$	$\omega(S_T - X_1)$	$\omega(S_T - X_1)$
	$-(1-\omega)C(X_3)$	0	0	0	$(1-\omega)(S_T - X_3)$

By entering into this position, by the assumption that

$$C(X_2) > \omega C(X_1) + (1 - \omega)C(X_3)$$

have a positive cashflow at time 0:

$$C(X_2) - (\omega C(X_1) + (1 - \omega)C(X_3))) > 0$$

If we can show that the total payoff in all of the above states is positive or zero, we are done. Consider the various outcomes for S_T:

- Case $S_T < X_1$: Total payoff is zero.

- Case $X_1 < S_T < X_2$: Total payoff is positive, $\omega(S_T - X_1) > 0$

- Case $X_2 < S_T < X_3$: Total payoff is positive. This takes a bit of algebra to show.
  Total payoff:

  $$-(S_T - X_2) + \omega(S_T - X_1) = X_2 - S_T + \omega(S_T - X_1)$$

  First, using the definition of ω, substitute for ω in terms of the X'es:

  $$X_2 = \omega X_1 + (1 - \omega)X_3$$

  $$\omega = \frac{X_3 - X_2}{X_3 - X_1}$$

  Substitute this above and simplify

  $$-(S_T - X_2) + \omega(S_T - X_1) = X_2 - S_T + \left(\frac{X_3 - X_2}{X_3 - X_1}\right)(S_T - X_1)$$

  After some algebra, get this to equal

  $$\frac{(X_1 - X_2)(S_T - X_3)}{X_3 - X_1} > 0$$

- Case $X_3 < S_T$: Total payoff is zero:

  $$-(S_T - X_2) + \omega(S_T - X_1) + (1 - \omega)(S_T - X_3)$$

  $$= X_2 - S_T + (\omega + (1 - \omega))S_T + \omega X_1 + (1 - \omega)X_3$$

  $$= X_2 - S_T + S_T + \omega X_1 + (1 - \omega)X_3$$

  $$= X_2 + (\omega X_1 + (1 - \omega)X_3) = 0$$

The position gives positive cash flow at time zero, positive probability of positive cash flows at time T and zero probability of negative cashflows at time T. Arbitrage opportunity.

9.7 *Options* [4]

Note: Put and call have different strike prices, cannot blindly apply put-call parity.

Let $C(x)$ be the value of the call with exercise price x. $P(x)$ is the value of a put with exercise price x. S is the stock price, and R the risk free rate.

Then, if $X_1 < X_2$,

$$C(X_1) < C(X_2) = P(X_2) + S - \frac{X_2}{1 + R} = 1 + 30 - \frac{28}{1.2} = 7.7$$

But $C(X_1) = 8$!

9.8 *M\$ Option* [4]

Under absence of arbitrage opportunities,

$$C \geq S - K\frac{1}{(1 + r)^{T-t}}$$

$$8 \geq 104 - 100\frac{1}{1.1^{0.5}} = 8.65$$

No arbitrage is violated. Exploit by buying a call and investing in K bonds with face value \$1, sell short stock.

9.9 *BoA* [4]

Don't exercise if dividend D does not compensate for interest cost of strike price.

$$D < K \left(1 - \frac{1}{(1+r)^{T-t}} \right)$$

$$2 < 55 \left(1 - \frac{1}{(1+0.1)^{0.5}} \right) = 2.56.$$

OK.

Chapter 10

Pricing Derivatives

Problems

10.1 *MLK* [4]

The current price of security MLK is 78. Next period the security will either be worth 120 or 90. The risk free interest rate is 33.33%. There are two digital options traded. One pays $1 if MLK is at 120 next period. This option is trading at 0.35. The other pays $1 if MLK is at 90 next period. This option is trading at 0.40.

1. Price a put option on MLK with exercise price $K = 90$.

2. Price a put option on MLK with exercise price $K = 100$.

3. Price a call option on MLK with exercise price $K = 100$.

4. Price a call option on MLK with exercise price $K = 120$.

Solutions

**10.1** *MLK* [4]

Value of MLK
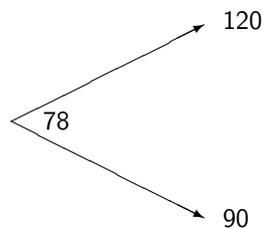
Digital option prices
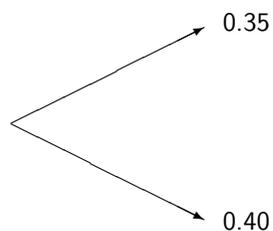
1. The call option with exercise price $K = 90$ has terminal payoffs:
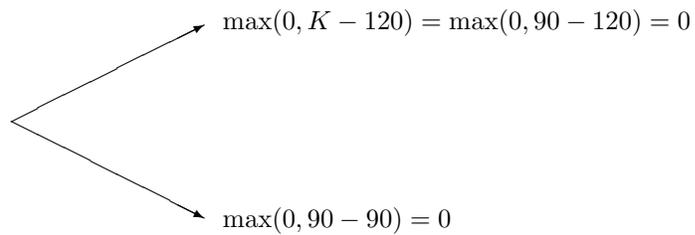
$$\max(0, K - 120) = \max(0, 90 - 120) = 0$$

$$\max(0, 90 - 90) = 0$$

Note that this option never has any future positive payoffs, its value today is zero.

2. The put option with exercise price $K = 100$ has terminal payoffs:

$$\max(0, 100 - 120) = 0$$

$$\max(0, 100 - 90) = 10$$
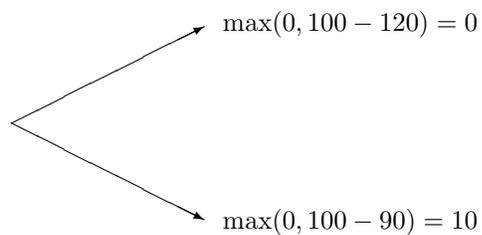
It value is calculated by noting that this option is equal to 10 units of the digital options paying off in the lowest state. This option has a price of 0.4.

$$\text{value} = 10 \cdot 0.4 = 4$$

3. The call option with exercise price $K = 100$ has terminal payoffs:

$$\max(0, 120 - 100) = 20$$

$$\max(0, 90 - 100) = 0$$
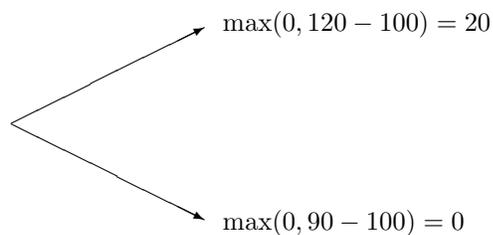
and value

$$\text{value} = 20 \cdot 0.35 = 7$$

4. The final call option is never in the money, and is therefore valued at zero today.

Chapter 11

Pricing of Multiperiod, Risky Investments

Problems

11.1 *MLK* [5]

The current price of security MLK is 89.50. Two periods from now the security will either be worth 120, 100 or 90. The one period risk free interest rate is 8.45%. There are three digital options traded. One pays $1 if MLK is at 120 two periods from now. This option is trading at 0.35. The second pays $1 if MLK is at 100 two periods from now. This option is trading at 0.25. The third pays $1 if MLK is at 90 two periods from now. This option is trading at 0.25.

1. Price a (two period) put option on MLK with exercise price $K = 90$.

2. Price a (two period) put option on MLK with exercise price $K = 100$.

3. Price a (two period) call option on MLK with exercise price $K = 100$.

4. Price a (two period) call option on MLK with exercise price $K = 120$.

Solutions

11.1 *MLK* [5]

Value of MLK
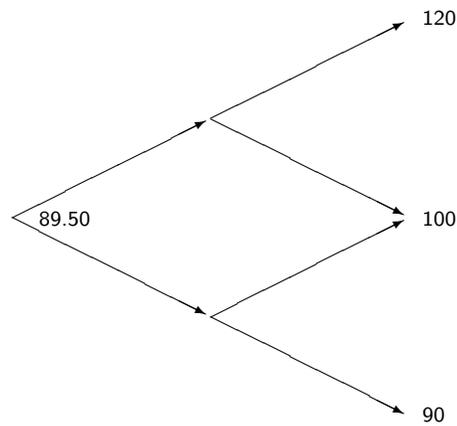
Digital option prices
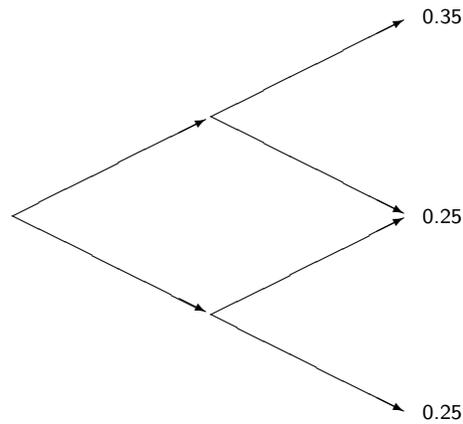
0.35

0.25

0.25

1. The put option with exercise price $K = 90$ has terminal payoffs:
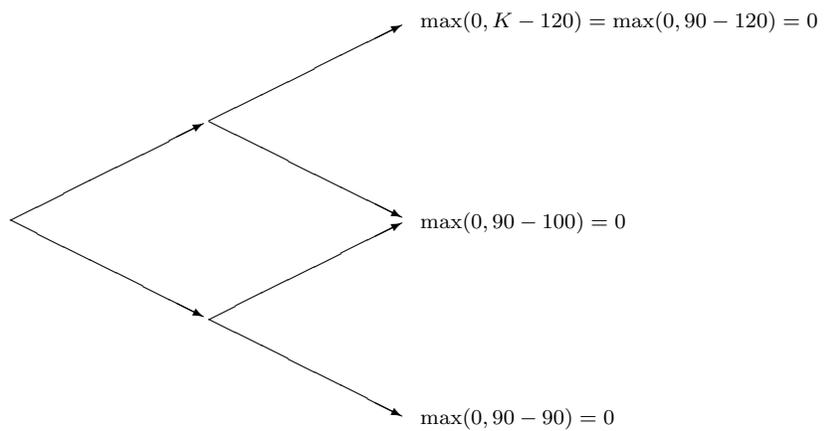
$\max(0, K - 120) = \max(0, 90 - 120) = 0$

$\max(0, 90 - 100) = 0$

$\max(0, 90 - 90) = 0$

Note that this option never has any future positive payoffs, its value today is zero.

2. The put option with exercise price $K = 100$ has terminal payoffs:

$\max(0, 100 - 120) = 0$

$\max(0, 100 - 100) = 0$

$\max(0, 100 - 90) = 10$
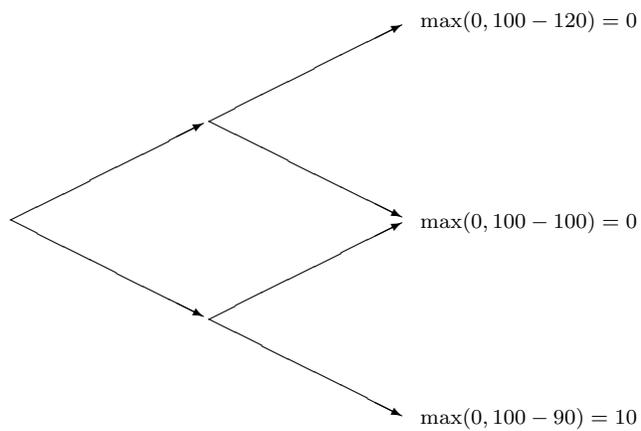
It value is calculated by noting that this option is equal to 10 units of the digital options paying off in the lowest state. This option has a price of 0.25.

$$\text{value} = 10 \cdot 0.25 = 2.50$$

3. The call option with exercise price $K = 100$ has terminal payoffs:

$$\text{max}(0, 120 - 100) = 20$$

$$\text{max}(0, 100 - 100) = 0$$

$$\text{max}(0, 90 - 100) = 0$$
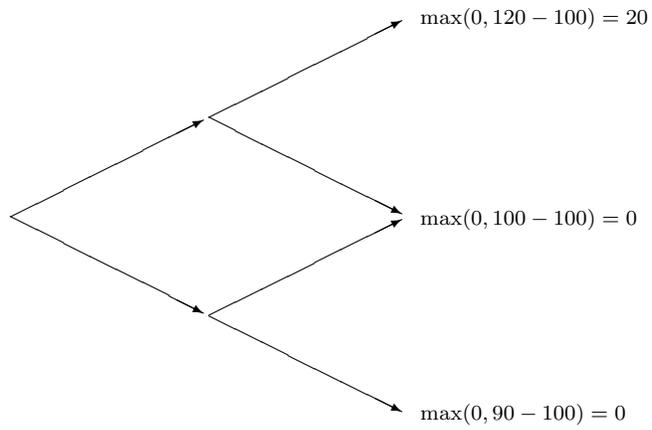
and value

$$\text{value} = 20 \cdot 0.35 = 7.00$$

4. The final call option is never in the money, and is therefore valued at zero today.

Chapter 12

Where To Get State Price Probabilities?

Problems

12.1 *MOP* [4]

The risk free interest rate is 4%. The current price of a stock in company MOP is 50. Next period the price of MOP stock will either be 40 or 90.

- Determine the state price probabilities for the two states.

- Determine the current price for a digital option that pays one dollar when the MOP price is 40 next period.

Solutions

12.1 *MOP* [4]

Given

$$r = 4\%$$

$$
\begin{array}{c}
S^u = 90 \\
S_0 = 50 \\
S^u = 40
\end{array}
$$
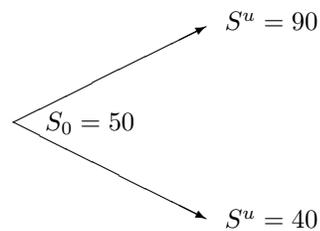
$$S_0 = 50 = \frac{1}{1+r}\left(p^u 90 + (1-p^u)50\right)$$

$$50(1 + 0.04) = p^u(90 - 40) + 40$$

$$p^u = \frac{50 \cdot 1.04 - 40}{50} = 0.24$$

Chapter 13

Warrants

Problems

13.1 *Option/Warrant* [2]

Consider a warrant and a call option, both written on IBM stock,

(a) Which of these securities has been issued by IBM?

Consider two scenarios.
(1) You own a warrant on IBM with maturity 6 months and exercise price 100.
(2) You own a call option on IBM with maturity 6 months and exercise price 100.

(b) Will you be indifferent between these two alternatives?

(c) If not, which one would you prefer?

13.2 *Warrants* [6]

A firm has issued 500 shares of stock, 100 warrants and a straight bond. The warrants are about to expire and all of them will be exercised. Each warrant entitles the holder to 5 shares at $25 per share. The market value of the firm's assets is 25,000. The market value of the straight bond is 8,000. That of equity is 15,000.

1. Determine the post–exercise value of a share of equity.

2. What is the mispricing of equity?

3. From the proceeds of immediate exercise, value the warrant.

4. Now assume that the market value of debt becomes $9,000, to reflect the increase in the value of the firm upon warrant exercise (which lowers the probability of bankruptcy). Re–compute the value of the warrants and of equity.

Solutions

13.1 *Option/Warrant* [2]

Warrants are call-options issued by corporations. They entitle buyers to purchase *newly created* stock of the firm at a given strike price at a certain date.

1. The warrant is issued by IBM, the call is a pure side-bet on the stock price.

2. The call. When a warrant is exercised, new stock is created, which decreases the value of each share. It is therefore less valuable to exercise a warrant than a (straight) option.

13.2 *Warrants* [6]

1. The new value of the firm V', equals the pre-exercise value of the firm, V, plus the proceeds from exercise:

$$V' = V + 500 \cdot 25 = 25,000 + 12,500 = 37,500$$

Using value additivity, the new value of equity, E', can be obtained from V' and the value of the bonds, B:

$$E' = V' - B = 37,500 - 8,000 = 29,500$$

So, the new per-share value, s', is

$$s' = \frac{E'}{500 + 100 \cdot 5} = \frac{29,500}{1,000} = 29.5$$

2. The old share value, s, is $15{,}000/500 = 30$. That is 0.50 too much, knowing that the shares will be worth only 29.5 after exercise of the warrants.

3. The warrants generate $5 \times \$29.5$ per piece, i.e., 147.5. For that, the older pays $5 \times \$25$, i.e., $\$125$. So the warrants value is $(147.5 - 125) = 22$.

4. Since B changes to $B' = 9000$, the new value of equity is

$$E' = V' - B' = 37,500 - 9,000 = 28,500$$

Hence, the per share value is $s' = 28.5$. The new value of the warrants is

$$5 \times \$28.5 - 5 \times \$25 = 142.5 - 125 = \$17.5$$

Chapter 14

The Dynamic Hedge Argument

Problems

14.1 *ud* [1]

The current price of the underlying is 50. This price will next period move to either 48 or 60. In a binomial setup, find the constants u and d.

14.2 *Call Option* [3]

The current price of the underlying is 50. This price will next period move to uS or dS, where $u = 1.1$ or $d = 0.95$. If the risk free interest rate is 5%, what is the price of a call option with exercise price 50?

14.3 *Call Option* [4]

A stock's current price is $160. There are two possible prices that may occur next period: $150 or $175. The interest rate on risk-free investments is 6% per period.

1. Assume that a (European) call option exists on this stock having on exercise price of $155.

    (a) How could you form a portfolio based on the stock and the call so as to achieve a risk-free hedge ?

    (b) Compute the price of the call.

2. Answer the above two questions if the exercise price was $180.

14.4 *Calls, Hedge* [6]

A stock's current price is $100. There are two possible prices at the end of the year: $150 or $ 75. A call option to buy one share at $100 at the end of the year sells for $20. Suppose that you are told that

1. writing 3 calls,

2. buying 2 stocks, and

3. borrowing $140

is a perfect hedge portfolio, i.e. a risk free portfolio. What is the risk free rate of interest?

14.5 *Call Option* [5]

You bought a call contract three weeks ago. The expiry date of the calls is five weeks from today. On that date, the price of the underlying stock will be either 120 or 95. The two states are equally likely to occur. Currently, the stock sells for 96. The exercise price of the call is 112. Each call gives you the right to buy 100 shares at the exercise price. You are able to borrow money at 10% per annum. What is the value of your call contract?

14.6 *A* [4]

The price of stocks in the "A" company is currently 40. At the end of one month it will be either 42 or 38. The risk free interest rate is 8% per annum. What is the value of a one–month European call option with a strike price of $39?

14.7 *Arbitrage* [8]

Consider the the binomial option pricing model, where the constants u and d are used to generate future states $S^u = uS$ and $S^d = dS$, and where r is the risk free interest rate. Show that if $d < u < (1+r)$ an arbitrage opportunity (free lunch) exists.

14.8 *MC* [4]

MC stock is selling for 30 per share. It is expected that the stock price will be either 25 or 35 in 6 months. Treasury bills that mature in 6 months yield 5%. (p.a.). Use a state–price probability (for the "up" state) of 0.5741.

1. What is the price of an MC option with strike 32?

2. Is the stock priced correctly?

Solutions

14.1 *ud* [1]
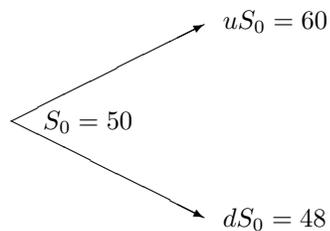
$$S^u = 60 = uS_0 = u50$$

$$u = \frac{60}{50} = 1.2$$

$$d = \frac{48}{50} = 0.96$$

14.2 *Call Option* [3]

First we find the possible evolution of the price of the underlying

$$S^u = uS_0 = 1.1 \cdot 50 = 55$$

$$S^d = dS_0 = 0.95 \cdot 50 = 47.50$$

and
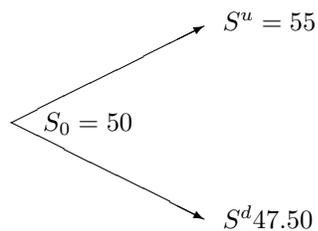
$$p^u = \frac{1.05 - 0.9}{1.1 - 0.9} = 0.75$$

$$p^d = 1 - p^u = 1 - 0.75 = 0.25$$

First find the call payoff at the terminal date:

$$C^u = \max(0, 55.5 - 50) = 5$$

$$C^d = \max(0, 47.5 - 50) = 0$$

$$C^u = 5$$

$$C^d = 0$$
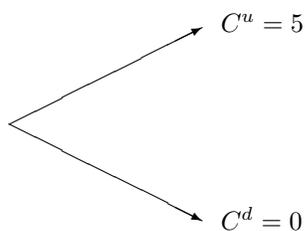
and then find the call value the first date

$$C_0 = \frac{1}{1+r}\left(p^u C^u + p^d C^d\right) = \frac{1}{1.05}\left(0.75 \cdot 5 + 0.25 \cdot 0\right) = 3.33$$
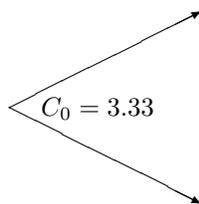
$$C_0 = 3.33$$

Note: The interest rate was originally given as 10% in the first edition of the book. Can you see why this information would give you problems in solving this question?

14.3 *Call Option* [4]

We have the following payoff next period:

$$S^u = 175$$

$$S_0 = 160$$

$$S^d = 150$$

1. Call option with exercise price 155.

$$C^u = \max(0, 175 - 155) = 20$$

$$C_0 = ?$$

$$C^d = \max(0, 150 - 155) = 0$$
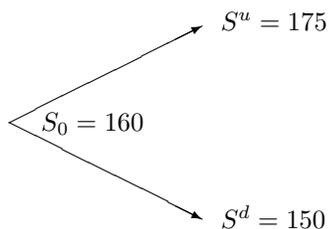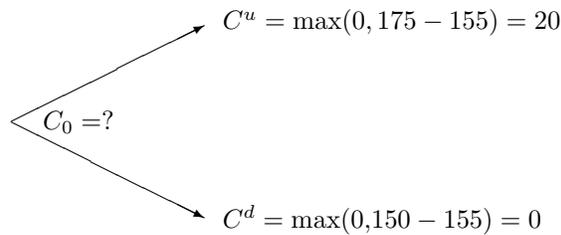
(a) How to get a risk free hedge by buying m call options? Need payoff in each period to be equal

$$S^d + mC^d = S^u + mC^u$$

$$m = \frac{S^d - S^u}{C^u - C^d} = \frac{150 - 175}{20 - 0} = -1.25$$

Need to sell 1.25 options to create a risk free hedge.

(b) To price the option, we find the parameters of the binomial option pricing model.
Start by finding u and d:

$$S^u = uS_0 = u160 = 175$$

$$S^d = dS_0 = d160 = 150$$

Solve for u and d:

$$u = \frac{175}{160} = 1.09375$$

$$d = \frac{150}{160} = 0.9375$$

Then find the state price probabilities:

$$p^u = \frac{(1+r) - d}{u - d} = \frac{1.06 - 0.9375}{1.09375 - 0.9375} \approx 0.784$$

Find option payoffs

$$C^u = \max(S^u - X, 0) = \max(175 - 155, 0) = 20$$

$$C^d = \max(S^d - X, 0) = \max(150 - 155, 0) = 0$$

Roll back:

$$C_0 = \frac{1}{1+r}(p^u C^u + (1 - p^u)C^d) = \frac{1}{1 + 0.06}(0.784 \cdot 20) = 14.79$$

(c) With an exercise price of 180, the call will never be exercised. It is worthless.

14.4 *Calls, Hedge* [6]

A perfect hedge implies that total payoffs are equal in each state.

Cashflows are

Time:	now	Maturity	
State:		$S_T = 75$	$S_T = 150$
Sell 3 calls	60	0	-150
Buy stock	-200	150	300
Borrow	140	$-140(1+r)$	$-140(1+r)$
	0	$150 - 140(1+r)$	

Something that costs nothing to establish now, should have a zero payoff in the future. (By no arbitrage). Hence,

$$150 - 140(1 + r) = 0$$

$$140(1 + r) = 150$$

$$1 + r = \frac{150}{140}$$

$$r = \left(\frac{150}{140}\right) = 17.1\%$$

14.5 *Call Option* [5]

Let us find the price of a call on one share.

$$S_0 = 96 \quad\nearrow\quad S^u = 120$$
$$\searrow\quad S^d = 95$$
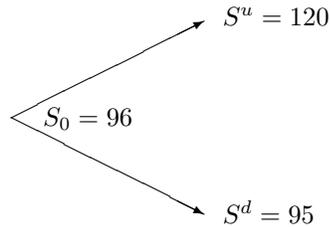
Find u and d:

$$u = \frac{120}{96} = 1.25$$

$$d = \frac{95}{96} = 0.989583333333$$

Then find p^u. Need to find the risk free rate for a 5 week period, approximate as

$$r \approx \frac{5 \cdot 0.06}{52} = 0.005769$$

$$p^u = \frac{(1 + r) - d}{u - d} = \frac{1.005769 - 0.989}{1.25 - 0.989} = 0.077$$

Then find terminal payoffs

$$C^u = \max(120 - 112, 0) = 8$$

$$C^d = \max(95 - 112, 0) = 0$$

and calculate option price

$$C_0 = \frac{1}{1 + r}(p^u C^u + (1 - p^u)C^d) = \frac{1}{1 + r}(0.077 \cdot 8 + 0) = 0.6109$$

The call on 100 shares is then worth

$$100 \cdot 0.6109 = 61.09$$

14.6 *A* [4]
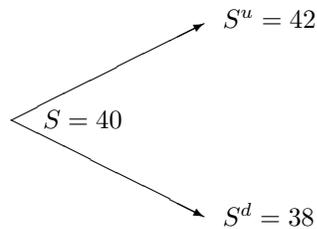
$$r = 8\%$$

$$X = 39$$

$$T - t = 1 \text{ month}$$

$$u = \frac{S^u}{S} = \frac{42}{40} = 1.05$$

$$d = \frac{S^d}{S} = \frac{38}{40} = 0.95$$

$$p^u = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.08 \cdot \frac{1}{12}} - 0.95}{1.05 - 0.95} = 0.567$$

Note that this uses continuous discouting. Alternatively approximate the one month interest rate as $r/12$.

$$1 - p^u = 0.433$$

$$C_0 = e^{-r\Delta t}(qC^u + (1 - q)C^d)$$

$$C^u = 3$$

$$C^d = 0$$

$$C_0 = e^{-0.08\frac{1}{12}}(0.567 \cdot 3 + 0.433 \cdot 0) = 1.69$$

14.7 *Arbitrage* [8]

Consider two alternative investments. One is to buy one unit of the risky asset, receiving either u or d times the initial investment. The other is to invest the same amount in the risk free asset, receiving $(1 + r)$ times the initial investment. Since $(1 + r) > u$, no matter what, the risk free payoff is higher than the risky payoff. The arbitrage opportunity is to short the risky asset and use the proceeds from the short to buy the risk free asset.

14.8 *MC* [4]

Using the fundamental valuation formula, and the fact that

$$p^u = 0.5741$$

$$P = \frac{1}{\sqrt{1 + R}} E^* \left[\max(32 - S, 0)\right]$$

where S is the stock price next period, find

$$P = \frac{1}{\sqrt{1.05}}(0.5741 \cdot 0 + (1 - 0.5741)(32 - 25)) = 2.91$$

To check that the stock is priced correctly:

$$S_0 = \frac{1}{(1+r)^{\frac{1}{2}}}E^*[S] = \frac{1}{\sqrt{1.05}}(0.5741 \cdot 35 + (1 - 0.5741)25) = 30.00$$

The stock is correctly priced.

Chapter 15

Multiple Periods in the Binomial Option Pricing Model

Problems

15.1 *ud* [2]

The current price of the underlying is 100. This price will two periods from now move to either 121, 99 or 81. Find the constants u and d by which prices move each period.

15.2 *Call* [3]

The current price of the underlying is 50. This price will each period move to uS or dS, where $u = 1.1$ or $d = 0.95$. If the per period risk free interest rate is 5%, what is the price of a two period call option with exercise price 50?

15.3 *HAL* [6]

You are interested in the computer company HAL computers. Its stock is currently priced at 9000. The stock price is expected to either go up by 25% or down by 20% each six months. The annual risk free interest rate is 20%.

Your broker now calls you with an interesting offer.

You pay C_0 now for the following opportunity: In month 6 you can choose whether or not to buy a call option on HAL computers with 6 months maturity (i.e. expiry is 12 months from now). This option has an exercise price of $9000, and costs $1,500. (You have an option on an option.)

1. If C_0 is the *fair* price for this "compound option," find C_0.

2. If you do not have any choice after 6 months, you *have to* buy the option, what is then the value of the contract?

15.4 *HS* [4]

The current price of a stock in the "Hello Sailor" entertainment company (HS) is $100. Each period the stock price either moves up by a factor $u = 1.5$ or down by a factor $d = 1/u$. (HS is in a highly variable industry.) The per period interest rate is 5%.

Consider the following "compound option" written on HS: You buy the option at time 0. The compound option gives you the right to at time 1 choose to receive one of the following two options:

- A call option on HS with exercise price 100 and expiring in period 2.

- A put option on HS with exercise price 100 and expiring in period 2.

Calculate the price of this "compound option."

Solutions

15.1 *ud* [2]

$$S_0 = 100$$

$$uuS_0 = S^{uu}$$

$$uu100 = 121$$

$$u = \sqrt{\frac{121}{100}} = 1.1$$

$$d = \sqrt{\frac{81}{100}} = 0.9$$
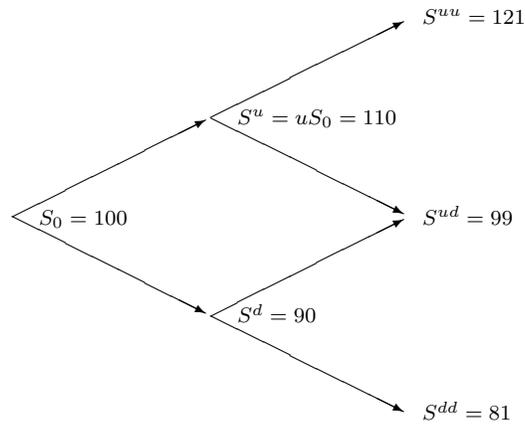
15.2 *Call* [3]

First we find the possible evolution of the price of the underlying

$$S^u = uS_0 = 1.1 \cdot 50 = 55$$

$$S^d = dS_0 = 0.95 \cdot 50 = 47.50$$

and
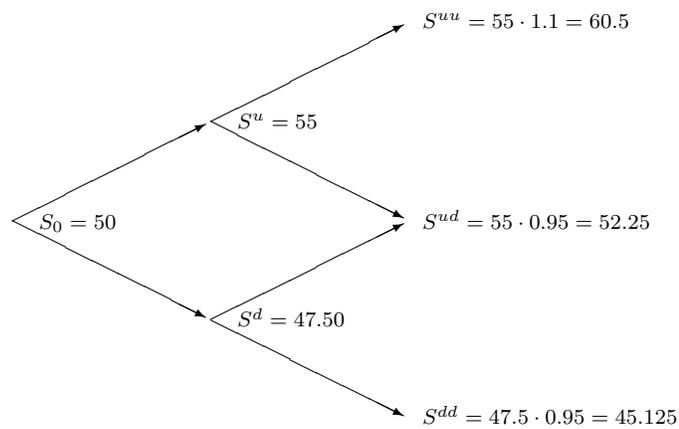
$$p^u = \frac{1.05 - 0.9}{1.1 - 0.9} = 0.75$$

$$p^d = 1 - p^u = 1 - 0.75 = 0.25$$

First find the call payoff at the terminal date:

$$C^{uu} = \max(0, 60.5 - 50) = 10.5$$

$$C^{ud} = \max(0, 52.2 - 50) = 2.25$$

$$C^{dd} = \max(0, 45.125 - 50) = 0$$

$$C^{uu} = 10.5$$

$$C^{ud} = 2.25$$

$$C^{dd} = 0$$
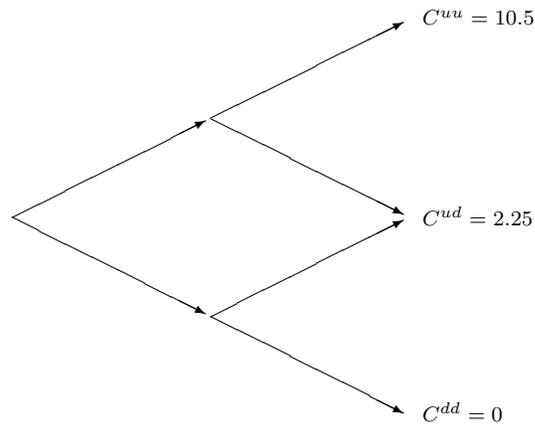
And then "roll back" by first finding the two possible call values in the first period

$$C^u = \frac{1}{1+r}\left(p^u C^{uu} + p^d C^{ud}\right) = \frac{1}{1.05}(0.75 \cdot 10.5 + 0.25 \cdot 2.25) = 8.04$$

$$C^d = \frac{1}{1+r}\left(p^u C^{du} + p^d C^{dd}\right) = \frac{1}{1.05}(0.75 \cdot 2.25 + 0.25 \cdot 0) = 1.61$$

$$C^u = 8.04$$

$$C^d = 1.61$$
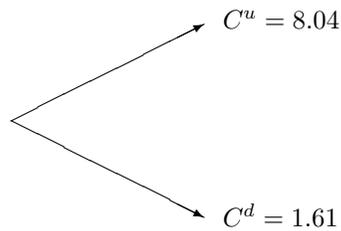
before finding the call value the first date

$$C_0 = \frac{1}{1+r}\left(p^u C^u + p^d C^d\right) = \frac{1}{1.05}(0.75 \cdot 8.04 + 0.25 \cdot 1.61) = 6.13$$
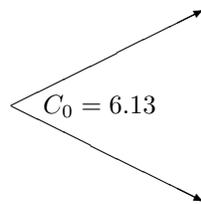
$$C_0 = 6.13$$

Note: The interest rate was originally given as 10% in the first edition of the book. Can you see why this information would give you problems in solving this question?

15.3 *HAL* [6]

Stock price movement, $u = 1.25$, $d = 0.80$.
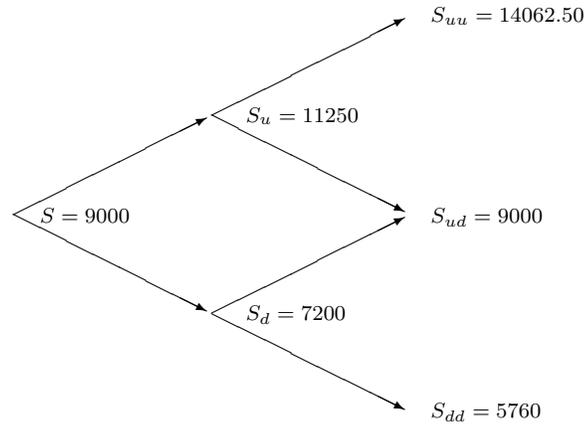
Start by calculating the value of an option at time 1, with exercise price 9000.

$$p^u = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.2 \cdot \frac{6}{12}} - 0.8}{1.25 - 0.8} \approx 0.678$$

$$1 - p^u = 0.322$$

Note that this uses continuous discouting. Alternatively approximate the discrete interest rate.

$$
\begin{aligned}
C_u &= e^{-r\Delta t}\left(p^u C_{uu} + (1 - p^u) C_{ud}\right) \\
&= e^{-0.2 \cdot 0.5}(0.678 \cdot 5062.50 + 0.322 \cdot 0) \\
&= 3,105.74
\end{aligned}
$$

$$C_d = 0$$

You are now offered to pay C_0 for the ability to choose to pay \$1,500 in month 6 for this option.

This can be analyzed on a tree. Clearly, you will not pay 1,500 in the down state, but will pay it in the up-state.
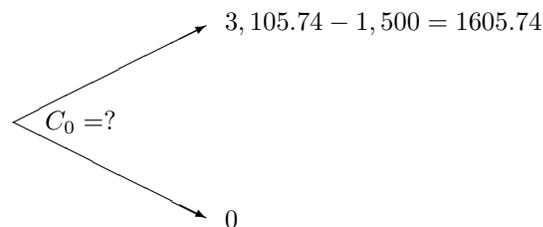

Calculate the value the usual way

$$
\begin{aligned}
C_0 &= e^{-r\Delta t}(p^u \cdot 1605.74 + (1 - p^u) \cdot 0) \\
&= e^{-0.2 \cdot 0.5}(0.678 \cdot 1605.74 + 0) \\
&= 985.08
\end{aligned}
$$

This is the current value of the "option on an option."

For the second part of the question, if you do not have any choice, this is the picture

$$3,105.74 - 1,500 = 1605.74$$

$$C_0 =?$$
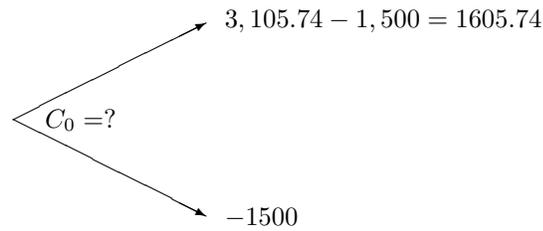
$$-1500$$

$$
\begin{aligned}
C_0 &= e^{-r\Delta t}(p^u \cdot 1605.74 + (1 - p^u) \cdot (-1500)) \\
&= e^{-0.2 \cdot 0.5}(0.678 \cdot 1605.74 + 0.322 \cdot (-1500)) \\
&= 548.05
\end{aligned}
$$

15.4 *HS* [4]

Consider the possible evolution of the underlying stock.

225

150

100   100

66.66

44.44
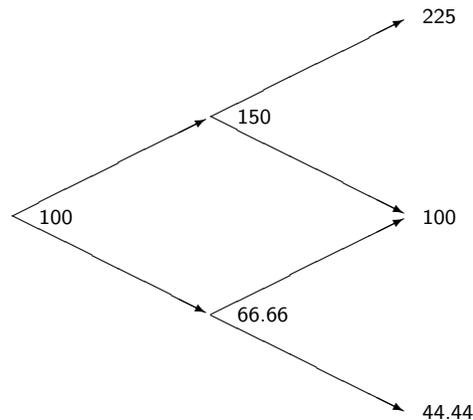
Clearly one would at time 1 want to hold a call option when in the up state and a put option in the down state. The terminal payments for the "compound option" is thus

125

?

?   0

?

55.55
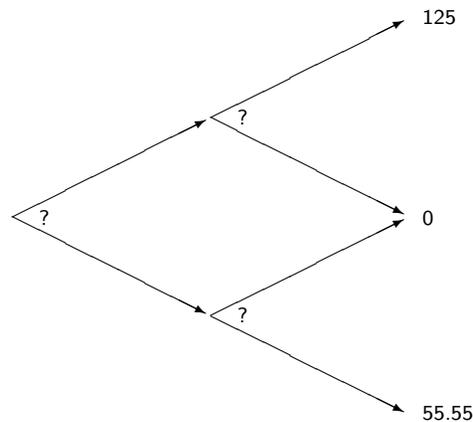
To find the current value of this, just use the usual backward induction.

$$p^u = \frac{e^r - d}{u - d} = \frac{e^{0.05} - 0.66667}{1.5 - 0.66667} \approx 0.46$$

Note that this uses continuous discouting. Alternatively approximate the discrete interest rate.

$$1 - p^u = 0.54$$

Let P be the value of the "compound option"

$$P_u = e^{-0.05}(0.46 \cdot 125 + 0.54 \cdot 0) \approx 54.70$$

$$P_d = e^{-0.05}(0.46 \cdot 0 + 0.54 \cdot 55.55) \approx 28.53$$

$$P_0 = e^{-0.05}(0.46 \cdot 54.70 + 0.54 \cdot 28.53) \approx 38.59$$
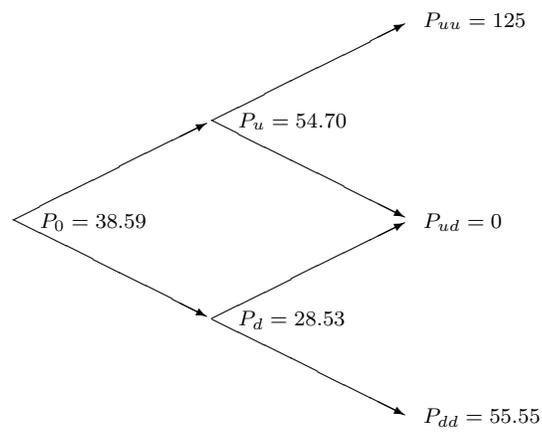
Chapter 16

An Application: Pricing Corporate Bonds

Problems

16.1 *Conversion* [1]
Why does conversion of convertible bonds not affect the value of the firm?

16.2 *Bond Covenants* [3]
In one or two sentences, answer the following.

1. Who benefits from the covenants in bond contracts when the firm is in financial trouble? Why?

2. Who benefits from the covenants in bond contracts when the firm is issuing debt? Why?

16.3 [3]
The Q corporation will next period realize a project that will have value either 100 or 20. This project is the only assets that Q corporation have. Q has issued a bond with face value of 50, due next period. The risk free interest rate is 10% and the current value of equity in Q is 40. Determine the current value of the bond.

16.4 *Projects* [7]
A start-up company considers two investment projects that require a $90 investment. Both are zero NPV projects (hence, the value of the project is $90). Only one project can be implemented. Part of the necessary funds are to be acquired through a zero-coupon debt issue, with face value of $85. The remainder is collected through an equity issue. The specifics of the two projects are

- Project A:

$$\text{Final value } = \begin{cases} 110 & \text{with probability } 0.8 \\ 80 & \text{with probability } 0.2 \end{cases}$$

- Project B:

$$\text{Final value } = \begin{cases} 150 & \text{with probability } 0.5 \\ 40 & \text{with probability } 0.5 \end{cases}$$

1. Which project has the highest expected return?

2. Assume management randomly picks a project, and decides to choose Project A. (Management should not have any particular preference, because both projects have a zero NPV.) How much money will the company raise from the debt issue? How much equity will be raised? Assume that the risk free interest rate equals 5%.

3. Assume now that the debt and equity issues to finance project A have been completed. Now the management turns around and does not implement project A, but, instead, goes ahead with project B. What is the loss in value to the debtholders? Would the equityholders applaud this move?

4. If the bondholders expect management to pick projects in the interest of shareholders, under which terms will bondholders finance project A?

16.5 *Convertible* [6]

Suppose the firms end of period value will be:

$$\text{Value} = \begin{cases} 1500 & \text{with probability } 0.6 \\ 800 & \text{with probability } 0.4 \end{cases}$$

Today's firm value is 1000. The risk free rate is 5%. The firm has 10 shares of equity and 100 convertible bonds with a face value of 10 each. The bond pays no coupon. One bond can be converted into one share.

Compute the value of the convertible bond and of the equity.

16.6 *Yazee* [4]

Yazee is valued at $V = 100$. Tomorrow's value V', will be either 150 or 50, with equal chance, Yazee has issued a corporate bond with face value 100 and no coupon, to be paid tomorrow. The bonds presently give a 15% yield. Is the bond mispriced? The risk free rate is 10%.

16.7 *AoB* [4]

AOB, Inc., has issued 2 shares of common stock and 1 convertible bond. AOB is valued at $V = 100$. Tomorrows value, V', will be either 150 or 50, with equal chance. The bond has a face value of \$30 and carries a coupon of 10%. The conversion ratio is 1:1. The bond is due tomorrow. The risk free rate is 10%. Value the bond.

Solutions

16.1 *Conversion* [1]

There are no cash flow consequences for the firm, it only changes the distribution of claims between the owners.

16.2 *Bond Covenants* [3]

1. The covenants protect *bondholders* from managers acting in behalf of shareholders and undertaking inefficient investment proficiencies during the time of financial distress.

2. These benefit the *stockholders* by allowing them to borrow from the bondholders at a reduced interest rate.

16.3 [3]

Can use the equity to determine implied probabilities.

Value of equity in the various states.
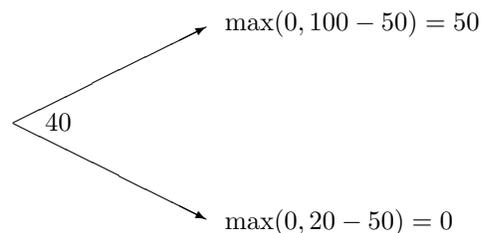
$$40 = \frac{1}{1+r} \left(p^u 50 + (1 - p^u)0 \right)$$

$$40 = \frac{1}{1.1} p^u 50$$

$$p^u = \frac{40}{50} \cdot 1.1 = 0.88$$

Then this p^u is used to determine the bond value

Bond payoffs:

$$\min(50, 100) = 50$$

B_0

$$\min(50, 20) = 20$$
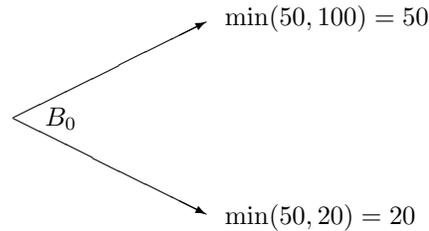
$$B_0 = \frac{1}{1 + 0.1} \left(p^u 50 + (1 - p^u)20\right) = 42.18$$

16.4 *Projects* [7]

1. The expected returns are

$$E[r_A] = 0.8\frac{110}{90} + 0.2\frac{80}{90} - 1 = 16\%$$

$$E[r_B] = 0.5\frac{150}{90} + 0.5\frac{40}{90} - 1 = 6\%$$

Project A has the higher expected return.

2. Apparently, the market charges P ($= V$, the value of the firm) $= \$90$ for a payoff pattern with $X^u = \$110$ and $X^d = \$80$. With an interest rate of 5%, this means that the state-price probability p^u solves:

$$90 = \frac{1}{1 + 0.05} \left(p^u 110 + (1 - p^u)80\right).$$

Hence $p^u = 0.483$. The value of debt, and hence the money raised from the debt issue, is

$$D = \frac{1}{1 + 0.05} \left(p^u 85 + (1 - p^u)80\right) = \frac{1}{1 + 0.05} \left(0.483 \cdot 85 + 0.517 \cdot 80\right) = 78$$

The value of equity (and the amount raised from the equity issue) is:

$$E = V - B = 90 - 78 = 12$$

3. Because the company changes projects, the state price probabilities also change.

Now p^u solves:

$$90 = \frac{1}{1 + 0.05} \left(p^u 150 + (1 - p^u)40\right).$$

Hence, $p^u = 0.496$. The value of the debt decreases to

$$D = \frac{1}{1 + 0.05} \left(p^u 85 + (1 - p^u)40\right) = \frac{1}{1 + 0.05} \left(0.496 \cdot 85 + 0.504 \cdot 40\right) = 59$$

Equity, in contrast, surges to

$$E = V - B = 90 - 59 = 31$$

Shareholders cheer, bondholders lament.

4. Given the answer to the last question, bondholders will not believe that project A will be carried out as proposed, and only accept terms for project B.

16.5 *Convertible* [6]

1. Let V' denote the end-of-period firm value. We have two cases. When $V' = \$1,500$, a share of equity is worth \$50 and an unconverted bond \$10. Since the conversion price is \$10, bondholders obviously decide to convert. The new capital structure will be: 110 shares of equity valued at \$1,500/100 = \$13.64 each. In contrast, when $V' = \$800$, the firm bankrupts. Hence, the shares are worth \$0, and the bonds \$8 (=800/100). Bondholders obviously don't convert.

So, the convertible bond is a security that pays \$13.64 in the "up state" u, and \$8 in the "down state" d. We can get the state-price probability p^u from our standard pricing formula and from the fact that the value of the firm equals \$1000:

$$1000 = \frac{1}{1+r} E^*[V] = \frac{1}{1.05} (p^u 1500 + (1 - p^u) 800).$$

Hence, $p^u = 0.357$.

Applying our formula to value the convertible bonds (100 of them), we obtain:

$$B = 100 \frac{1}{1+r} (p^u 13.64 + (1 - p^u) 8) = 100 \frac{1}{1.05} (0.357 \cdot 13.64 + (1 - 0.357) 8) = \$954$$

We can do the same for equity (10 shares)

$$E = 10 \frac{1}{1+r} (p^u 13.64 + (1 - p^u) 0) = 100 \frac{1}{1.05} (0.357 \cdot 13.64 + 0.643 \cdot 0) = \$46$$

D and E add up to the value of the firm, confirming value additivity:

$$B + E = 954 + 46 = 1,000$$

16.6 *Yazee* [4]

Price of Bond $B = \frac{100}{1.15} = 86.96$.

State price probabilities solves

$$100 = \frac{1}{1.1} (p^u 150 + (1 - p^u) 50)$$

$$p^u = 0.6$$

Hence B should be

$$B = \frac{1}{1.1} (p^u 100 + (1 - p^u) 150) = \frac{1}{1.1} (0.6 \cdot 100 + 0.4 \cdot 150) = 72.77$$

The bond is mispriced.

16.7 *AoB* [4]

Value of bond

| | Action | |
State	Convert	Don't convert
$V' = 150$	$\frac{150}{3} = 50$	33
$V' = 50$	$\frac{50}{3} = 16.7$	33

The optimal action is to convert when $V' = 150$ and not when $V' = 50$.

The bond is worth

$$B = \text{ risk free bond } + \text{option} = \frac{1}{1.1}33 + \frac{1}{1.1}(p^u(50 - 33) + (1 - p^u)0) = 39.28$$

The probability p^u is found from

$$100 = V = \frac{1}{1.1}(p^u 150 + (1 - p^u)50)$$

$$p^u = 0.6$$

Chapter 17

Are capital structure decisions relevant?

Problems

17.1 *Debt/Equity* [7]
Firm Z and Y have identical cash flows. Firm Z is 40% debt financed and 60% equity financed, while firm Y is 100% equity financed. The same required rate of return on their debt equals 10%. (Assume debt is perpetual)

1. Next period's cash flows for each firm are $100. Assume both firms pay out all excess cash in the form of dividends. What cash flows go to the debt and equity holders of both firms? Assume no corporate taxes. (Use D_z for the value of firm Z's debt).

2. You own 10% of firm Z's stock. What cash flow will you get in the future? What combination of other assets will give you the same cash flow?

3. Suppose the value of firm Z is greater than firm Y. How can you become very rich? (You may assume no transactions costs, or other market imperfections)

4. Now, suppose there is a corporate tax rate of 40%. What should the value of each firm be?

17.2 *Frisky* [4]
Frisky, Inc is financed entirely by common stock which is priced according to a 15% expected return. If the company re-purchases 25% of the common stock and substitutes an equal value of debt, yielding 6%, what is the expected return on the common stock after the re-financing?

17.3 *JB* [4]
JB Manufacturing is currently an all-equity firm. The equity of firm is worth $2 million. The cost of that equity is 18%. JB pays no taxes. JB plans to issue $400,000 in debt and use the proceeds to repurchase equity. The cost of debt is 10%.

1. After the repurchase the stock, what will the overall cost of capital be?

2. After the repurchase, what will the cost of equity be?

17.4 *LRC* [3]
You invest $100,000 in the Liana Rope Company. To make the investment, you borrowed $75,000 from a friend at a cost of 10%. You expect your equity investment to return 20%. There are no taxes. What would your return be if you did not use leverage?

17.5 *OFC* [5]
Old Fashion Corp. is an all-equity firm famous for its antique furniture business. If the firm uses 36% leverage through issuance of long–term debt, the CFO predicts that there is a 20% chance that the ROE(Return on Equity) will be 10%, 40% chance that the ROE will be 15%, and 40% chance that the ROE will be 20%. The firm is tax-exempt. Explain whether the firm should change its capital structure if the forecast of the CFO changes to 30%, 50% and 20% chances respective for the three ROE possibilities. That is, tell us whether the value of assets and equity change as a result of the changes in ROEs.

17.6 *V&M* [5]

Note: In the question you are asked to assume risk neutrality. This means that the state price probabilities are not colored by risk aversion (fear) so they are equal to the estimated probabilities in the question.

VanSant Corporation and Matta, Inc., are identical firms except that Matta, Inc., is more levered than VanSant. The companies' economists agree that the probability of a recession next year is 20% and the probability of a continuation of the current expansion is 80%. If the expansion continues, each firm will have EBIT of 2 million. If a recession occurs, each firm will have EBIT of 0.8 million. VanSant's debt obligation required the firm to make 750,000 in payments. Because Matta carries more debt, its debt payment obligations are 1 million. *Note:* EBIT is short for *Earnings Before Interest and Taxes.* Often this is considered a good estimate of cash flows before interest and tax payments.

Assume that the investors in these firms are risk-neutral and that they discount the firms' cash flows at 15%. Assume a one–period example. Also assume there are no taxes.

1. Duane, the president of VanSant, commented to Matta's president, Deb, that his firm has a higher value than Matta, Inc, because VanSant has less debt and, therefore, less bankruptcy risk. Is Duane correct?

2. Using the data of the two firms, prove your answer.

3. What might cause the firms to be valued differently?

17.7 *Negative NPV?* [3]

Do you agree or disagree with the following statement? Explain your answer.

A firm's stockholders would never want the firm to invest in projects with negative NPV.

Solutions

17.1 *Debt/Equity* [7]

1. Cash flows.

	Firm Z	Firm Y
Cash flow	100	100
Debt payment	$0.10 D_Z$	–
Equity payment	$100 - .10 D_Z$	100

2. Future cash flows:

$$0.10 \cdot (100 - 0.10 \cdot D_Z)$$

Alternative assets to replicate these cash flows:

Buy 10% of Y	$0.10 \cdot 100$
Borrow 10% of D_Z at 10% rate	$-0.1 \cdot (0.10 \cdot D_Z)$
Net	$0.1 \cdot (100 - 0.1 \cdot D_Z)$

3. We then recognise an arbitrage opportunity.

	Cashflows	
Time:	Now	Next period
Buy Y	$-V_Y$	100
Short Z	$V_Z - D_Z$	$-(100 - 0.1 D_Z)$
Borrow D_Z	D_Z	$-0.1 D_Z$
Net	$-V_Y + V_Z > 0$	0

4. New firm values after tax of 40%. Let V_Y, V_Z be the no tax values of Y and Z.

   Since Y has no debt, its value goes down by 40%, to $0.60 \cdot V_Y$.

   For firm Z, its value is adjusted for the tax shelter of debt:

   Firm Z goes to $0.6V_Z + 0.4D_Z = 0.6V_Y + 0.4D_Z$.

17.2 *Frisky* [4]

$$r = r^* + (r^* - r_D)\frac{D}{E}$$

$$r = 0.15 + (0.15 - 0.06)\frac{1}{3} = 18\%$$

17.3 *JB* [4]

1. When the firm is an all-equity firm, the cost of equity capital (18%) is the same as the cost of capital for the firm. When the firm does not pay taxes, the cost of capital for the firm is not affected by the debt-equity mix.

   The cost of capital thus remains the same, 18%.

2. To find equity capital, use

$$
\begin{aligned}
r_E &= r^* + (r^* - r_D)\frac{D}{E} \\
&= 0.18 + (0.18 - 0.10)\frac{D}{E} \\
&= 0.18 + (0.18 - 0.10)\frac{400}{2,000 - 400} \\
&= 0.18 + 0.08\frac{400}{1,600} \\
&= 0.18 + 0.02 = 20\%.
\end{aligned}
$$

17.4 *LRC* [3]

The equity returns $5,000(=20% of 25,000); the loan requires $7,500 (=10% of $75,000). Hence, the investment returns in total $12,500, which is 12.5% on $100,000. The would be the return on investment if it were totally financed by equity.

17.5 *OFC* [5]

There are no changes to the cash flows from the assets of the firm if the capital structure is changed. Hence, the value of the firm does not change. The new estimates will change the relative value of debt and equity, but shareholders will be no better off if the firm raises more debt or retires some debt. This is M&M irrelevance.

17.6 *V&M* [5]

V', next year's value of either firm, is $2 (in millions) in the "up" state or $0.8 in the "down" state. the state-price probability for the "up" state is 0.8. Debt has a face value of $D^V = 0.75$ for Van Sant, and $D^M = 1$ for Matta. The risk free rate equals 15%.

1. Duane is wrong. Both firms have the same cash flows from assets, and hence, the same firm value.

2. The value of Van Sant, V^V, is

$$V^V = \frac{1}{1.15} \left(0.8 \cdot 2 + 0.2 \cdot 0.8\right) = 1.53$$

The value of Matta, V^M, is the same

$$V^M = \frac{1}{1.15} \left(0.8 \cdot 2 + 0.2 \cdot 0.8\right) = 1.53$$

3. Suppose there are bankruptcy costs of $A = 0.1$. Since VanSant never defaults (in the down state, it has 0.8 to cover a debt of 0.75), its value is not affected by the presence of A. Matta, on the other hand, changes value, down to

$$V^M = \frac{1}{1.15} \left(0.8 \cdot 2 + 0.2 \cdot (0.8 - 0.1)\right) = 1.51$$

17.7 *Negative NPV?* [3]

If bonds are in place (the firm has collected the money), then shareholders may have the incentive to change projects, towards a more volatile one. This may even be a negative NPV project. Covenants, warrants and conversion rights attached to the bonds will keep shareholders from doing so.

Chapter 18

Maybe capital structure affects firm value after all?

Problems

18.1 *Leverage* [6]

A firm has expected net operating income (X) of $600. Its value as an unlevered firm (V_U) is $2,000. The firm is facing a tax rate of 40%. Suppose the firm changes it ratio of debt to equity ratio to equal 1. The cost of debt capital in this situation is 10%. Use the MM propositions to:

1. Calculate the after–tax cost of equity capital for both the levered and the unlevered firm.

2. Calculate the after–tax weighed average cost of capital for each.

3. Why is the cost of equity capital higher for the levered firm, but the weighted average cost of capital lower?

18.2 *GTC* [5]

Note: In the question you are asked to assume risk neutrality. This means that the state price probabilities are not colored by risk aversion (fear) so they are equal to the estimated probabilities in the question.

Good Time Co. is a regional chain department store. It will remain in business for one more year. The estimated probability of boom year is 60% and that of recession is 40%. It is projected that Good Time will have total cash flows of $250 million in a boom year and $100 million in a recession. Its required debt payment is $150 million per annum. Assume a one-period model.

Assume risk neutrality and an annual discount rate of 12% for both the stock and the bond.

1. What is the total stock value of the firm?

2. If the total value of bond outstanding for Good Time is $108.93 million, what is the expected bankruptcy cost in the case of recession?

3. What is the total value of the firm?

4. What is the promised return on the bond?

18.3 *Bond Issue* [4]

An firm that is currently all–equity is subject to a 30% corporate tax rate. The firm's equityholders require a 20% return. The firm's initial market value is $3,500,000, and it has 175,000 shares outstanding. Suppose the firm issues $1 million of bonds at 10% and uses the proceeds to repurchase common stock.

Assume there is no change in the cost of financial distress for the firm. According to MM, what is the new market value of the equity of the firm?

18.4 *LMN* [3]

LMN is currently all equity financed. The equity of the firm is worth 7 million. LMN is planning to issue bonds with a value of 4 million and a 10% coupon. LMN is paying 30% corporate tax. Individual investors are paying 20% tax on capital gains and dividends, and 25% tax on interest income. In a Miller equilibrium, what is the new value of the firm after the bond issue?

18.5 *Bond* [3]

A company is issuing a 3 year bond with a face value of 25 million and a coupon of 10%. The company is paying taxes with 28%. The company's cost of capital is 15%. What is the value of the bond issue for the company?

18.6 *Tax Shield Value* [5]

The general expression for the value of a leveraged firm in a world in which $\tau_S = 0$ is

$$V_L = V_U + \left(\frac{1 - (1 - \tau_C)}{1 - \tau_B} \right) B - C(B)$$

where V_U is the value of an unlevered firm, τ_C is the effective corporate tax rate for the firm, τ_B is the personal tax rate of the marginal bondholder, B is the debt level of the firm, and $C(B)$ is the present value of the costs of financial distress for the firm as a function of its debt level. (Note: $C(B)$ encompasses all non-tax-related effects of leverage on the firm's value.)

Assume all investors are risk neutral.

1. In their no-tax model, what do Modigliani and Miller assume about τ_C, τ_B and $C(B)$? What do these assumptions imply about a firm's optimal debt–equity ratio?

2. In their model that includes corporate taxes, what do Modigliani and Miller assume about τ_C, τ_B and $C(B)$? What do these assumptions imply about a firm's optimal debt–equity ratio?

3. Assume that IBM is certain to be able to use its interest deductions to reduce its corporate tax bill. What would the change in the value of IBM be if the company issued \$1 billion in debt and used the proceeds to repurchase equity? Assume that the personal tax rate on bond income is 20%, the corporate tax rate is 34%, and the costs of financial distress are zero.

18.7 *Infty.com* [4]

Infty.com will generate forever a before–tax cash flow of \$15. The corporate tax rate is 50%. The risk free rate is 10%.

1. Value the firm if it is all–equity

2. Value the firm if it issues a perpetual bond with coupon \$5.

Solutions

18.1 *Leverage* [6]

1. Let us use the data for the unlevered firm to find r^*. We are given the (before-tax) operating income X. The value of the unlevered firm is

$$V_U = \frac{\text{after-tax income}}{r^*}$$

$$\rightarrow \quad r^* = \frac{\text{after-tax income}}{V_U}$$

$$r^* = \frac{X(1 - \tau)}{V_U} = \frac{600(1 - 0.4)}{2,000} = 18\%$$

For the unlevered firm, this is also the cost of equity capital, $r_E = r^* = 18\%$.

Use this to find the value of the levered firm.

$$\begin{aligned} r_E &= r^* + (r^* - r_D)(1 - \tau)\frac{D}{E} \\ &= 0.18 + (0.18 - 0.10)(1 - 0.4) \cdot 1 \\ &= 22.8\%. \end{aligned}$$

2. Calculate weighted costs of capital of two alternatives:

$$WACC_U = 18\%$$

$$WACC_L = \frac{1}{2}r_E + \frac{1}{2}(1-\tau)r_D = \frac{1}{2}0.228 + \frac{1}{2}(1-0.4)0.10 = 14.4\%$$

3. The equity is riskier, hence the return on equity for the levered firm is higher. The lower $WACC$ reflects the tax savings from the leverage.

18.2 *GTC* [5]

1. V', next year's value of the firm, is \$250 (in millions) in the "up" state or \$100 minus (as yet unknown) bankruptcy costs in the "down" state. The state-price probability for the "up" state is 0.6. Debt has a face value of $D = 150$. The risk free rate equals 12%.

   Using our formula, the value of equity E equals

   $$E = \frac{1}{1+r}E^*\left[\max(V'-D,0)\right] = \frac{1}{1.12}(0.6 \cdot 150 + 0.4 \cdot 0) = 54$$

2. Without bankruptcy costs, the value of debt B^{nocost}, would have been:

   $$B^{nocost} = \frac{1}{1.12}(0.6 \cdot 150 + 0.4 \cdot 100) = 116$$

   Since the bonds are actually selling for $B = 190$ (rounded), the *value of the bankruptcy cost*, BC, equals

   $$BC = B^{nocost} - B = 116 - 109 = 7$$

3. Using value additivity, the value of the firm, V, equals

   $$V = E + B = 54 + 109 = 163$$

4. The promised return, i.e., the yield, y, equals

   $$y = \frac{150}{109} - 1 = 38\%$$

   Note: one can infer the *actual bankruptcy costs upon default*, A, from the value of the firm. By our valuation formula

   $$V = \frac{1}{1+r}E^*[V'] = \frac{1}{1.12}(0.6 \cdot 250 + 0.4 \cdot (100 - A)) = 163$$

   Hence $A = 18.6$.

18.3 *Bond Issue* [4]

Currently, the value of the firm is \$3.5 million, the same as the value of equity. The repurchase of \$1 million decreases the value of equity by \$1 million, to 2,500 million.

But this does not account for the tax shield of the new bond issue. If the bonds are perpetual, the value of the firm increases by 1 million $\times \tau$, where τ is the tax rate. In this case $\tau = 30\%$. Thus the tax shield is 300,000, and the value of equity is 2.5 mill $+$ 300,000 $=$ 2,800,000.

18.4 *LMN* [3]

The value without "leverage" is

$$V_U = 7 \text{ mill}$$

The tax advantage of issuing 10 mill worth of debt.
Given

$$r_D = 10\%$$

$$\tau_C = 30\%$$

$$\tau_{PD} = 25\%$$

$$\tau_{PE} = 20\%$$

With everybody taxable, find the value with leverage by multiplying D with

$$\left(1 - \frac{(1 - \tau_C)(1 - \tau_{PE})}{(1 - \tau_{PD}}\right)$$

$$1 - \frac{(1 - 0.3)(1 - 0.2)}{1 - 0.25} = 0.25$$

The tax advantage of debt is

$$0.25 \cdot D = 1 \text{ mill.}$$

18.5 *Bond* [3]

1.740.800

18.6 *Tax Shield Value* [5]

1. $\tau_C = \tau_B = C(B) = 0$. There is no optimal debt-equity ratio.

2. τ_c is nonzero. $\tau_B = C(B) = 0$. It is optimal to issue 100% debt.

3. Let V_U stand for the original firm value, and V_L the firm value after the increase in debt.

$$V_L - V_U = \frac{1 - (1 - \tau_C)}{1 - \tau_B} B = \frac{1 - (1 - 0.34)}{1 - 0.2} 1 = \$0.425 (billion)$$

18.7 *Infty.com* [4]

$$V_U = \frac{15 \cdot 0.5}{0.1} = 75$$

$$V_L = 75 + 0.5\frac{5}{0.1} = 100$$

Chapter 19

Valuation Of Projects Financed Partly With Debt

Problems

19.1 *Project* [3]

A company is considering a project with the following after-tax cashflows:

t	0	1	2	3
X_t	$-150,000$	$60,000$	$60,000$	$60,000$

If the project is all–equity financed it has a required rate of return of 15%. To finance the project the firm issues a 4 year bond with face value of 100,000 and an interest rate of 5%. Remaining investments are financed by the firm's current operations. The company is facing a tax rate of 30%.

Determine the NPV of the project.

19.2 *Bond Issue* [2]

A company issues a one year, zero coupon bond with face value of 25 million. The bond has an interest rate of 8%. The company is paying tax with 28%. Determine the value of the debt tax shield.

19.3 *NORSK* [7]

NORSK, Inc., is valued at $V = 100$. Tomorrows value, V', will be either 150 ("up" state) or 50 ("down" state), with equal chance. NORSK is presently an all-equity firm, but they are considering issuing a corporate bond with face value \$100 and coupon \$10, because they were told that they can reduce their taxable earnings with the amount of the coupon. The corporate tax rate is 50%. According to their accountants, NORSK will have \$10 in taxable earnings in the "up" state, and \$5 in the "down" state. The risk free rate is 10%.

1. How does the value of the firm change upon the bond issue?

2. Value the bond. Use this to re-value the firm, now using the APV formula, unlike in your previous answer.

3. Why is there a discrepancy between the values of the (levered) firm you obtained in the previous two answers?

Solutions

19.1 *Project* [3]

Without the tax advantage

t	=	0	1	2	3
C_t	=	-150	60	60	60

$$NPV = -150 + \frac{60}{(1+0.15)^1} + \frac{60}{(1+0.15)^2} + \frac{60}{(1+0.15)^3} = -13.0065$$

Annual interest tax shield: $0.3 \cdot 5000 = 1500$

PV of tax shield

t	$=$	0	1	2	3	4
C_t	$=$	0	1500	1500	1500	1500

$$NPV = \frac{1500}{(1+0.05)^1} + \frac{1500}{(1+0.05)^2} + \frac{1500}{(1+0.05)^3} + \frac{1500}{(1+0.05)^4} = 5318.93$$

Project NPV is

$$NPV = -13000 + 5318 = -7682$$

19.2 *Bond Issue* [2]

560,000

19.3 *NORSK* [7]

1. The probability p^u is found from

$$100 = V = \frac{1}{1.1}(p^u 150 + (1 - p^u)50)$$

$$p^u = 0.6$$

The increase in the value of the firm is the PV of the tax shields

$$= \frac{1}{1.1}\left(0.6 \cdot (50\% \times 10) + 0.4 \cdot (50\% \times 5)\right) = 3.64$$

2. Bond

$$B = \frac{1}{1.1}(0.6 \cdot 110 + 0.4 \cdot 50) = 78.18.$$

$$V^{APV} = V + \tau B = 100 + 0.5 \cdot 78.18 = 139.09$$

3.
   - Cash flows are not risk free
   - Debt is not perpetual
   - Cash flows are not perpetual.

Chapter 20

And What About Dividends?

Problems

20.1 *Dividend Inc* [2]

The stock price of Dividend, Inc is 30 November 139. 1 December the stock goes ex dividend, paying a cash dividend of 11. The owners of stock in Dividend, Inc do not pay tax on their dividend income. What is your estimate of the stock price 2 December?

20.2 *Dividends and Taxes* [4]

A given share is sold for $30 just before time t_0. If the firm pays a $3 dividend per share, the price will immediately drop to $27. Suppose you own 100 shares. If the firm decides not to distribute dividends, you would need to sell 10 shares (at$30 a share) since you need to have a $300 cash income (pre tax) Assume that the shares were originally bought for $20 each.

1. If both ordinary personal tax rate and capital gains tax are 28%, what is your after-tax wealth under the two alternative situations?

2. Suppose now the tax rates for capital gains are lower, you pay 40% tax on ordinary income, and 16% tax on capital gains. What is your after-tax wealth under the two alternative situations?

20.3 *Dividends* [4]

The firm is capitalized by 100,000 shares of common stock which trade at the beginning of the period at 10 per share. The expected net income in period one is $X_1 = 200,000$ and the firm has declared a cash dividend of 1 per share, to be paid at the end of the period. The firm's cost of equity is 20%.

Ignore personal taxes.

1. What is the ex-dividend share price? What would have been the end-of-period stock price if the firm skipped the dividend?

2. How many shares of commons stock will the firm have to sell at the ex-dividend price in order to undertake an investment project which requires an investment $I_1 = 200,000$?

3. What is the value of the firm just after the new issue? What would have been the value of the firm if it skipped the dividend and used the retained earnings to finance the investment?

20.4 *Stable Rest* [3]

Stable, Inc has for the last ten years been paying out 5% of the book value of equity as dividend. Rest, Inc has for the last fifteen years been paying 80% of after tax income as dividend. This year both Stable, Inc and Rest, Inc is lowering dividend payments by $10.

If the signaling hypothesis is correct, which of the following statements is *correct*?

1. The stock price of Stable, Inc goes down by less than the dividend amount.

2. The market thinks profitability of Stable, Inc is lessened.

3. The market thinks profitability of Rest, Inc is improved.

20.5 *Dividend Amount* [4]

A company is expecting after tax income of 3 million next year. The company currently has a debt/equity ratio of 80%. The company has the possibility of investing in a project with at total investment of 4 million. If the company wants to keep it's current debt/equity ratio, how much should the company pay as dividend next year?

20.6 *Stock* [2]

The current price of a stock is 50. The cost of capital for the company is 10%, and the company dividend is expected to grow by 5% annually. What is the expected dividend payment next period?

Solutions

20.1 *Dividend Inc* [2]

$$P = 139 - 11 = \underline{128}$$

20.2 *Dividends and Taxes* [4]

The tax you pay are calculated as follows:

Dividends:

$$\text{Tax} = \tau_i(3 \cdot 100) = \tau_i \cdot 300$$

$$\text{Wealth} = 100(27 + 3(1 - \tau_i))$$

$$\text{Wealth} = 2700 + 300(1 - \tau_i))$$

Sell shares:

$$\text{Tax} = \tau_g(10 \cdot (30 - 20)) = \tau_g \cdot 100$$

$$\text{Wealth} = 90 \cdot 30 + 30 \cdot 10 - 10 \cdot 10\tau_g$$

$$\text{Wealth} = 2700 + 300 - 100\tau_g$$

$$\text{Wealth} = 3000 - 100\tau_g$$

To find the exact wealth, plug in the tax rates.

1. $\tau_i = 0.28$, $\tau_g = 0.28$.

$$\text{Wealth} = 2700 + 300(1 - \tau_i)) = 2916$$

$$\text{Wealth} = 3000 - 100\tau_g = 2972$$

2. $\tau_i = 0.40$, $\tau_g = 0.16$.

$$\text{Wealth} = 2700 + 300(1 - \tau_i)) = 2880$$

$$\text{Wealth} = 3000 - 100\tau_g = 2984$$

20.3 *Dividends* [4]

1. Remember

$$P_0 = \frac{E[d_1] + E[P_1]}{1 + r}$$

Here, we have all but $E[P_1]$:

$$10 = \frac{1 + E[P_1]}{1 + 0.20}$$

and

$$E[P_1] = 10(1 + 0.20) - 1 = \$11$$

If the firm skipped the dividend, would have

$$E[P_1] = 10 \cdot (1 + 0.20) = \$12$$

2. Investment of $200,000. The firm pays dividends of $100,000 \cdot 1 = \$100,000$ out of earnings of $200,000. It will have to get $100,000 of new capital to finance the investment. At an ex-dividend price of $11, this means an issue of

$$100,000/11 \approx 9091 \text{ stocks.}$$

3. The new value of the firm

$$(100,000 + 9091) \text{ stocks } \cdot \$11 = 1,200,000$$

Without the dividend, the value of the firm would have been

$$100,000 \text{ stocks } \cdot \$12 = \$1,200,000$$

20.4 *Stable Rest* [3]

The market thinks profitability of Stable, Inc is lessened.

20.5 *Dividend Amount* [4]

0.77 mill

20.6 *Stock* [2]

$$50 = \frac{E[X_1]}{0.1 - 0.05}$$

$$E[X_1] = 2.5$$

Chapter 21

Risk And Incentive Management

Problems

21.1 *Binomial Options* [4]

Consider a case where the stock price S follows a binomial process. Currently, the stock price is $S_0 = 100$. Each period, the stock price either moves down 10% or up 15%. The (one period) risk free interest rate is 2.5%.

Consider a one-period call option with exercise price $X = 100$.

1. If you own one stock, how many call options do you need to buy at time 0 for the cashflows in period 1 to be riskless?

2. If you own one call, how many units of the underlying do you need to buy at time 0 for the cashflows in period 1 to be riskless?

Consider now a two period American Call option, with exercise price $X = 100$.

3. If you own one stock, how many of the two period call options do you need to buy at time 0 for the cashflows in period 1 to be riskless?

21.2 *Option* [4]

The current stock price is 160. Next period the price will be one of 150 or 175. The current risk free interest rate is 6%. You buy 1 stock and issue m call options on the stock with an exercise price of 155. What must be m be for the portfolio to be risk free?

21.3 *Futures Price* [5]

Suppose that storing the physical asset has a cost of c in the period from t to T. Show that the futures price satisfies

$$F = S(1 + r)^{(T-t)} + c$$

21.4 *Hedging* [3]

The stock price can next period be either 100 or 200. The stock price today is 150. You have a put option that expires next period. The exercise price of the put option is 100. The price of a discount bond that matures next period is 0.8. You would like to eliminate the risk of holding the put option. What position do you need to take in the stock?

21.5 *VaR* [4]

You own a portfolio with a value today of 10 million. The standard deviation of the weekly return is 0.01. Assuming that weekly returns are normally distributed, estimate Value at Risk (VaR) on a weekly basis with a confidence level of 5%.

21.6 *VaR* [3]

Your current portfolio of equities has a market value of 100,000. Assume normally distributed returns.

1. Suppose the whole portfolio is invested in one stock. The stock has an annual expected return of 10% and an annual standard deviation of 25%.

   Estimate Value at Risk for your portfolio on a daily horizon and a confidence level of 1%.

2. Suppose instead that the portfolio is invested with equal weights in two stocks, each with the same expected return and standard deviation 25%. If the correlation between the two shares is positive, but less than one, will the VaR of the portfolio be smaller or larger than the previous VaR? What if the correlation is negative?

21.7 *Are You Lucky* [5]

The current value of equity in the Are You Lucky Gold Mine (AYLGM) is 50. One period from now the stock price will be one of 58.2 or 45.8. The risk free interest rate is 5% per period.

1. Calculate the price of a call option on one AYLGM stock with exercise price 50.

2. What is the forward price for delivery of one AYLGM stock one period from now?

3. If you want to create a portfolio of forward contracts and risk free borrowing and lending that has the same payoffs as the call option with exercise price of 50, what is your position in forwards and risk free borrowing and lending?

21.8 *T Bill Future* [3]

Today, a Treasury bill (which carries no coupon) is selling for 91.5% (of par value). A futures contract for the delivery of the Treasury bill tomorrow carries a futures price of 91.5% (of par value). You don't know the actual one–period risk free rate, but you know it is positive. Is there a free lunch?

Solutions

21.1 *Binomial Options* [4]

$S_{uu} = 132.25$

$S_u = 115$

$S_0 = 100$

$S_{ud} = 103.50$

$S_d = 90$

$S_{dd} = 81$
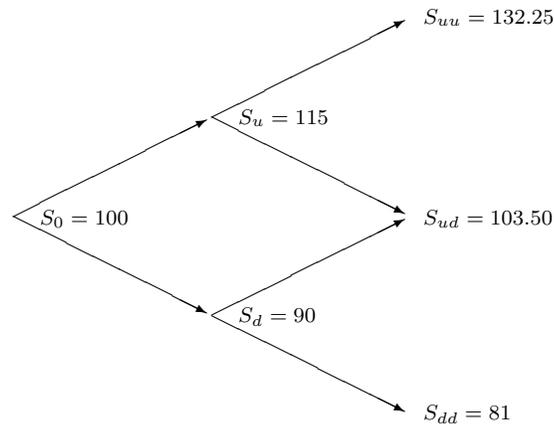
1. Let ω_C be the number of call options necessary to hedge 1 stock. ω_C solves

$$S_u + \omega_C C_u = S_d + \omega_C C_d$$

$$\omega_C = \frac{S_d - S_u}{C_u - C_d} = \frac{90 - 115}{15} = -1.66$$

2. Similarly, let ω_C be the number of stocks options necessary to hedge 1 call option

$$\omega_S S_u + C_u = \omega_S S_d + C_d$$

$$\omega_S = \frac{C_d - C_u}{S_u - S_d} = \frac{0 - 15}{115 - 90} = \frac{-15}{25} = -0.6$$

3. Next want to find the number of stocks ω_S to hedge one call at time 2. Need to know the values of the call option at time 1.

$$p^u = \frac{e^r - d}{u - d} = \frac{e^{0.025} - 0.9}{1.15 - 0.9} = 0.501$$

$$1 - p^u = 0.499$$

Note that this uses continuous discouting. Alternatively approximate the discrete interest rate.

$$C_u = e^{-r}(p^u C_{uu} + (1 - p^u)C_{ud}) = e^{-0.025}(0.501 \cdot 32.25 + 0.499 \cdot 3.50) = 17.46$$

$$C_d = e^{-r}(p^u C_{ud} + (1 - p^u)C_{uu}) = e^{-0.025}(0.501 \cdot 3.50 + 0) = 1.7$$

$$\omega_S = \frac{C_d - C_u}{S_u - S_d} = \frac{1.70 - 17.46}{115 - 90} \approx -0.63$$

$$\omega_C = \frac{S_d - S_u}{C_u - C_d} = \frac{90 - 115}{17.46 - 1.70} \approx -1.59$$

21.2 *Option* [4]

$$S_d - mC_d = S_u - mC_u$$

$$-m = \frac{S_d - S_u}{C_u - C_d} = \frac{150 - 175}{20 - 0} = -1.25$$

$$m = 1.25$$

21.3 *Futures Price* [5]

The case without storage cost c has already been discussed in the book

We considered a forward contract on an underlying asset that provided no income. There are also no restrictions on shortselling of the underlying asset. Then the forward price F has to satisfy

$$F = S(1 + r)^{(T-t)},$$

the forward price is the future value of the current price of the underlying. It is easy to show that violations of this will lead to free lunches. Let us start with the case where

$$F > S(1 + r)^{(T-t)}$$

The tables below illustrates how we would set up a portfolio to exploit this free lunch.

Arbitrage strategy for case $F > S(1 + r)^{(T-t)}$

	Time	
	t	T
Sell forward	0	$F - S_T$
Borrow S_t	S_t	$-S(1 + r)^{(T-t)}$
Buy underlying	$-S_t$	S_T
Total	0	$F - S(1 + r)^{(T-t)} > 0$

On the other hand, if $F < S(1 + r)^{(T-t)}$, it is also easy to exploit the free lunch, as the next table illustrates

Arbitrage strategy for case $F < S(1 + r)^{(T-t)}$

	Time	
	t	T
Buy forward	0	$S_T - F$
Invest S	$-S$	$S(1+r)^{(T-t)}$
Short underlying	S	$-S_T$
Total	0	$S(1+r)^{(T-t)} - F > 0$

To avoid arbitrage we need an exact inequality

$$F = S(1+r)^{(T-t)}$$

Now, the storing cost will enter into the arbitrage relations in the places where one will be buying the underlying security.

Arbitrage strategy for case $F > S(1+r)^{(T-t)} - c$

	Time	
	t	T
Sell forward	0	$F - S_T$
Borrow S_t	S_t	$-S(1+r)^{(T-t)}$
Buy underlying	$-S_t$	$S_T - c$
Total	0	$F - S(1+r)^{(T-t)} - c > 0$

Arbitrage strategy for case $F < S(1+r)^{(T-t)} - c$

	Time	
	t	T
Buy forward	0	$S_T - F$
Invest S	$-S$	$S(1+r)^{(T-t)}$
Short underlying	S	$-(S_T - c)$
Total	0	$S(1+r)^{(T-t)} - F - c > 0$

Note that we are making assumptions about what happens to the cost of storing when you short the underlying asset. What do we assume?

21.4 *Hedging* [3]

The right to sell the stock for 10 is worthless, since the option will never be in the money.

21.5 *VaR* [4]

One piece of information not given here concerns the expected return. A reasonable approximation is to put this at zero. One then uses the 5% confidence level to find the factor $k = 1.64$ from a normal table, and estimate VaR as

$$VaR = 10 \text{ million } \times 1.64 \times 0.01 = 164,000.$$

21.6 *VaR* [3]

1. VaR at the 1% level on a daily horizon. If we work with trading days (most reasonable), calculate

$$SD_{daily} = \frac{1}{\sqrt{250}} 0.25 = 0.01581$$

Another acceptable interpretation of this could be to use actual days, in which case you would calculate

$$SD_{daily} = \frac{1}{\sqrt{365}} 0.25 = 0.013$$

From a normal table, find multiplicative factor 2.33 corresponding to 1% level.

The *return* corresponding to the required VaR is

$$E[r] - k\sigma(r),$$

where k is the multiplicative factor, taken from a normal table, and $E[]$ and σ are with the right frequencies.

If the annual return is 10%, a reasonable approximation is

$$\bar{r}_{daily} = \frac{0.1}{250} = 0.0004$$

Use this with the standard deviations above to get

Daily VaR:

$$E[r] - k\sigma(r)$$

$$0.0004 - 2.33 \cdot 0.01581 = -3.64\%$$

giving a VaR of 3.64 million.

2. As long as the correlation is less than $+1$, the standard deviation (and hence the VaR) of the portfolio will be smaller, and smaller the lower the correlation.

21.7 *Are You Lucky* [5]

Assumptions of the problem about the underlying security:
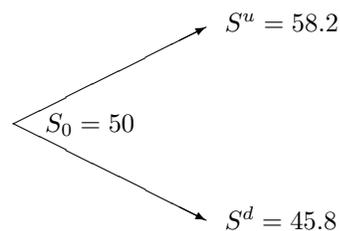
Price of option

$$u = 1.165$$

$$d = 0.916$$

$$p^u = \frac{1.05 - 0.916}{1.165 - 0.916} = 0.54$$

$$C_0 = \frac{1}{1+r}(p^u C_u + (1 - p^u)C_d) = 4.22$$

Forward price

$$F = 50 \cdot 1.05 = 52.4$$

To set up a hedge portfolio involving a forward, solve the following set of equations

$$C_u = w_F(S_u - F) + w_B \cdot 1$$

$$C_d = w_F(S_d - F) + w_B \cdot 1$$

Here w_F is the number of forward contracts, and w_B the number of risk free discount bonds to buy.
Solve for w_F and w_B:

$$w_F = -\frac{C_u - C_d}{S_u - S_d}$$

$$w_B = C_u - w_F(S_u - F)$$

21.8 *T Bill Future* [3]

Yes, use the following strategy

Today:

	NOW	TOMORROW
Long futures	$+0$	$B - 0.915$
Short t bill	$+0.915$	$-B$
Invest at R	$-0.915\left(\frac{1}{1+R}\right)$	0.915
	$0.915\frac{R}{1+R} > 0$	zero no matter what B is